CS719 Quiz

Max marks: **Time: 1.5 hours**

- *The exam is open-book, open-notes and open-material-brought-to-exam-hall.*
- *Be brief, complete and stick to what has been asked. Unnecessarily lengthy solutions may be penalized.*
- *If you need to make any reasonable assumptions, state them clearly. Unreasonable assumptions run the risk of attracting penalty.*
- *If you need to use/cite results covered in class, you may simply cite the result, without going into a formal proof.*
- ***Do not copy from others or indulge in unfair means.***
  Students found indulging in such activities will be summarily awarded the FR grade.

1. (a) *[10 marks]* Show that a Boolean lattice is finite if and only if 1 (the top element) is the join of a finite number of atoms.

   (b) *[10 marks]* Show that a Boolean lattice with ACC also has DCC.

2. This question remedies the problem we had with our discussion of de Morgan's laws for a pair of posets with a Galois Connection between them.

   Let P and Q be lattices and let (α, γ) be a Galois Connection between them, where $\alpha : P \to Q$ and $\gamma : Q \to P$. Define $\alpha(P) = \{y \mid \exists x \in P.\, y = \alpha(x)\}$ and $\gamma(Q) = \{x \mid \exists y \in Q.\, x = \gamma(y)\}$.

   (a) *[10 marks]* Show that $\alpha(P)$ and $\gamma(Q)$ are lattices (though not necessarily sub-lattices of Q and P, respectively).

   (b) *[5 + 5 marks]* Let $\wedge_2$ be the meet operation in $\alpha(P)$ and $\vee_1$ be the join operation in $\gamma(Q)$. Show that for all $x, y \in \alpha(P)$, $\gamma(x \vee_Q y) = \gamma(x) \vee_1 \gamma(y)$. Similarly show that for all $x, y \in \gamma(Q)$, $\alpha(x \wedge_P y) = \alpha(x) \wedge_2 \alpha(y)$.

   It is easy to show (and you are not required to show this as part of this quiz question) that the join operation in $\alpha(P)$ is $\vee_Q$ and the meet operation in $\gamma(Q)$ is $\wedge_P$.

3. *[10 marks]* Find a lattice for which the set of join-irreducible elements is the same as the set of atoms, but the lattice is not Boolean.