**CS 228M : Logic for Computer Science** Spring 2026

# Tutorial 4

## Instructions:

- The following problems are meant for you to practice, so that your understanding of the topic improves.
- You must solve all problems to get the maximum benefit from practice problems.
- You must not submit your solutions to these problems. These are not going to be graded.
- A problem may have multiple solution techniques. Discussion among students is strongly encouraged in order to understand different perspectives.

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1. A directed graph $G = (V, E)$ contains a set $V$ of vertices, and a set $E \subseteq V \times V$ of directed edges. Such a graph can be viewed as a $\mathcal{V}$-structure, where the vocabulary $\mathcal{V} = \{R, =\}$, with $R$ being a binary predicate. Specifically, we let the set of vertices $V$ of $G$ be the universe, and interpret the relation $R$ on $V$ as $R(u, v) = \mathsf{true}$ iff $(u, v) \in E$.
   (a) Consider a graph $G = (V, E)$ that is a directed cycle of 5 nodes. Assume there are no self-loops on vertices.
   Which of the following sentences are true of this graph? Give justification.
      - $\forall x \exists y \ \big(\neg R(x, y) \wedge \ \exists z \ (R(x, z) \wedge \neg R(z, y))\ \big)$
      - $\forall x \exists y \ \big(\neg R(x, y) \wedge \ \exists z \ (R(x, z) \wedge R(y, z))\ \big)$
      - $\exists x \forall y \ \big(\neg R(x, y) \wedge \ \exists z \ (R(x, z) \wedge \neg R(z, y))\ \big)$

(b) Write a sentence (formula with no free variables) on vocabulary $\mathcal{V}$ that evaluates to true on only those $\mathcal{V}$-structures, i.e. graphs, with the following properties. Write a separate sentence for each property.

- Star shaped (one hub, with spokes to all other vertices)
- Do not have any cycle of length 4 or less
- Have a subgraph of at least 4 nodes that is a clique.

(c) Is it possible to write a sentence that is satisfied only by graphs with an even number of nodes?