Richer representation of games

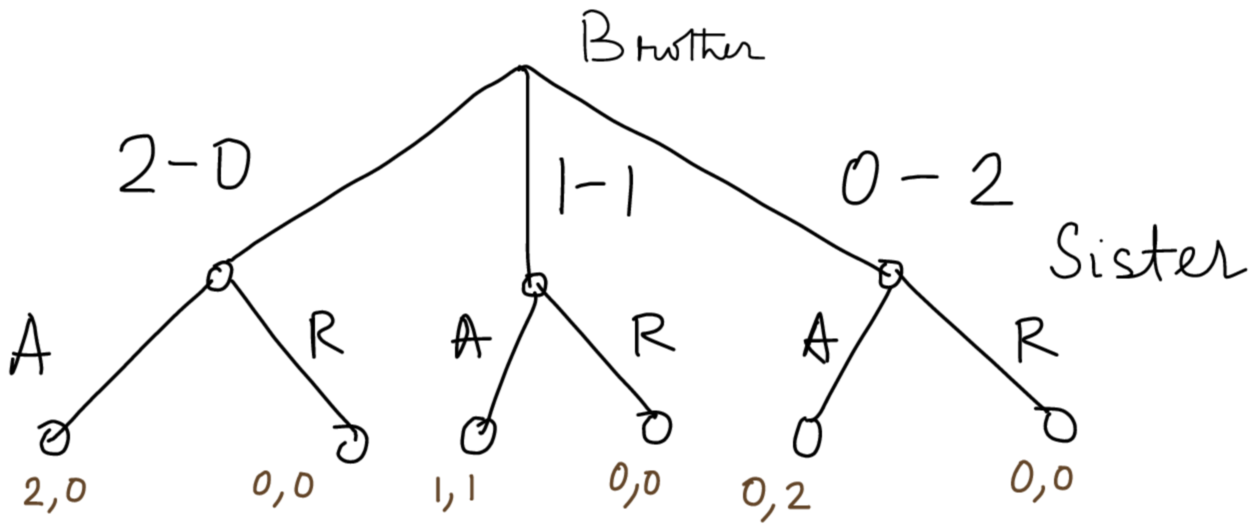
More appropriate for multi-stage games, e.g., chess

Players interact in a sequence - the sequence of actions is the history of the game

Perfect Information Extensive Form Games (PIEFG)

Warning: more notation :)

Ex. Brother-Sister chocolate division



Disagreement → both chocolates taken away

Formal capture: PIEFG $\langle N, A, \mathcal{H}, \mathcal{X}, P, (u_i)_{i \in N} \rangle$

- N : set of players
- A : set of all possible actions (of all players)
- $\mathcal{H}$: set of all sequence of actions (histories) satisfying
  - empty $\emptyset \in \mathcal{H}$
  - if $h \in \mathcal{H}$, any sub-sequence h' of h starting at the root must be in $\mathcal{H}$
  - $h = (a^{(0)}, a^{(1)}, \ldots, a^{(T-1)})$ is terminal if $\nexists\, a^{(T)} \in A$ s.t. $(a^{(0)}, a^{(1)}, \ldots, a^{(T)}) \in \mathcal{H}$

- $Z \subseteq \mathcal{H}$: set of all terminal histories
- $\mathcal{X} : \mathcal{H} \setminus Z \to 2^A$: action set selection function
- $P : \mathcal{H} \setminus Z \to N$: player function
- $u_i : Z \to \mathbb{R}$: utility of i

The strategy of a player in an EFG is a tuple of actions at every history where the player plays.

$$S_i = \bigtimes_{\{h \in \mathcal{H} : P(h) = i\}} \mathcal{X}(h)$$

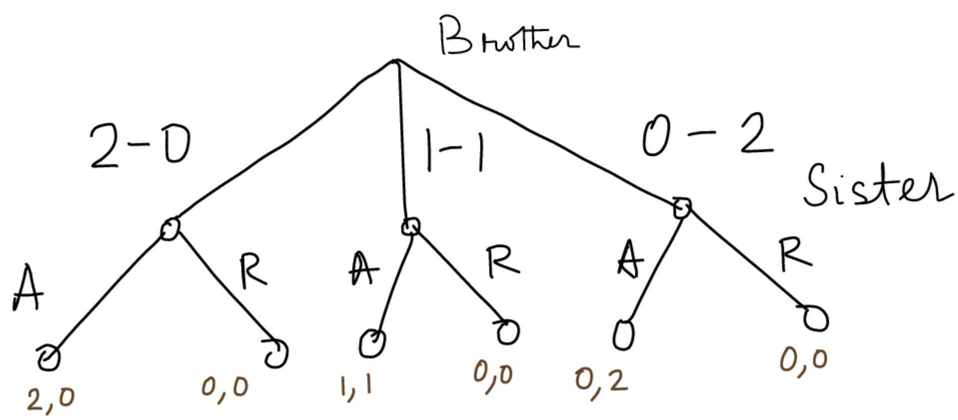
Remember: strategy is a complete contingency plan of the player.

It enumerates potential actions a player can take at every node where she can play, even though some combination of actions may never be executed together.

$N = \{1\,(B),\ 2\,(S)\}$

$A = \{2\text{-}0, 1\text{-}1, 0\text{-}2, A, R\}$

$\mathcal{H} = \{\phi, (2\text{-}0), (1\text{-}1), (0\text{-}2), (2\text{-}0, A), (2\text{-}0, R), (1\text{-}1, A), (1\text{-}1, R), (0\text{-}2, A), (0\text{-}2, R)\}$



$Z = \{(2\text{-}0, A), (2\text{-}0, R), (1\text{-}1, A), (1\text{-}1, R), (0\text{-}2, A), (0\text{-}2, R)\}$

$\mathcal{X}(\phi) = \{(2\text{-}0), (1\text{-}1), (0\text{-}2)\}$, $\mathcal{X}(2\text{-}0) = \mathcal{X}(1\text{-}1) = \mathcal{X}(0\text{-}2) = \{A, R\}$

$P(\phi) = 1$, $P(2\text{-}0) = P(1\text{-}1) = P(0\text{-}2) = 2$

$u_1(2\text{-}0, A) = 2$, $u_1(1\text{-}1, A) = 1$, $u_2(1\text{-}1, A) = 1$, $u_2(0\text{-}2, A) = 2$ [utilities are zero at other terminal histories]

$S_1 = \{2\text{-}0, 1\text{-}1, 0\text{-}2\}$

$S_2 = \{A,R\} \times \{A,R\} \times \{A,R\} = \{AAA, AAR, ARA, ARR, RAA, RAR, RRA, RRR\}$

Transforming PIEFG into NFG

Once we have the S_1 and S_2, the game can be represented as an NFG

	AAA	AAR	ARA	ARR	RAA	RAR	RRA	RRR
2-0	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0
1-1	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
0-2	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0

Nash equilibrium like {2-0, RRA} not quite reasonable -

why R at 1-1? {2-0, RRR} is not a credible threat

hence this equilibrium concept is not good enough for predicting outcomes in PIEFGs.

Also, the representation has huge redundancy. EFG is succinct.