Generalization of VCG mechanism

Need: incorporate a larger class of DSIC mechanisms in the quasi-linear domain.

Affine Maximizer Allocation Rule

$$f^{AM}(\theta) \in \underset{a \in A}{\arg\max} \left(\sum_{i \in N} w_i \theta_i(a) + K(a) \right)$$

where $w_i \geqslant 0 \;\forall i \in N$, not all zero – different weights for players

$K : A \to \mathbb{R}$ is any arbitrary function – translation

Special cases: $K \equiv 0$ and (i) $w_i = 1 \;\forall i \in N$ (Efficient)

(ii) $w_d = 1$ and $w_i = 0 \;\forall i \neq d$ (Dictatorial)

w_i's are different $\Rightarrow$ not ANON

K is a non-constant function $\Rightarrow$ different importance is given to different allocations

- AM is a superclass of VCG/efficient allocations. Hence it can satisfy more properties.
- We can ask a characterization question (like GS theorem) in the quasi-linear setting with public goods.

Defn: An AM rule f^{AM} with weights w_i, $i \in N$ and the function K satisfies independence of non-influential agents (INA) if for all $i \in N$ with $w_i = 0$ we have $f^{AM}(\theta_i, \underline{\theta}_{-i}) = f^{AM}(\theta_i', \underline{\theta}_{-i})$, $\forall \theta_i, \theta_i', \underline{\theta}_{-i}$.

Remark: this is a tie-breaking requirement. The weight zero agent does not influence the allocation decision, hence it should not break any tie either.

If INA was not satisfied, then the AM can be manipulated.
E.g., suppose there is a tie when $w_i = 0$ for some valuation profile, but the allocation is the less preferred one for agent i.

Theorem: An AM rule satisfying INA is implementable in dominant strategies.

Proof sketch: we need to construct a payment function to make (f^{AM}, p^{AM}) DSIC. Consider

$$p_i^{AM}(\theta_i, \underline{\theta}_{-i}) = \begin{cases} \frac{1}{w_i}\left[h_i(\underline{\theta}_{-i}) - \left(\sum_{j \neq i} w_j \theta_j \left(f^{AM}(\theta)\right) + K\left(f^{AM}(\theta)\right) \right) \right] & \forall i : w_i > 0 \\ 0 , & \forall i : w_i = 0 . \end{cases}$$

Payoff of i if $w_i > 0$

$$\theta_i\left(f^{AM}(\theta_i, \underline{\theta}_{-i})\right) - p_i^{AM}(\theta_i, \underline{\theta}_{-i})$$

$$= \frac{1}{w_i}\left[\left(\sum_{j \in N} w_j \theta_j \left(f^{AM}(\theta_i, \underline{\theta}_{-i})\right) + K\left(f^{AM}(\theta_i, \underline{\theta}_{-i})\right) \right) - h_i(\underline{\theta}_{-i}) \right]$$

$$\geqslant \frac{1}{w_i}\left[\left(\sum_{j \in N} w_j \theta_j \left(f^{AM}(\theta_i', \underline{\theta}_{-i})\right) + K\left(f^{AM}(\theta_i', \underline{\theta}_{-i})\right) \right) - h_i(\underline{\theta}_{-i}) \right]$$

$$= \theta_i\left(f^{AM}(\theta_i', \underline{\theta}_{-i})\right) - p_i^{AM}(\theta_i', \underline{\theta}_{-i})$$

for i where $w_i = 0$, the payments are zero and

$$f^{AM}(\theta_i, \underline{\theta}_{-i}) = f^{AM}(\theta_i', \underline{\theta}_{-i}) \quad \forall \theta_i, \theta_i', \underline{\theta}_{-i}$$

hence the payoffs are identical for all types of i.

Similar to GS theorem, we ask what if the valuations are unrestricted.

$\theta_i : A \to \mathbb{R}$; Θ_i contains all such valuation functions, no restriction on the functions is imposed.

With this unrestricted space of valuations, we can characterize the class of DSIC mechanisms in the quasi-linear domain.

Theorem (Roberts 1979)

Let A be finite with $|A| \geq 3$. If the type space is unrestricted, then every ONTO and dominant strategy implementable allocation rule must be an affine maximizer.

Similarity with GS theorem: There it is restricting the class to dictatorships here to affine maximizers.

Restricted domains are open research domains.

Proof reference: Lavi, Mualem, Nisan (2009): Two Simplified Proofs of Roberts' theorem.