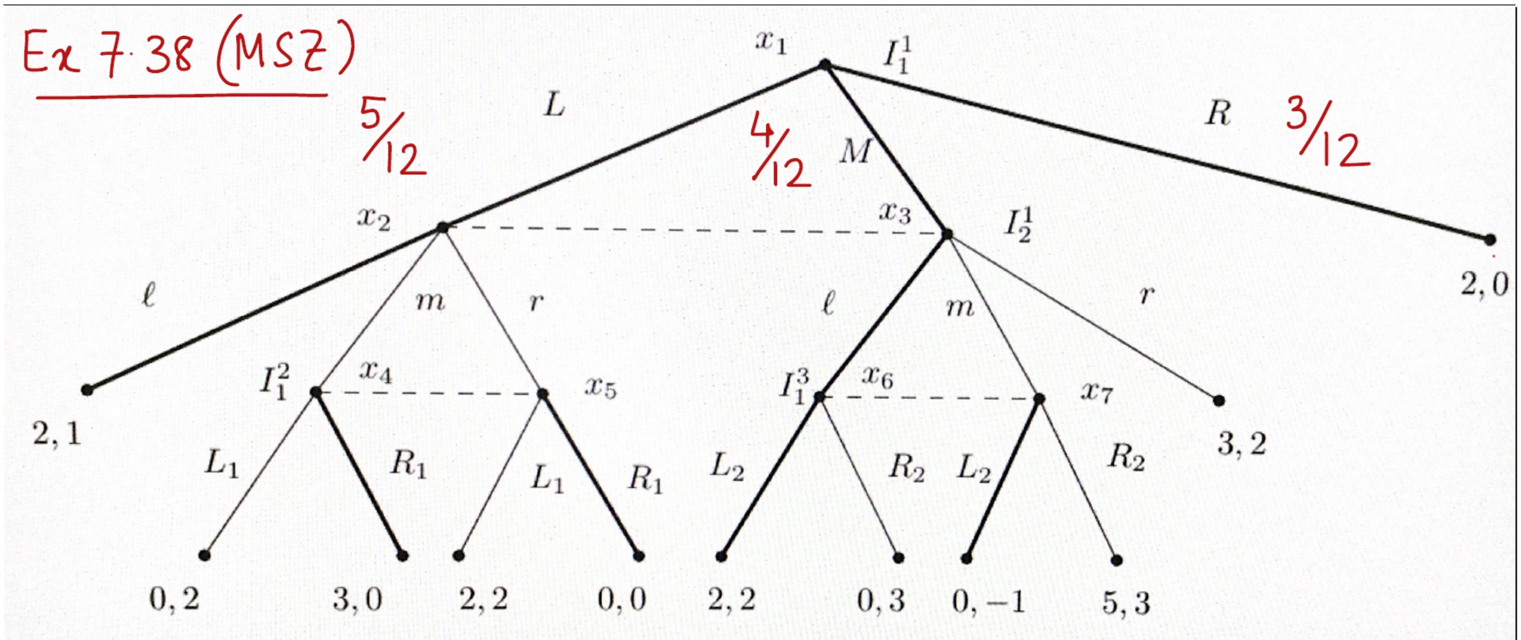
Equilibrium notions in IIEFGs

- Can extend the subgame perfection of PIEFG, but since the nodes/histories are uncertain, we need to extend to mixed strategies
- Because of the information sets, best response cannot be defined without the belief of each player

Belief is a conditional probability distribution over the histories in an information set – conditioned on reaching the information set.

Example: an IIEFG with perfect recall, i.e., mixed and behavioral strategies are equiv.



Consider the behavioral strategy profile: σ_1, at I_1^1 $\left(\frac{5}{12} L, \frac{4}{12} M, \frac{3}{12} R\right)$

at I_1^2, choose R_1, at I_1^3, choose L_2

σ_2 : choose ℓ

Q: Is this an equilibrium? Which implies

- are the Bayesian beliefs consistent with P_σ – that visits vertex x w.p. $P_\sigma(x)$
- The actions and beliefs are consistent for every player, i.e., maximizes their expected utility

- player 1, at I_1^3, believes that x_6 is reached w.p. 1

  if the belief was $> \frac{2}{7}$ in favor of x_7, should have chosen R_2

Choose an action maximizing expected utility at each information set

  — sequential rationality

The strategy vector σ induces the following probabilities to the vertices

$$P_\sigma(x_2) = \frac{5}{12}, \quad P_\sigma(x_3) = \frac{4}{12}, \quad P_\sigma(x_4) = P_\sigma(x_5) = P_\sigma(x_7) = 0, \quad P_\sigma(x_6) = \frac{4}{12}$$

- player 2, at I_2^1, believes that x_3 is reached w.p.

$$P_\sigma(x_3 \mid I_2^1) = \frac{P_\sigma(x_3)}{P_\sigma(x_2) + P_\sigma(x_3)} = \frac{4/12}{4/12 + 5/12} = 4/9$$

Similarly, $P_\sigma(x_2 \mid I_2^1) = 5/9$

Is the action of player 2 sequentially rational wrt her belief?

by picking l, her expected utility $= \frac{5}{9} \times 1 + \frac{4}{9} \times 2 = \frac{13}{9}$, this is larger than any other choice of actions.

- Given this, what will be the sequentially rational strategy of player 1 at I_1^1?

- L, M, R all gives the same expected utility for 1 (utility = 2)

mixed/behavioral strategy profile σ is sequentially rational for all players

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Formal definitions

① **Belief**: Let the information sets of player i be $I_i = \{I_i^1, \dots, I_i^{k(i)}\}$

The belief of player i is a mapping $\mu_i^j : I_i^j \to [0,1]$, s.t.

$$\sum_{x \in I_i^j} \mu_i^j(x) = 1 .$$

② **Bayesian belief**: A belief $\mu_i = (\mu_i^j, j = 1, \dots, k(i))$ of player i is Bayesian wrt the behavioral strategy σ, if it is derived from σ using Bayes rule, i.e.,

$$\mu_i^j(x) = \frac{P_\sigma(x)}{\sum_{y \in I_i^j} P_\sigma(y)} , \quad \forall x \in I_i^j , \ \forall j = 1, 2, \dots, k(i) .$$

③ **Sequential rationality**:

A strategy σ_i of player i at an information set I_i^j is sequentially rational given $\underline{\sigma}_{-i}$ and partial belief μ_i^j if

$$\sum_{x \in I_i^j} \mu_i^j(x) \, u_i(\sigma_i, \underline{\sigma}_{-i} \mid x) \geq \sum_{x \in I_i^j} \mu_i^j(x) \, u_i(\sigma_i', \underline{\sigma}_{-i} \mid x) .$$

The tuple (σ, μ) is sequentially rational if it is sequentially rational for every player at every information set.

The tuple (σ, μ) is also called an assessment.

Sequential rationality is a refinement of Nash equilibrium

The notion coincides with SPNE when applied to PIEFGs

Theorem: In a PIEFG, a behavioral strategy profile σ is an SPNE iff the tuple $(\sigma, \hat{\mu})$ is sequentially rational.

[In PIEFG, every information set is a singleton; $\hat{\mu}$ is the degenerate distribution at that singleton]

Equilibrium with sequential rationality

Perfect Bayesian equilibrium

An assessment (σ, μ) is a perfect Bayesian equilibrium (PBE) if for every player $i \in N$

1. μ_i is Bayesian wrt σ, and
2. σ_i is sequentially rational given σ_{-i} and μ_i

Often represented only with σ, since μ is obtained from σ.

Self-enforcing (like the SPNE) in a Bayesian way.