Mechanism Design with Transfers

Social Choice Function \( F : \Theta \rightarrow \mathcal{X} \)

\( \mathcal{X} \) : space of all outcomes

In this domain, an outcome \( \mathcal{X} \) has two components

allocation \( a \) and payment vector \( \pi = (\pi_1, \ldots, \pi_n), \pi_i \in \mathbb{R} \)

Examples of allocations

1. A public decision of building a bridge, park, or museum.
   \( a \in A = \{ \text{park, bridge, } \ldots \} \)

2. Allocation of a divisible good, e.g., a shared spectrum
   \( a = (a_1, a_2, \ldots, a_n), a_i \in [0, 1], \sum_{i \in \mathbb{N}} a_i = 1 \)
   \( a_i \) : fraction of the resource \( i \) gets.

3. Single indivisible object allocation
   \( a = (a_1, \ldots, a_n), a_i \in \{0, 1\}, \sum_{i \in \mathbb{N}} a_i \leq 1 \)

4. Partition of indivisible objects
   \( S \) : set of objects
   \( A = \{ (A_1, \ldots, A_n) : A_i \subseteq S \forall i \in \mathbb{N}, A_i \cap A_j = \emptyset \forall i \neq j \} \)

Type of an agent \( i \) is \( \Theta_i \in \Theta_i \), this is a private information of \( i \).

Agents' benefit from an allocation is defined via the valuation function

Valuation depends on the allocation and type of the player

\( v_i : A \times \Theta_i \rightarrow \mathbb{R} \) [independent private values]
E.g., if \( i \) has a type “environment saver” \( \Theta_i^{\text{env}} \) and \( a \in \{ \text{Bridge, Park} \} \), then \( v_i(B, \Theta_i^{\text{env}}) < v_i(P, \Theta_i^{\text{env}}) \). The value can change if the type changes to “business friendly” \( \Theta_i^{\text{bus}} \):

\[
v_i(B, \Theta_i^{\text{bus}}) > v_i(P, \Theta_i^{\text{bus}})
\]

Payments \( \pi_i \in \mathbb{R} \), \( \forall i \in \mathbb{N} \)

Payment vector \( \Pi = (\pi_1, \pi_2, \ldots, \pi_n) \)

Utility of player \( i \), when its type is \( \Theta_i \) and the outcome is \((a, \pi)\) is given by

\[
U_i((a, \pi), \Theta_i) = v_i(a, \Theta_i) - \pi_i
\]

possibly non-linear in payment

Quasi-linear payoffs

Q: Why is this a domain restriction?

Consider two alternatives \((a, \pi)\) and \((a, \pi')\)

Suppose \( \pi'_i < \pi_i \), there cannot be any preference profile in the quasi-linear domain where \((a, \pi)\) is more preferred than \((a, \pi')\) for agent \( i \).

The utilities are \( v_i(a, \Theta_i) - \pi'_i > v_i(a, \Theta_i) - \pi_i \).

In the complete domain both orders would have been feasible.

This simple restriction opens up the opportunity for a lot of SCFs to satisfy interesting properties.