CS719 Quiz

Max marks: 30 **Time: 1 hour**

- *The exam is open-book, open-notes and open-material-brought-to-exam-hall.*
- *Be brief, complete and stick to what has been asked. Unnecessarily lengthy solutions may be penalized.*
- *If you need to make any reasonable assumptions, state them clearly. Unreasonable assumptions run the risk of attracting penalty.*
- *If you need to use/cite results covered in class, you may simply cite the result, without going into a formal proof.*
- ***Do not copy from others or indulge in unfair means.***
  Students found indulging in such activities will be summarily awarded the FR grade.

1. *[10 + 10 marks]* Let $(L; \leq)$ be a bounded lattice, i.e. a lattice with top ($\top$) and bottom ($\bot$). For every $u, v \in L$ such that $u < v$, we define the *closed interval* $[u, v]$ to be the poset $(\{x \mid u \leq x \leq v\}; \leq)$. For every closed interval $[u, v]$ in L and for every $a \in [u, v]$, a *relative complement* of a with respect to $[u, v]$ is an element $b \in [u, v]$ such that $a \wedge b = u$ and $a \vee b = v$. The *complement* of a in L is simply its relative complement with respect to $[\bot, \top]$.

   A closed interval $[u, v]$ in L is said to be complemented if every $a \in [u, v]$ has a relative complement with respect to $[u, v]$. The lattice L is said to be relatively complemented if every closed interval $[u, v]$ in L is complemented.

   (a) Is $[u, v]$ a lattice for every $u, v \in L$ such that $u < v$? Either give a proof or provide a counterexample.

   (b) Is it possible for L to be complemented (i.e. every element has a complement), but not be relatively complemented? Either give a proof or provide a counter-example.

2. *[10 marks]* Show that if L is a lattice and if every two element subset of L is a sub-lattice, then L must be a chain.