CS719 Practice Homework #4

- *Be brief, complete and stick to what has been asked.*
- ***Do not turn in your solutions. These problems are for you practice only.***

1. **Problems on partial orders:**
   - From *Introduction to Lattices and Order, by Davey and Priestley:* Problem nos. 1.5, 1.6, 1.12, 1.16, 1.23, 1.24, 1.29, 1.30 (i) and (ii).
   - Let $(P, \leq)$ be a poset of size at least $a.b + 1$, for $a, b \in \aleph$. Show that P either contains a chain of size at least $a + 1$ or an anti-chain of size at least $b + 1$.
   - Let $(X, \leq)$ be a non-empty poset and let $P = \wp(X)$, i.e., the powerset of X. Define a relation $\ll$ on P as follows. For $A, B \in P$, $A \ll B$ iff for every $a \in A$, there exists $b \in B$ such that $a \leq b$. Prove that if $A \ll B$ and $B \ll A$, then A and B have the same set of maximal elements.
   - Given a poset $(P, \leq)$, the *down map* $\phi_\downarrow$, and *up map* $\phi_\uparrow$ are defined as follows:
     - $\phi_\downarrow : P \rightarrow \wp(P)$ is given by $\phi_\downarrow(a) = \downarrow a = \{b \mid b \in P, b \leq a\}$
     - $\phi_\uparrow : P \rightarrow \wp(P)$ is given by $\phi_\uparrow(a) = \uparrow a = \{b \mid b \in P, a \leq b\}$

     Show that $\phi_\downarrow$ is an order embedding of $(P, \leq)$ into $(\wp(P), \subseteq)$, i.e. $a \leq b \;\leftrightarrow\; \phi_\downarrow(a) \subseteq \phi_\downarrow(b)$. Similarly, show that $\phi_\uparrow$ is an order anti-embedding of $(P, \leq)$ into $(\wp(P), \subseteq)$, i.e. $a \leq b \;\leftrightarrow\; \phi_\uparrow(b) \subseteq \phi_\uparrow(a)$.

2. **Problems on preliminary topics in lattices:**
   - From *Introduction to Lattices and Order, by Davey and Priestley:* Problem nos. 2.2, 2.5, 2.6, 2.9, 2.11, 2.13 (ii) and (iii), 2.15, 2.17, 2.19