What happens to equilibrium after iterative elimination?

**Theorem:** Consider $G$ and $\hat{G}$ are games before and after elimination of a strategy [not necessarily dominated]. If $s^*$ is a PSNE in $G$ and survives in $\hat{G}$, then $s^*$ is a PSNE in $\hat{G}$ too.

**Intuition:** PSNE strategy was the maxima, removing others will continue keeping this as maxima. **Proof:** exercise.

Can new equilibrium be generated?

**Theorem:** Consider NFG $G$. Let $\hat{s}_j$ be a weakly dominated strategy of $j$. If $\hat{G}$ is obtained from $G$ eliminating $\hat{s}_j$, every PSNE of $\hat{G}$ is a PSNE of $G$.

No new PSNE if the eliminated strategy is dominated.

**Proof:**

$\hat{G} : \hat{s}_j = s_j \setminus \{\hat{s}_j\}$, $\hat{s}_i = s_i$, $\forall i \neq j$.

**TST:** If $s^* = (s^*_j, s^*_i)$ is a PSNE in $\hat{G}$, it is a PSNE in $G$.

$u_i(s^*) > u_i(s_i, s^*_i)$, $\forall i \neq j$, $\forall s_i \in \hat{s}_i = s_i$.

$u_j(s^*) > u_j(s^*_j, s^*_i)$, $\forall s_j \in \hat{s}_j$ - this has one less need to show that there is no profitable deviation for any player in $G$, for $i \neq j$, this is immediate — no strategies are removed for $j$, this is true for all strategies except $\hat{s}_j$. 
Since $\hat{\Delta}_j$ is dominated, $\exists t_j \in \hat{\Delta}_j \setminus \{\hat{\Delta}_j\}$

s.t. $u_j(t_j, \Delta_j) > u_j(\hat{\Delta}_j, \Delta_j), \forall \Delta_j \in S_j$

so, in particular, $u_j(t_j, \Delta_j^*) > u_j(\hat{\Delta}_j, \Delta_j^*)$

Since $\Delta_j^*$ is a PSNE in $\hat{G}$ and $t_j \in \hat{\Delta}_j$,

$u_j(\Delta_j^*, \Delta_j^*) > u_j(t_j, \Delta_j^*) > u_j(\hat{\Delta}_j, \Delta_j^*)$

Summary:

- Elimination of strictly dominated strategies have no effect on PSNE.
- Elimination of weakly dominated strategies may reduce the set of PSNEs, but never adds new.
- The maxmin value is unaffected by the elimination of strictly or weakly dominated strategies.