

Tutorial 6

Instructions:

- The following problems are meant for you to practice, so that your understanding of the topic improves.
- You must solve all problems to get the maximum benefit from practice problems.
- You must not submit your solutions to these problems. These are not going to be graded.
- A problem may have multiple solution techniques. Discussion among students is strongly encouraged in order to understand different perspectives.

-
1. For each of the first-order logic formulas given below, write an equivalent Prenex Conjunctive Normal Form (PCNF) formula, and then use it to write an equi-satisfiable Skolem Normal Form (SNF) formula. Indicate clearly which Skolem function symbol is used for which variable when constructing the SNF formula. Try to minimize the number of Skolem functions used for each formula.

1. $\exists x (P(x, y) \wedge (\exists y Q(x, y) \rightarrow \exists z R(x, y, z)))$
2. $\exists x (P(f(x), x) \vee \exists x \forall y Q(y, g(x, y)))$
3. $\forall x (P(x, y, z) \rightarrow (\exists y Q(x, y, z) \rightarrow \exists z R(x, y, z)))$
4. $\forall x ((z > x) \rightarrow (\exists y ((y > x) \wedge (z > y))))$
5. $\exists x (\forall y P(x, y) \rightarrow \forall z \exists w (Q(x, z, w) \vee P(z, w)))$

2. Show that in each of the following sub-questions, the given set of formulas is consistent.

1. $\{\forall x \neg S(x, x), \forall x \exists y S(x, y), \forall x \forall y \forall z (S(x, y) \wedge S(y, z) \rightarrow S(x, z))\}$
2. $\{\exists x S(x, x), \forall x \forall y (S(x, y) \rightarrow (x = y))\}$
3. $\{\varphi_i \mid i \geq 1\}$, where $\varphi_i \equiv \exists x_1 \dots \exists x_i \bigwedge_{1 \leq s < t \leq i} \neg(x_s = x_t) \wedge \bigwedge_{s=1}^{i-1} E(x_s, x_{s+1})$
4. $\{\forall x \neg(f(x) = x), \forall x \exists y (x = f(y)), \forall x \forall y (f(x) = f(y) \rightarrow (x = y))\}$
5. $\{\forall x \exists y (P(x) \rightarrow Q(y)), \forall x (Q(x) \rightarrow \neg P(x)), \exists x \forall y (Q(y) \rightarrow (x = y))\}$