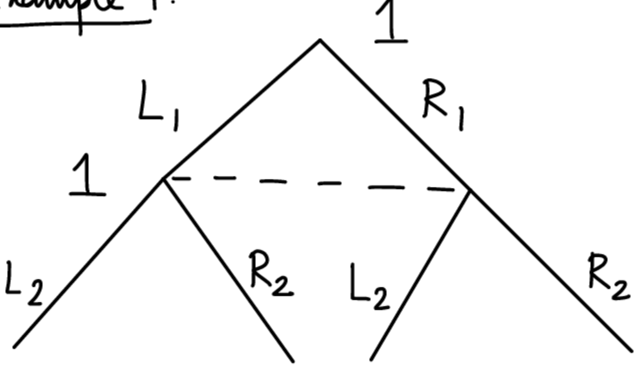
Why behavioral strategies are desirable?

1. More natural in large IIEFGs
   - players plan at a stage (information set) rather than a master plan

2. Smaller number of variables to deal with
   - consider a player having 4 information sets with 2 actions each
   - needs (2^4-1) variables to represent mixed strategies
   - needs 4 variables for behavioral strategies

Question: can one construct one from the other? OR does equivalence always hold?

Example 1:



mixed strategy

$\sigma_1(L_1, L_2)$, $\sigma_1(L_1, R_2)$, $\sigma_1(R_1 L_2)$, $\sigma_1(R_1, R_2)$

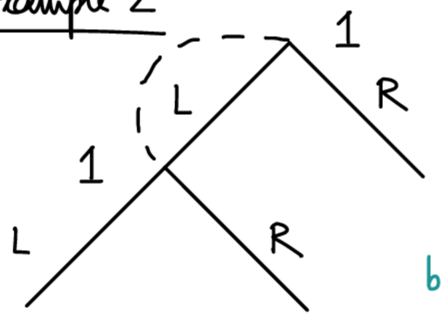
behavioral strategy

$b_1(L_1)$, $b_1(L_2)$

mixed strategy has more control over the profiles, e.g., $\sigma_1(L_1 R_2) = \sigma_1(R_1 L_2) = 0$ but not possible in behavioral strategies

mixed strategy with no equivalent behavioral strategies

Example 2



A behavioral strategy can have positive mass on LR, but mixed strategy cannot

behavioral strategy with no equivalent mixed strategy

Ex 1: player remembers that it made a move but forgets what move.

Ex 2: player forgets whether it made a move or not

The equivalence does not hold if players are forgetful.

When does behavioral strategy has no equivalent mixed strategy?

Let x be a non-root node

action at x_1 leading to x : The unique edge emanating from x_1 that is on the path from root to x.

In ex 2, there is a node which has a path from root to itself that crosses the same information set twice

if the path from root to x passes through vertices x_1 and $\hat{x}_1$ that are in the same information set of player i, and

the action leading to x at x_1 and $\hat{x}_1$ are different, then no pure strategy can ever lead to x

mixed strategy is randomization over pure strategies, every mixed strategy will put zero mass on x.

but behavioral strategy randomizes on every vertex independently, hence x can be reached in behavioral strategies with positive probability

The above observation can be stated as a lemma

Lemma: If there exists a path from the root to some vertex x that passes through the same information set at least twice, and if the action leading to x is not the same at each of those vertices, then the player of the information set has a behavioral strategy that has no equivalent mixed strategy.

The lemma helps us in proving the following characterization result of equivalence

Theorem (6.11 of MSZ)

Consider an IIEFG s.t. every vertex has at least two actions. Every behavioral strategy has an equivalent mixed strategy iff each information set of a player intersects every path emanating from the root at most once.

Proof: reading exercise from MSZ.