Roll No.: ____________________

CS 228M: Logic in Computer Science Spring 2026

Quiz 3

Time: 8:30 - 9:30 *Max Marks: 30*

Instructions:

- ***Please write your roll number on all pages in the space provided at the top.***
- *Be brief, complete, and stick to what has been asked.*
- ***You must write your answer for every question only in the space allocated for answering the question. Answers written outside the allocated space risk not being graded.***
- ***You can use an extra answer book for rough calculations. You must write your roll number on the extra answer book if you are using one.***
- ***You must submit this question+answer book in its entirey along with any extra answer book for rough calculations (if you used one).***
- *Untidy presentation of answers, and random ramblings will be penalized by negative marks.*
- *Unless asked for explicitly, you may cite results/proofs covered in class without reproducing them.*
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others. All detected cases of copying will be reported to DADAC with names and roll nos. of all involved. The stakes are high if you get reported to DADAC, so you are strongly advised not to risk this.***

DO NOT TURN THIS PAGE UNTIL YOU ARE ASKED TO.

THIS PAGE IS INTENTIONALLY LEFT BLANK AND CAN BE USED FOR ROUGH WORK

Roll No.: ______________________

1. **In search of consistency** [10]

   Consider the vocabulary $\mathcal{V} = \{E, =\}$, where E is a binary predicate. Recall that every $\mathcal{V}$-structure can be interpreted as a directed graph.

   In each sub-question below, you are given a set of first-order logic sentences over $\mathcal{V}$. You are required to determine whether each set is consistent or not.

   - If you think a set is consistent, you must provide a $\mathcal{V}$-structure, i.e. a directed graph, that serves as a model for all sentences in the set. Providing a directed graph that serves as a model for some, but not all, sentences in the set, will fetch you no marks.
   - If you think a set is inconsistent, you must find a smallest subset of sentences that is inconsistent. Providing an inconsistent subset of sentences that is not a smallest inconsistent subset, will fetch you no marks.

   (a) [5]
   $$\{\ \forall x\, \exists y\, \exists z\, \big(\neg(y = z) \wedge E(y, x) \wedge E(x, z)\big),$$
   $$\forall x\, \forall y\, \big(E(x, y) \rightarrow \neg E(y, x)\big),$$
   $$\exists x\, \exists y\, \exists z\, \big(\neg(y = z) \wedge E(x, y) \wedge E(x, z)\ \ \}$$

(b) $\{ \;\; \forall x \forall y \big(E(x,y) \rightarrow \neg E(y,x)\big),$ 5
$\forall x \exists y \big(\neg E(x,y) \wedge \neg E(y,x)\big),$
$\forall x \exists y \exists z \big(E(x,y) \wedge E(y,z) \wedge E(z,x)\big),$
$\forall x \forall y \forall z \big(\neg(x=y) \rightarrow \big((x=z) \vee (y=z)\big)\big) \;\; \}$

2. **Normal forms for FOL** [10]

   Consider the first-order logic sentence φ given below

   $$\exists x \left(P(x,y) \to \left((\exists y\, Q(x,y)) \to (\exists x\, R(x,y)) \right) \right)$$

   Please note the parenthesization carefully.

   (a) Give a Prenex Conjunctive Normal Form (PCNF) formula that is semantically equivalent to φ. Show all steps in the construction of the PCNF formula. [5]

(b) A student wishes to construct a Skolem Normal Form (SNF) formula that is equisatisfiable with φ, and in which all Skolem functions are of arity 0, i.e. all Skolem functions are Skolem constants. If you think this is possible, give such an SNF formula, indicating which Skolem constant corresponds to which existentially quantified variable. Otherwise, explain why the student's wish cannot be fulfilled. [5]

3. **What can we express?** [10]

For this question, assume once again that the vocabulary $\mathcal{V}$ is $\{E, =\}$, where E is a binary predicate. Recall once again that every $\mathcal{V}$-structure can be interpreted as a directed graph.

(a) Give a first-order logic sentence φ_2 that evaluates to true on a directed graph if and only if the graph has exactly two edges in it. Note that for distinct vertices x and y in a directed graph G, an edge from x to y and an edge from y to x count as two different directed edges in G. [5]

(b) You are given a set of first-order logic sentences $\mathcal{F} = \{\varphi_{2k} \mid k \geq 1\}$, where (a) each φ_{2k} is a first-order logic sentence over the vocabulary $\mathcal{V}$, and (b) φ_{2k} evaluates to true over a directed graph G if and only if G has exactly $2k$ edges. 5

Armed with the set $\mathcal{F}$ (you don't need to write down explicitly what each formula φ_{2k} is), show that there does not exist any first order logic sentence φ_e over $\mathcal{V}$ such that φ_e evaluates to true on a graph G if and only if G has an even number of edges.

EMPTY PAGE: MAY BE USED FOR ROUGH WORK

Roll No.: ______________________

EMPTY PAGE: MAY BE USED FOR ROUGH WORK