CS 228M : Logic for Computer Science Spring 2026

Tutorial 5

Instructions:

- The following problems are meant for you to practice, so that your understanding of the topic improves.
- You must solve all problems to get the maximum benefit from practice problems.
- You must not submit your solutions to these problems. These are not going to be graded.
- A problem may have multiple solution techniques. Discussion among students is strongly encouraged in order to understand different perspectives.

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1. For each of the following sentences, either give a natural deduction proof of its validity, or show that there is a structure over which the sentence evaluates to false.
   1. $\big((\forall x\ P(x))\ \vee\ (\forall x\ Q(x))\big)\ \rightarrow\ \forall x\ (P(x) \vee Q(x))$
   2. $\forall x\ \big((\exists y\, R(x,y))\ \vee\ \neg R(x,x)\big)$
   3. $\forall x \forall y\ \big(S(x,y)\ \rightarrow\ \big(\exists z\, (S(x,z) \wedge S(z,y))\big)\big)$
   4. $\exists y \big((\forall x\, P(x))\ \rightarrow\ P(y)\big)$
   5. $\forall x \forall y\ \big(S(x,y) \rightarrow (x = y)\big)\ \rightarrow \exists x\, S(x,x)$

2. Consider the vocabulary $\mathcal{V} = \{E, =\}$. We have seen earlier that every $\mathcal{V}$-structure can be viewed as a directed graph. A $\mathcal{V}$-sentence φ therefore defines a family of directed graphs, namely all those graphs, which when viewed as $\mathcal{V}$-structures satisfy φ.

   For each of the following families of graphs, either give a first-order logic sentence that defines the family precisely, or use the Compactness Theorem to show that there cannot exist any first-order logic sentence that defines the family.

   1. $F_1 = \{G \mid \text{G is a graph with no even length cycle}\}$.
   2. $F_2 = \{G \mid \text{G is a graph in which every vertex has an even number of edges going out of it}\}$.
   3. $F_3 = \{G \mid \text{G is a graph in which the longest path length is even}\}$.
   4. $F_4 = \{G \mid \text{every cycle in } G \text{ is of length atmost } 2\}$.
   5. $F_5 = \{G \mid \text{There is at most one vertex that lies on a cycle of length } 2\}$.