Roll No.: ______________________

CS781 In-class Evaluation 1 (Autumn 2025)

Max marks: 10 **Duration: 10 mins**

- *You are required to answer each question only in the sheet provided to you.*
- *Only material written in the sheet will be graded.*
- *The exam is open book and notes. However, you are not allowed to consult others for your answers.*
- *Be brief, complete and stick to what has been asked.*
- *Unless asked for explicitly, you may cite results/proofs covered in class without reproducing them.*
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others. Penalty for offenders: FR grade.***

Consider the humble single-neuron network shown below. Assume that the inputs x_1, x_2 always satisfy the constraint

$$(x_1 \geq -1) \wedge (x_2 \leq 1) \wedge (-0.5 \leq x_2 - x_1)$$

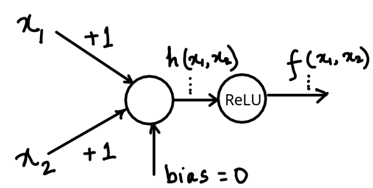


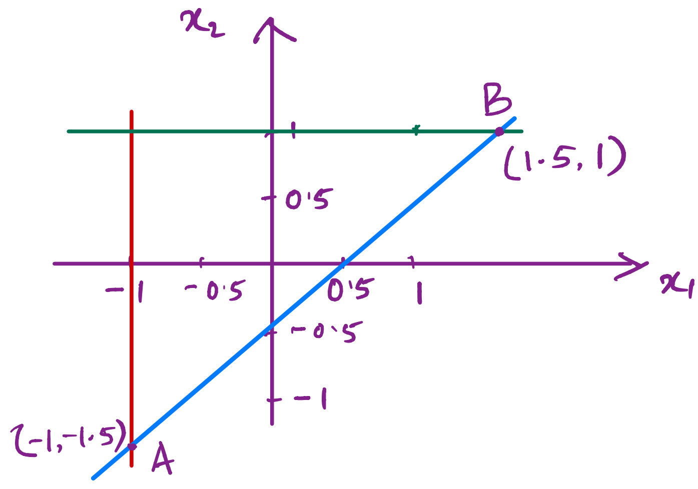
Figure 1: A humble neuron

Derive as good linear bounds as you can on $f(x_1, x_2)$ using the α-CROWN technique. Show all steps in your derivation. Your final answer should be of the form $A_0 + A_1 x_1 + A_2 x_2 \leq f(x_1, x_2) \leq B_0 + B_1 x_1 + B_2 x_2$, where the A_i's and B_j's are expressions in terms of α (the parameter used in α-CROWN).

$h(x_1, x_2) = x_1 + x_2$.

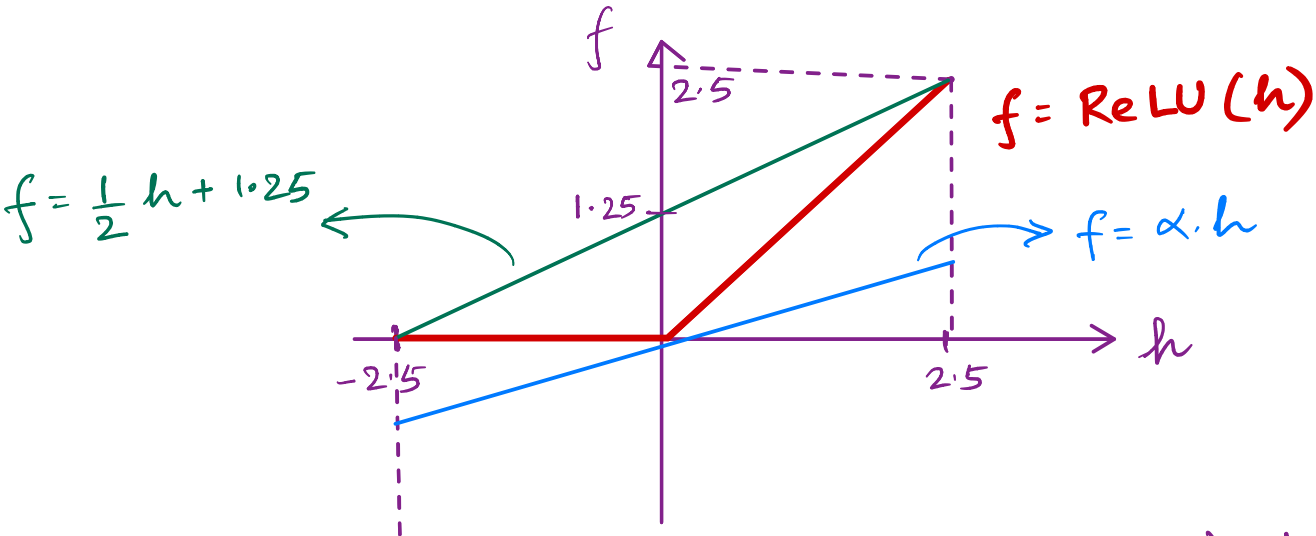
To find the bounds of $x_1 + x_2$, we can plot the feasible region x_1, x_2.

This is the triangular region enclosed by the red, green and blue lines



The max. value of $x_1 + x_2$ is achieved at point B (1.5, 1) and the min. value is achieved at point A (−1, −1.5).

$$-1-1.5 = -2.5 \le h(x_1, x_2) \le 1 + 1.5 = 2.5$$



$$\therefore \quad \alpha . h(x_1, x_2) \le f(x_1, x_2) \le \frac{2.5}{5.0} h(x_1, x_2) + 1.25$$

$$0 \le \alpha \le 1$$

Simplifying:

$$\alpha . (x_1 + x_2) \le f(x_1, x_2) \le \frac{1}{2}(x_1 + x_2) + 1.25$$

or, $\alpha . x_1 + \alpha . x_2 \le f(x_1, x_2) \le \frac{1}{2} x_1 + \frac{1}{2} x_2 + 1.25$