What happens to equilibrium after iterative elimination ?

Theorem: Consider G and $\hat{G}$ are games before and after elimination of a strategy [not necessarily dominated]. If s^* is a PSNE in G and survives in $\hat{G}$, The s^* is a PSNE in $\hat{G}$ too.

Intuition: PSNE strategy was the maxima, removing others will continue keeping this as maxima. Proof: exercise.

Can new equilibrium be generated ?

Theorem: Consider NFG G. Let $\hat{s}_j$ be a weakly dominated strategy of j. If $\hat{G}$ is obtained from G eliminating $\hat{s}_j$, every PSNE of $\hat{G}$ is a PSNE of G

No new PSNE if the eliminated strategy is dominated.

Proof: $\hat{G}$: $\hat{S}_j = S_j \setminus \{\hat{s}_j\}$, $\hat{S}_i = S_i$, $\forall i \neq j$.

TST: if $s^* = (s_j^*, s_{-j}^*)$ is a PSNE in $\hat{G}$, it is a PSNE in G

$u_i(s^*) \geqslant u_i(s_i, s_{-i}^*)$, $\forall i \neq j$, $\forall s_i \in \hat{S}_i = S_i$

$u_j(s^*) \geqslant u_j(s_j, s_{-j}^*)$, $\forall s_j \in \hat{S}_j$ - this has one less

need to show that there is no profitable deviation for any player in G

for $i \neq j$, this is immediate — no strategies are removed

for j, this is true for all strategies except $\hat{s}_j$

Since $\hat{s}_j$ is dominated, $\exists\, t_j \in \hat{S}_j = S_j \setminus \{\hat{s}_j\}$

s.t. $u_j(t_j, \underline{s}_{-j}) \geq u_j(\hat{s}_j, \underline{s}_{-j}),\ \forall \underline{s}_{-j} \in \underline{S}_{-j}$

so, in particular, $u_j(t_j, \underline{s}^*_{-j}) \geq u_j(\hat{s}_j, \underline{s}^*_{-j})$

since s^* is a PSNE in $\hat{G}$ and $t_j \in \hat{S}_j$,

$$u_j(s^*_j, \underline{s}^*_{-j}) \geq u_j(t_j, \underline{s}^*_{-j}) \geq u_j(\hat{s}_j, \underline{s}^*_{-j})$$

Summary:

- Elimination of strictly dominated strategies have no effect on PSNE .
- Elimination of weakly dominated strategies may reduce the set of PSNEs, but never adds new.
- The maxmin value is unaffected by the elimination of strictly or weakly dominated strategies