Recap: 1. Iterated elimination of dominated strategies
2. Preservation of equilibrium
3. Stability & security coincide for matrix games
4. Limited to pure strategies - PSNE may not exist

\[
\begin{array}{c|cc}
 & L & R \\
\hline
L & -1,1 & 1,-1 \\
R & 1,-1 & -1,1 \\
\end{array}
\]

Mixed strategies

Probability distribution over the set of strategies

Consider a finite set \( A \)

Define \( \Delta A = \{ p \in [0,1]^{|A|} : \sum_{a \in A} p_a = 1 \} \)

Set of all probability distributions over \( A \).

\( \sigma_i \) is a mixed strategy of player \( i \)

\( \sigma_i \in \Delta(S_i) \), i.e., \( \sigma_i : S_i \to [0,1] \wedge \sum_{s_i \in S_i} \sigma_i(s_i) = 1 \).

We are discussing non-cooperative games, the players choose their strategies independently.
The joint probability of 1 picking $s_1$ and 2 picking $s_2 = \sigma_1(s_1) \sigma_2(s_2)$ utility of player $i$ at a mixed strategy profile $(\sigma_i, \sigma_i)$ is

$$U_i(\sigma_i, \sigma_i) = \sum_{\lambda_1 \in S_1} \sum_{\lambda_2 \in S_2} \sum_{\lambda_i \in S_i} \sigma_i(s_i) \sigma_2(s_2) \cdots \sigma_n(s_n) U_i(s_1, s_2, \ldots, s_n)$$

we are overloading $U_i$ to denote the utility at pure and mixed strategies.

Utility at a mixed strategy is the expectation of the utilities at pure strategies.

So, all the rules of expectation hold, e.g., linearity.

**Example:**

<table>
<thead>
<tr>
<th></th>
<th>$\frac{4}{5}$</th>
<th>$\frac{1}{5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2}{3}$</td>
<td>L</td>
<td>1, -1</td>
</tr>
<tr>
<td>$\frac{1}{3}$</td>
<td>R</td>
<td>1, -1</td>
</tr>
</tbody>
</table>

$$U_i(\sigma_1, \sigma_2) = \frac{2}{3} \cdot \frac{4}{5} \cdot (-1) + \frac{2}{3} \cdot \frac{1}{5} \cdot 1 + \frac{1}{3} \cdot \frac{4}{5} \cdot 1 + \frac{1}{3} \cdot \frac{1}{5} \cdot (-1)$$

mixture of mixed strategies

$$U_i(\lambda \sigma_i + (1-\lambda) \sigma_i', \sigma_i) = \lambda U_i(\sigma_i, \sigma_i) + (1-\lambda) U_i(\sigma_i', \sigma_i).$$