Why behavioral strategies are desirable?

1. More natural in large NIEFGs
   - players plan at a stage (information set) rather than a master plan
2. Smaller number of variables to deal with
   - consider a player having 4 information sets with 2 actions each
   - needs \(2^4-1\) variables to represent mixed strategies
   - needs 4 variables for behavioral strategies

Question: can one construct one from the other? OR does equivalence always hold?

**Example 1:**

\[
\begin{array}{c}
L_1 \\
1 \\
L_2 \\
R_2 \\
1 \\
R_1
\end{array}
\]

mixed strategy
\[
\sigma_1(L_1, L_2), \sigma_1(L_1, R_2), \sigma_1(R_1, L_2), \\
\sigma_1(R_1, R_2)
\]

behavioral strategy
\[
b_1(L_1), b_1(L_2)
\]

mixed strategy has more control over the profiles, e.g., \(\sigma_1(L_1, R_2) = \sigma_1(R_1, L_2) = 0\)
but not possible in behavioral strategies
mixed strategy with no equivalent behavioral strategies

**Example 2**

\[
\begin{array}{c}
1 \\
L \\
1 \\
R \\
L
\end{array}
\]

A behavioral strategy can have positive mass on LR, but mixed strategy cannot

behavioral strategy with no equivalent mixed strategy

Ex 1: player remembers that it made a move but forgets what move.
Ex 2: player forgets whether it made a move or not
The equivalence does not hold if players are forgetful.

When does behavioral strategy have no equivalent mixed strategy?

Let \( x \) be a non-root node

action at \( x_1 \) leading to \( x \): The unique edge emanating from \( x_1 \) that is on the path from root to \( x \).

In ex 2, there is a node which has a path from root to itself that crosses the same information set twice

if the path from root to \( x \) passes through vertices \( x_1 \) and \( x_2 \) that are in the same information set of player \( i \), and

the action leading to \( x \) at \( x_1 \) and \( x_2 \) are different, then no pure strategy can ever lead to \( x \).

mixed strategy is randomization over pure strategies, every mixed strategy will put zero mass on \( x \).

but behavioral strategy randomizes on every vertex independently, hence \( x \) can be reached in behavioral strategies with positive probability.

The above observation can be stated as a lemma

Lemma: If there exists a path from the root to some vertex \( x \) that passes through the same information set at least twice, and if the action leading to \( x \) is not the same at each of those vertices, then the player of the information set has a behavioral strategy that has no equivalent mixed strategy.

The lemma helps us in proving the following characterization result of equivalence

*Theorem (6.11 of MSZ)*

Consider an IIEFG s.t. every vertex has at least two actions. Every behavioral strategy has an equivalent mixed strategy iff each
information set of a player intersects every path emanating from the root at most once.

Proof: reading exercise from MSZ.