Strategies in NIEFGs

Strategy set of $i$: $S_i = \prod_{j=1}^{k(i)} X(I_i^j)$

Randomized strategies in NIEFG

In NIEFGs, mixed strategies randomize over pure strategies.

In EFGs, randomization can happen in different ways:
- Randomize over the strategies defined at the beginning of the game.
- Randomize over the action at an information set - behavioral strategy.

Definition: Behavioral Strategy

A behavioral strategy of a player in an NIEFG is a function that maps each of her information sets to a probability distribution over the set of possible actions at that information set.

Question: What is the relation between mixed and behavioral strategies?

In this example: MSs live in $\mathbb{R}^4$, BSs live in two $\mathbb{R}^2$ spaces.
Definition: equivalence of mixed strategies are equivalent if every mixed-strategy vector of a player, i.e., in an IIEFG, of the form 
\[ (\frac{3}{2} \cdot 9, \frac{1}{2} \cdot 5) = (\frac{3}{2} \cdot a, \frac{1}{2} \cdot b) \]

\[ \phi(x; 1, b) = (\frac{3}{2} \cdot (R_1 + b_2) + \frac{1}{2} \cdot (R_2 + b_2)) \cdot b_2 \]

Important: different players can choose different kind of strategies.

Example: if 1 chooses \( a \) and 2 chooses \( b \).

\[ \phi(x; 1, b) = b_1(x; 1)R_1 + b_2(x; 1)R_2 \]

\[ \phi(x; 1, b) = b_1(x; 1)R_1 + b_2(x; 1)R_2 \]

Equivalence in terms of the probability of reaching a vertex/strategy:

Say \( \phi(x; 1, b) \) is the probability of reaching a vertex/strategy 1.

Equivalently, \( \phi(x; 1, b) \) in the same for behavioral strategy profile 1.

Similarly, \( \phi(x; 1, b) \) in the same for behavioral strategy profile 2.
Example: in the game above
\[ b_1(I_1')(L_1) = \sigma_1(L_1L_2) + \sigma_1(L_1R_2) \]
\[ b_1(I_1')(R_1) = \sigma_1(R_1L_2) + \sigma_1(R_1R_2) \]
\[ b_1(I_2')(L_2) = \sigma_1(L_2|R_1) \]
\[ b_1(I_2')(R_2) = \sigma_1(R_2|R_1) \]

The strategies induce the same probability of reaching a vertex

More on equivalent strategies

The equivalence, by definition, holds at the leaf nodes too.

Claim: it is enough to check the equivalence only at the leaf nodes.

Reason: pick an arbitrary non-leaf node, the probability of reaching that node is equal to the sum of the probabilities of reaching the leaf nodes in its subtree.

This argument can be extended further.

Theorem (Utility equivalence)

If \( \sigma_i \) and \( b_i \) are equivalent, then for every mixed/behavioral strategy vector of the other players \( \xi_i \), the following holds

\[ u_j(\sigma_i, \xi_i) = u_j(b_i, \xi_i), \forall j \in \mathbb{N}. \]

Repeat the argument for any equivalent mixed and behavioral strategy profiles.

Corollary: Let \( \sigma \) and \( b \) are equivalent, i.e., \( \sigma_i \) and \( b_i \) are equivalent \( \forall i \in \mathbb{N} \).

Then \( u_i(\sigma) = u_i(b). \)