Conversion of a Turing machine to one with 3 states

by P Ashok Kumar

Problem: Given a TM M that accepts a recursively enumerable language by final state, make a 3 state TM M' that accepts the same language by final state.

Solution: Let the states in M' be q_0, q_1, q_2. Let q_2 be the final state.
Let s_0 to s_{k-1} be the states in M.
Assume the input tape to contain $k+3$ tracks where k is the number of states in M. The first track contains the input. The next k tracks will be used to store information about the state of M being simulated(the number of 1's in these k tracks in the current position indicate the state of M' being simulated). Initially these tracks contain only blanks. The $(k+2)$th track contains the symbol R initially. The $(k+3)$th track contains information about the operation going on. This track initially contains only blanks.

Let q_0 be the initial state of M'. The transitions required to be defined for M' are defined in words below.

Initialisation:
When q_0 sees a R and B on $k+2$th and $k+3$th track(the lowest two tracks), it replaces the B with P, moves right and enters state q_1. When q_1 sees a R and B on the lowest two tracks, it replaces the B with X, moves left and enters state q_0.
Therefore after the first two moves of the TM M', the head will be on the first position of the tape in state q_0. The lowest two tracks at the first position are RP(R in the $k+2$th tape and P in $k+3$ tape). The position to the right of the current position contains RX in the lowest two tracks.
Let at any general moment, the positions to the left of the head contain LB in the lowest two tracks, RX to the immediate right, and RB to the right beynond this (Initially there is nothing to the left of the head).

Processing:
Idea: When q_0 see a P on the lowest track, it simulates state s_i of M, where i is the number of 1s in the present position from the 2nd to $(k+1)$th track. There are four possible cases

1. Let s_i write b and move left and enter state s_j on seeing the input a on first track in M. For that case, define q_0 to write b on the first track, put j 1s followed by $(k-j)$ Bs on the next k tracks and RS' on the last two tracks and moves right, and enters q_1. q_1 on seeing RX on the lowest two tracks writes RB over it, moves left and enters state q_0. q_0 on seeing RS' writes RS over it, moves left and enters state q_0. If q_0 sees LB in the lowermost track, it writes LC over it, moves right and enters state q_0. If q_0 sees RS on the lowest tracks, and there exists a one on the above k tracks, one 1 is replaced by a blank, moves left and enters q_0. If q_0 sees LC on the lowest tracks,

writes a 1 over a blank in the upper k tracks, moves right and enters q_0. If q_0 sees RS on the lowest tracks, but no 1 in the 2nd to $(k+1)$th track, it writes B over S in the lowermost track, moves left and enters state q_1. q_1 on seeing a C on the lowest track replaces it with a P, moves right, and enters q_1. Copying of state-information is done. Now, we have alredy defined q_1 on seeing RB on the lowest track. It replaces RB with RX, moves left and enters state q_0. The next simulation will start

2. If in M s_i had written b, moved right and entered s_j on seeing a on 1st track, define transition for q_0 for that case as: write b on first track, write j 1s followed by $(k-j)$ Bs on the next k tracks, and LS on the lowest tracks, moves right and enters q_0. q_0 on seeing RX on the lowest tracks is defined to replace that by RC, move left and enter q_0. q_0 on seeing LS or RC on the lowest track may be defined as for RS and LC in previous case.

3. If in M, s_i had entered an accepting state after writing b, define q_0 for this case to write b on uppermost tape, enter q_2 and move right. q_2 halts, whatever be the symbols on the tracks at current position. Therefore if input string is accepted by M, M' halts in q_2

4. If in M, s_i would have halted on seeing a in the input, define q_0 to simply halt in this case.

The above may be converted to a two state TM that accepts the given recursively enumerable language by halting.

A figure follows on the next page.

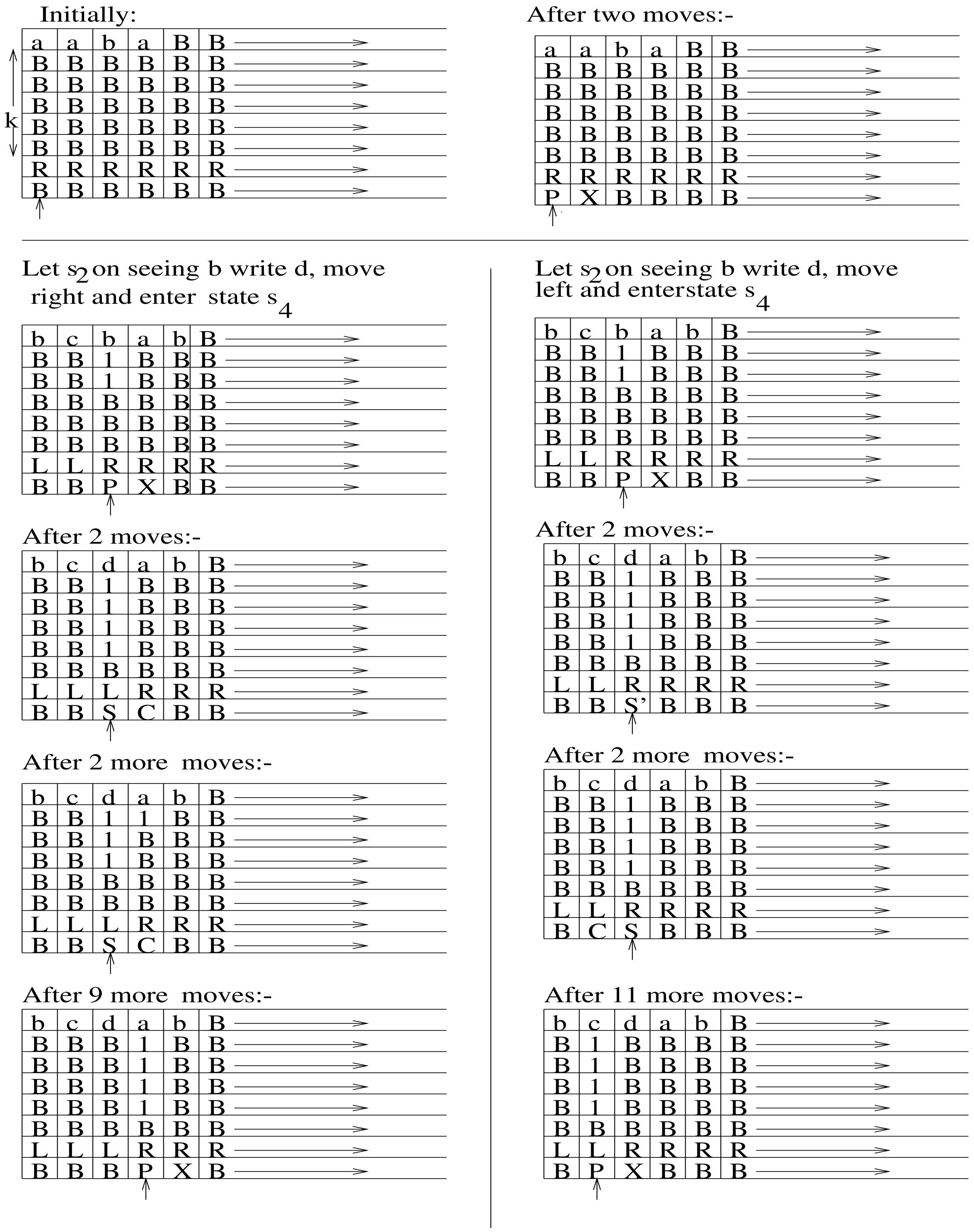

Figure 1: Some sample moves of M'