Mixed strategy Nash equilibrium (MSNE)

<u>Defn</u>: MSNE is a mixed strategy profile $(\sigma_i^*, \sigma_{-i}^*)$, s.t.

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i', \sigma_{-i}^*), \quad \forall \sigma_i' \in \Delta(S_i), \forall i \in N$$

Relation between PSNE and MSNE?

An alternative definition

<u>Theorem</u>: A mixed strategy profile $(\sigma_i^*, \sigma_{-i}^*)$ is an MSNE if and only if $u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(s_i, \sigma_{-i}^*)$, $\forall s_i \in S_i$, $\forall i \in N$

<u>Proof</u>: $(\Rightarrow)$ s_i is a special case of the mixed strategy
The mixed strategy with s_i having prob. 1. Inequality holds by definition of MSNE.

$(\Leftarrow)$ Pick an arbitrary mixed strategy σ_i of player i

$$u_i(\sigma_i, \sigma_{-i}^*) = \sum_{s_i \in S_i} \sigma_i(s_i) \, u_i(s_i, \sigma_{-i}^*)$$

$$\leq \sum_{s_i \in S_i} \sigma_i(s_i) \, u_i(\sigma_i^*, \sigma_{-i}^*) \quad \text{(Given)}$$

$$= u_i(\sigma_i^*, \sigma_{-i}^*) \sum_{s_i \in S_i} \sigma_i(s_i) = u_i(\sigma_i^*, \sigma_{-i}^*)$$

Example of MSNE

Is the mixed strategy profile an MSNE?

		$4/5$	$1/5$
		L	R
$2/3$	L	$-1, 1$	$1, -1$
$1/3$	R	$1, -1$	$-1, 1$

to prove this, need to show there does not exist any better mixed strategy for the player.

expected utility of player 2 from $L = \frac{2}{3} \cdot 1 + \frac{1}{3}(-1) = \frac{1}{3}$,

from $R = -\frac{1}{3}$

expected utility will increase if some probability is transferred from R to L $\Rightarrow$ the current profile is not an MSNE.

Some balance in the utilities is needed

Redo the calculations

		$1/2$	$1/2$
		L	R
$1/2$	L	$-1, 1$	$1, -1$
$1/2$	R	$1, -1$	$-1, 1$

does there exist any improving mixed strategy?