Games

- Cooperative (players form coalitions, utilities are defined over coalitions)
- Non-cooperative
  - Incomplete information (players do not deterministically know which game they're playing)
  - Complete information
    - Normal form representation, appropriate for simultaneous move
      Equilibrium notions: SDSE, WDSE, PSNE, MSNE, Correlated
    - Extensive form representation, appropriate for multistage games
      Equilibrium notions: SPNE (PIEFG), mixed and behavioral strategies (IIEFG), PBE

Other types of games: repeated, stochastic, etc.

Games with complete information -

Players deterministically know the game they are playing

- there can be some chance moves but those probabilities are known

Games with incomplete information

Players do not deterministically know which game they are playing

- players receive private signals / types
- To discuss: special subclass: games with incomplete information with common priors (Harsanyi 1967) - Bayesian Games

Example: Soccer game - two competing teams

Each can choose a gameplan: aim to WIN or aim to DRAW

We will call the gameplan as their type

these are private signals to them, often caused by external factors, e.g., weather condition, player injury, ground condition etc.

- there are four possible type profiles in this example WW, WD, DW, DD. The payoff matrices differ.

WW

	Attack	Defence
Attack	1,1	2,0
Defence	0,2	0,0

WD [DW is symmetrically opposite]

	Attack	Defence
Attack	2,0	2,1
Defence	0,1	1,0

DD

	Attack	Defence
Attack	0,0	1,0
Defence	0,1	-1,-1

Assumptions: ① The probabilities of choosing the different games (or the type profiles) come from a common prior distribution.

② The common prior is a common knowledge

Definition: A Bayesian game is represented by

$$\langle N, (\Theta_i)_{i\in N}, P, (\Gamma_\theta)_{\theta \in \times_{i\in N} \Theta_i} \rangle,$$

where N: set of players, Θ_i: set of types of player i, P: common prior distribution over $\Theta = \times_{i\in N} \Theta_i$, with the restriction that

$\sum_{\theta_{-i}\in\Theta_{-i}} P(\theta_i, \theta_{-i}) > 0, \forall \theta_i \in \Theta_i, \forall i \in N$, i.e., marginals for every type is positive (otherwise we can prune the type set)

Γ_θ: NFG for the type profile $\theta \in \Theta$, i.e.,

$$\Gamma_\theta = \langle N, (A_i(\theta))_{i\in N}, (u_i(\theta))_{i\in N} \rangle$$ [we assume $A_i(\theta) = A_i, \forall \theta$]

$u_i : A \times \Theta \to \mathbb{R}$, $A = \times_{i\in N} A_i$.

Stages of a Bayesian game

1. $\theta = (\theta_i, \theta_{-i})$ is chosen randomly according to P
2. Each player observes her own type θ_i
3. Player i picks action $a_i \in A_i$, $\forall i \in N$.
4. Player i's payoff realizes $u_i(a_i, a_{-i}; \theta_i, \theta_{-i})$.