
CS781 Quiz 1 (Autumn 2023)

Max marks: 30

Duration: 50 mins

- *The exam is open book and notes. However, you are not allowed to search on the internet or consult others over the internet for your answers.*
- *Be brief, complete and stick to what has been asked.*
- *Unless asked for explicitly, you may cite results/proofs covered in class without reproducing them.*
- *If you need to make any assumptions, state them clearly.*
- **Do not copy solutions from others. Penalty for offenders: FR grade.**

1. The neural network shown in Fig. 1 has 1 input layer, 1 hidden layer and 1 output layer. Assume that all hidden and output layer nodes have bias 0, and a non-linear activation function $\sigma(\cdot)$ is used in the hidden layer nodes (but not in output layer nodes). Thus, the output of node x_3 is given by $\sigma(x_1 - x_2)$ and that of node x_4 is given by $\sigma(x_2 - x_1)$. The function $\sigma(x)$ is defined as $\max(\alpha.x, \beta.x)$, where $0 \leq \alpha < 0.25$ and $0.75 \leq \beta \leq 1$.

Our goal in this question is to find out if $\min_{(x_1, x_2) \in \mathcal{C}} x_5 \geq 0$ and also if $\max_{(x_1, x_2) \in \mathcal{C}} x_5 \leq 0.5$, where \mathcal{C} denotes the p -norm unit ball around $(0, 0)$, i.e. $(x_1, x_2) \in \mathbb{B}_p((0, 0), 1)$ and $p > 1$

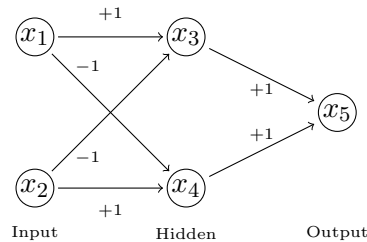


Figure 1: A neural network

[5 marks] Using the interval abstract domain studied in class and using only interval propagation, find upper and lower bounds for every node n in the neural network. Upper and lower bounds of x_3, x_4, x_5 can be in terms of the parameters α and β .

[10 marks] Next, suppose we wish to use the 4-tuple abstract domain used in the DEEPPOLY paper. Using the upper and lower bounds determined above, find the best linear expressions in x_1 and x_2 that bound x_3 and x_4 from above and below. Your expressions can include α and β and are not required to be linear in these parameters. As in the DEEPPOLY paper, you must use only a single linear expression (in x_1 and x_2) for each lower/upper bound.

[5 marks] Find linear expressions in x_1 and x_2 for the lower and upper bound of x_5 using the backward propagation of expressions all the way upto the input layer, as explained in the DEEPPOLY paper. Your expressions can include α and β and are not required to be linear in these parameters.

[5 marks] Use the expressions obtained above and the result about dual norms and Holder's inequality studied in class to find a lower bound of $\min_{(x_1, x_2) \in \mathcal{C}} x_5$ and an upper bound of $\max_{(x_1, x_2) \in \mathcal{C}} x_5$ in terms of α and β , where $\mathcal{C} = \mathbb{B}_p((0, 0), 1)$ and $p > 1$.

[5 marks] From the bounds obtained in the previous sub-question, can you find a (possibly non-linear) relation between α and β that ensures that $x_5 \geq 0$ for all $(x_1, x_2) \in \mathcal{C}$ for all $p > 1$? Give justification for your answer.