Examples of optimal mechanisms

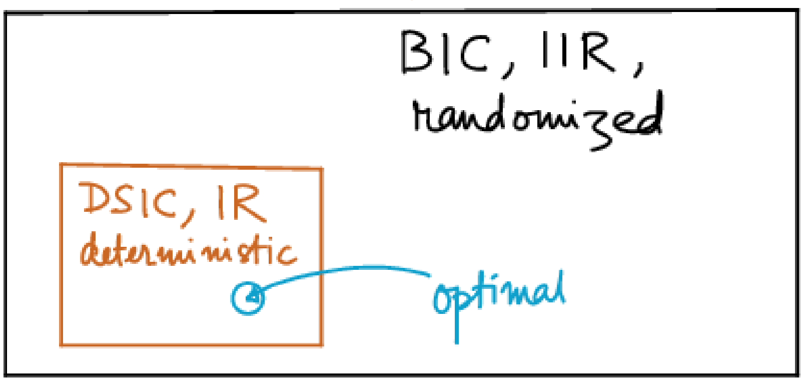
Optimal mechanism design problem:

$$\max \int_T \left(\sum_{i \in N} w_i(t_i)\, f_i(t) \right) g(t)\, dt \quad \text{s.t. } f \text{ is NDE.}$$

Solution for regular w_i's:

$$f_i(t) = \begin{cases} 1 & \text{if } w_i(t_i) \geq w_j(t_j) \ \forall j \\ 0 & \text{ow} \end{cases}$$

We wanted to find an allocation that is NDE, but found an f that is non-decreasing. Also, it is deterministic.



Space of regular virtual valuations

Theorem: Suppose every agent's valuation is regular. Then, for every type profile t,

if $w_i(t_i) < 0 \ \forall i \in N$, $f_i(t) = 0 \ \forall i \in N$.

otherwise, $f_i(t) = \begin{cases} 1 & \text{if } w_i(t_i) \geq w_j(t_j) \ \forall j \in N \\ 0 & \text{ow} \end{cases}$

ties are broken arbitrarily. Payments are given by

$$p_i(t) = \begin{cases} 0 & \text{if } f_i(t) = 0 \\ \max\{ w_i^{-1}(0), K_i^*(t_{-i}) \} & \text{if } f_i(t) = 1 \end{cases}$$

Then (f, p) is an optimal mechanism.

$w_i^{-1}(0)$: The value of t_i where $w_i(t_i) = 0$.

$K_i^*(t_{-i}) = \inf\{t_i : f_i(t_i, t_{-i}) = 1\}$

The minimum value of t_i where i begins to be the winner

<u>Example 1</u>: Two buyers : $T_1 = [0, 12]$, $T_2 = [0, 18]$

Uniform, independent prior.

$$w_1(t_1) = t_1 - \frac{1 - G_1(t_1)}{g_1(t_1)} = t_1 - \frac{1 - \frac{t_1}{12}}{\frac{1}{12}} = 2t_1 - 12$$

$$w_2(t_2) = 2t_2 - 18$$

t_1	t_2	action	p_1	p_2
4	8	unsold	0	0
2	12	sold to 2	0	9
6	6	sold to 1	6	0
9	9	sold to 1	6	0
8	15	sold to 2	0	11

<u>Example 2</u>: Symmetric bidders: The valuations are drawn from the same distribution, $g_i = g$, $T_i = T$, $\forall i \in N$

virtual valuation : $w_i = w$.

$w(t_i) > w(t_j)$ iff $t_i > t_j$

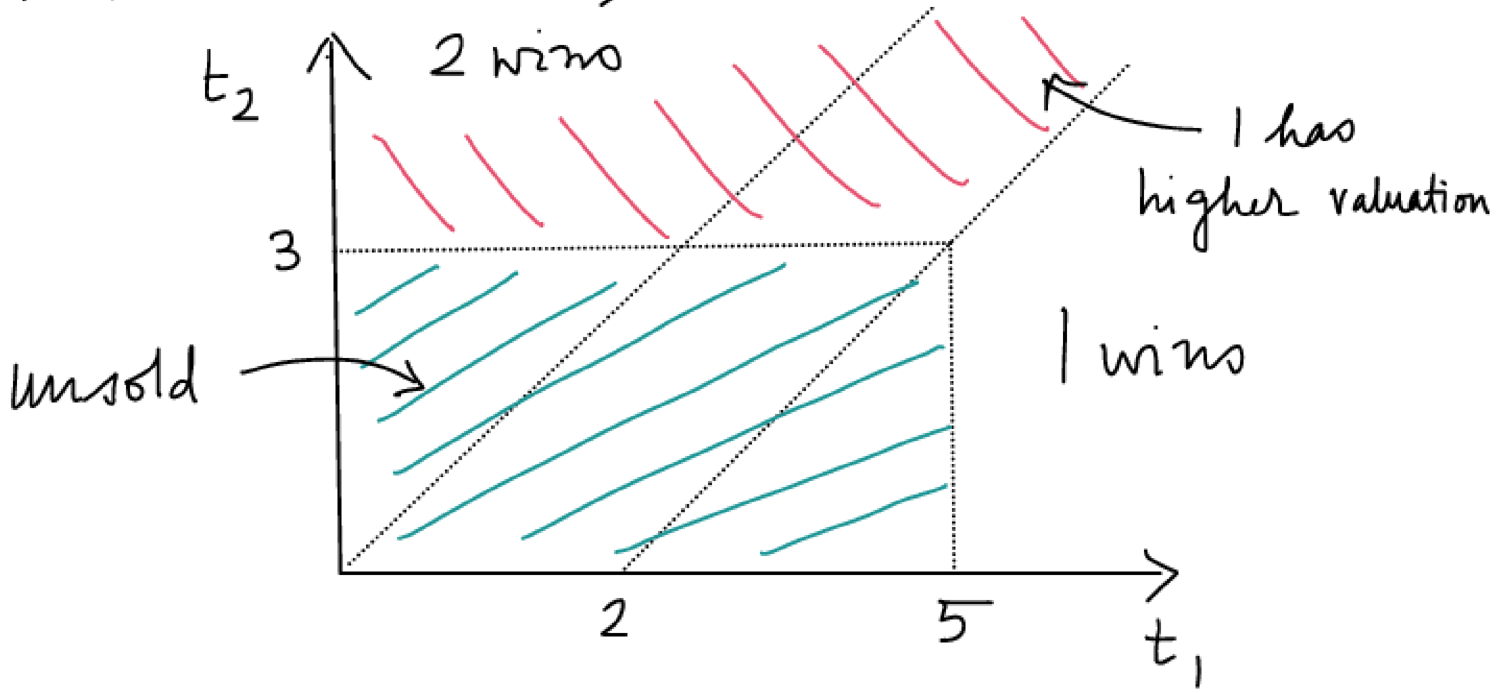
the object goes to the highest bidder. Not sold if $w^{-1}(0) > t_i$ $\forall i \in N$. $\text{Payment}_i = \max\{w^{-1}(0), \max_{j \neq i} t_j\}$

Second price auction with a reserve price, and is efficient when the object is sold.

Example 3: Efficiency and Optimality

$T_1 = [0, 10]$, $T_2 = [0, 6]$, uniform, independent prior

$w_1(t_1) = 2t_1 - 10$, $w_2(t_2) = 2t_2 - 6$



Unsold is inefficient, also in the region of the plane.