Addendum to transform Bayesian game to complete information NFG.

\[ N = \bigcup_{i \in N} \Theta_i = \{ \theta_1, \theta_2, \ldots, \theta_1^{G_1}, \ldots \} \]

**New player set**

Consider two players, type sets \( \Theta = \{ \theta_1, \theta_1^2 \} \), \( \Theta = \{ \theta_2, \theta_2^2 \} \)

original payoffs \( u_i \) Bayesian game

utility of player \( \theta_1 \)

\[
\bar{u}_{\theta_1}(a_{\theta_1}, a_{\theta_1^2}, a_{\theta_2}, a_{\theta_2^2}) = P(\theta_2^2 | \theta_1^1) u_1(a_{\theta_1}, a_{\theta_2^2}, \theta_1^1, \theta_2^2) + P(\theta_2 | \theta_1) u_1(a_{\theta_1}, a_{\theta_2}, \theta_1^1, \theta_2^2)
\]

[defining \( a_{\theta_i} = a_i(\theta_i) \), \( a_{\theta_i}^2 = a_i(\theta_i^2) \) etc.]

consider a mixed strategy \( (\sigma_1, \sigma_2, \sigma_1^2, \sigma_2^2) \) in this new game

\[
\bar{u}_{\theta_1}(\sigma_1, \sigma_2, \sigma_2^2, \sigma_2^2) = \sum_{a_{\theta_1}^1 \in A_1} \sum_{a_{\theta_1}^2 \in A_2} \sum_{a_{\theta_2}^1 \in A_1} \sum_{a_{\theta_2}^2 \in A_2} \sigma_1^1(a_{\theta_1}^1) \sigma_1^2(a_{\theta_1}^2) \sigma_2^1(a_{\theta_2}^1) \sigma_2^2(a_{\theta_2}^2) \times \bar{u}_{\theta_1}(a_{\theta_1}, a_{\theta_2}, a_{\theta_2}, a_{\theta_2})
\]

now plug this into from 1, irrelevant \( a_{\theta_i} \) terms will sum to 1

\[
= \sum_{a_{\theta_1} \in A_2} P(\theta_2 | \theta_1) \sigma_2^1(a_{\theta_2}) u_1(a_{\theta_1}, a_{\theta_2}, \theta_1, \theta_2)
\]

\[
= \sum_{a_{\theta_2} \in A_2} P(\theta_2 | \theta_1) \sigma_2^2(a_{\theta_2}) u_1(a_{\theta_1}, a_{\theta_2}, \theta_1, \theta_2)
\]

Hence a mixed strategy in the complete information game is a mixed strategy \( (\sigma_1, \sigma_2) \) in the Bayesian game.

It follows that the MSNE in that game will be a BE in this game.