CS620 Quiz 1 (Spring 2021)

Max marks: 25 **Due: Sun Jan 31, 11.59am**

- *Be brief, complete and stick to what has been asked.*
- *Untidy presentation of answers, and random ramblings will be penalized by negative marks.*
- *Unless asked for explicitly, you may cite results/proofs covered in class without reproducing them.*
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others. Penalty for offenders: FR grade.***
- **Expected time to solve:** ≤ 45 **mins.**

Consider a DNN N with $k \geq 3$ layers (layer 1 is the "input" layer, and layer k is the "output" layer). For layer i of the network, let $\mathcal{L}_{i-1}$ be the input domain and $\mathcal{L}_i$ be the output domain. Thus, the input domain of the overall network N is $\mathcal{L}_0$ and its output domain is $\mathcal{L}_k$.

Suppose the transformer associated with the slice of the network from layers i through j, both inclusive, is $\nu_{i,j} : \mathcal{L}_{i-1} \to \mathcal{L}_j$. So, the overall transformer of the DNN N is $\nu_{1,k}$.

Let $D_{i-1} : \mathcal{L}_{i-1} \times \mathcal{L}_{i-1} \to \mathbb{R}^{\geq 0}$ be a distance metric that maps a pair of inputs of the i^{th} layer to a non-negative real number, such that $D_{i-1}(I, I) = 0$ for every $I \in \mathcal{L}_{i-1}$. For $I, J \in \mathcal{L}_{i-1}$, we say $D_{i-1}(I, J)$ is the *distance* between the inputs I and J of layer i.

For every layer $i < k$, we also have four parameters $\varepsilon_{1,i}, \varepsilon_{i,k}, \delta_{1,i}, \delta_{i,k} \geq 0$ such that the following two Hoare triples hold:

- $\{D_0(I_0, J_0) \leq \varepsilon_{1,i}\}\ I_i \leftarrow \nu_{1,i}(I_0);\ J_i \leftarrow \nu_{1,i}(J_0);\ \{D_i(I_i, J_i) \leq \delta_{1,i}\}$
- $\{D_i(I_i, J_i) \leq \varepsilon_{i,k}\}\ I_k \leftarrow \nu_{i+1,k}(I_i);\ J_k \leftarrow \nu_{i+1,k}(J_i);\ \{I_k = J_k\}$.

1. Let $A = \{i \mid \delta_{1,i} \leq \varepsilon_{i,k}\}$. What can you say about the input-output behaviour of the overall network N if:

   (a) *[5 marks]* $A \neq \emptyset$?

   (b) *[5 marks]* $A = \emptyset$?

   Explain your answers clearly with reasons.

2. Suppose further that we know that there exist inputs $I^\star, J^\star$ such that $\nu_{1,k}(I^\star) \neq \nu_{1,k}(J^\star)$. Indicate with reasons which of the following are necessarily true.

   (a) *[5 marks]* There is at least one $i < k$ such that $D_i(\nu_{1,i}(I^\star), \nu_{1,i}(J^\star)) \geq \delta_{1,i}$.

   (b) *[5 marks]* There is at least one $i < k - 1$ such that $D_i(\nu_{1,i}(I^\star), \nu_{1,i}(J^\star)) \geq \varepsilon_{i+1,k}$.

   (c) *[5 marks]* $D_{k-1}(\nu_{1,k-1}(I^\star), \nu_{1,k-1}(J^\star))$ cannot be arbitrarily large.