GS Theorem holds for unrestricted preferences

$$f : \mathcal{P}^n \longrightarrow A$$

↑ all preferences admissible

One reason for a restrictive result like GS theorem is that the domain of the SCF is large – a potential manipulator has many options to manipulate.

Strategyproofness (defined alternatively)

$$f(P_i, \underline{P}_{-i}) \; P_i \; f(P_i', \underline{P}_{-i}), \; \forall P_i, P_i' \in \mathcal{P}, \forall i \in N$$

$$\text{OR} \quad f(P_i, \underline{P}_{-i}) = f(P_i', \underline{P}_{-i}). \qquad \forall \underline{P}_{-i} \in \mathcal{P}^{n-1}$$

If we reduce the set of feasible preferences from $\mathcal{P}$ to $\mathcal{S} \subset \mathcal{P}$

The SCF f strategyproof on $\mathcal{P}$ continues to be strategyproof over $\mathcal{S}$, but there can potentially be more f's that can be strategyproof, i.e., satisfy the condition above.

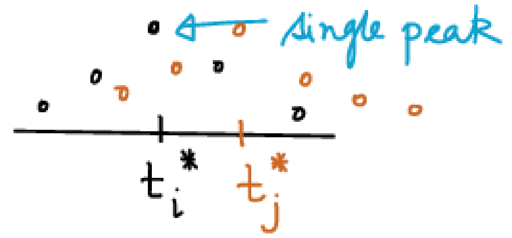
Domain restrictions

1. Single peaked preferences
2. Divisible goods allocation
3. Quasi-linear preferences

Each of these domains have interesting non-dictatorial SCFs that are strategyproof.

Single peaked preferences

Ex. temperature of a room – for every agent, most comfortable temperature t_i^* – anything above or below are monotonically less preferred.



One common order over the alternatives

Agent preferences are single peaked wrt that common order

Other examples:

① Facility location: School / Hospital / Post office

② Political ideology: Left, Center, Right

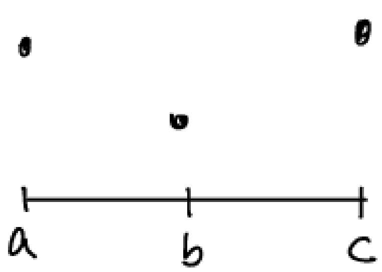
The natural ordering (or common ordering) of the alternatives is denoted via $<$ [as in real numbers]

in general, any relation over the alternatives that is transitive and antisymmetric. In this course, we will assume ① alternatives live on a real line

② consider only one-dimensional single-peakedness

How is it a domain restriction?

Consider $a < b < c$



all possible preferences

a	b	b	c	a	c
b	a	c	b	c	a
c	c	a	a	b	b

<u>Defn</u>. A preference ordering P_i (linear over A) of agent i is single-peaked wrt the common order $<$ of the alternatives if ① $\forall\, b, c \in A$ with $b < c \leq P_i(1)$, $c\, P_i\, b$

② $\forall\, b, c \in A$ with $P_i(1) \leq b < c$, $b\, P_i\, c$.

Let $\mathcal{S}$ be the set of single peaked preferences

The SCF : $f : \mathcal{S}^n \to A$.

How does it circumvent GS theorem?

Each player's preference has a peak. Suppose, f picks the leftmost peak. For the agent having the leftmost peak, no reason to misreport. For any other agent, the only way she can change the outcome is by reporting her peak to be left of the leftmost – but that is strictly worse than the current outcome.

Repeat this argument for any fixed k-th peak from left. Even the rightmost peak choosing SCF is also strategyproof, so is median $\left(k = \left\lceil \frac{n}{2} \right\rceil \right)$