CS 405/6001: Game Theory and Algorithmic Mechanism Design **Jul–Nov 2026**

Lecture 22: Myerson's Lemma and Optimal Mechanism Design

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Disclaimer: *These notes aggregate content from several texts and have not been subjected to the usual scrutiny deserved by formal publications. If you find errors, please bring to the notice of the Instructor.*

22.1 Recap

Last lecture we saw that the Vickrey utility is convex, and its slope is the win probability. We also reviewed convex functions. Two lemmas from that review do all the work today. Let $g : I \to \mathbb{R}$ be convex and $\phi : I \to \mathbb{R}$ satisfy $\phi(z) \in \partial g(z)$ for all $z \in I$.

Lemma 22.1 *For all $x, y \in I$ with $x > y$, we have $\phi(x) \geqslant \phi(y)$.*

Lemma 22.2 *For any $x, y \in I$, $g(x) = g(y) + \int_y^x \phi(z)\, dz$.*

The plan: show that DSIC makes an agent's utility convex in her type, with her win probability as a subgradient.

22.2 Single Object Allocation

We keep the setting of the previous lecture. Agent i has type space $T_i = [0, b_i]$. An allocation rule $f : T \to \Delta A$ gives each agent i a probability $f_i(t)$ of receiving the object at type profile $t = (t_1, \ldots, t_n)$, and $f_0(t)$ is the probability of not selling. Each agent has a payment $p_i(t)$. Valuations are in the product form, so agent i's utility when she reports s_i and her true type is t_i is

$$u_i(s_i; t_i, t_{-i}) = t_i f_i(s_i, t_{-i}) - p_i(s_i, t_{-i}).$$

Definition 22.3 (Non-decreasing allocation rule) *An allocation rule f is **non-decreasing** if for every agent $i \in N$ and every $t_{-i} \in T_{-i}$,*

$$f_i(t_i, t_{-i}) \geqslant f_i(s_i, t_{-i}), \quad \forall s_i, t_i \in T_i \text{ with } t_i > s_i.$$

In words, holding the others' types fixed, the probability of getting the object never decreases as an agent's valuation increases.

22.3 Myerson's Lemma

The following theorem, due to Myerson [Mye81], characterizes DSIC mechanisms in the single parameter domain.

Theorem 22.4 (Myerson's Lemma) *Suppose $T_i = [0, b_i]$ for all $i \in N$, and the valuations are in the product form. An allocation rule $f : T \to \Delta A$ and a payment rule $(p_1, \ldots, p_n)$ form a DSIC mechanism if and only if*

1. *the allocation rule f is non-decreasing, and*
2. *the payments satisfy*

$$p_i(t_i, t_{-i}) = p_i(0, t_{-i}) + t_i f_i(t_i, t_{-i}) - \int_0^{t_i} f_i(x, t_{-i})\, dx, \quad \forall i \in N,\ t_i \in T_i,\ t_{-i} \in T_{-i}. \tag{22.1}$$

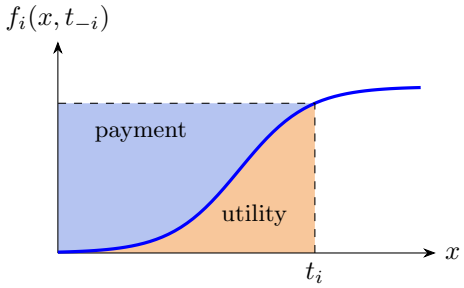


Figure 22.1: Myerson's payment with $p_i(0, t_{-i}) = 0$. The rectangle $t_i f_i(t_i, t_{-i})$ splits into the payment (blue, above the curve) and the utility (orange, the area under the curve).

Intuition. The lemma ties incentive compatibility to two conditions. Monotonicity: a higher valuation gives a weakly higher chance of winning. The payment formula: given the allocation curve, the payment is pinned down up to the constant $p_i(0, t_{-i})$. As Figure 22.1 shows, the agent keeps exactly the area under her allocation curve, and pays the rest of the rectangle.

Example 22.5 (The spot-GPU lottery) *A cloud provider allocates a spare GPU by a lottery that depends on the bids. Fix everyone else's bid. Your chance of winning, as a function of your own bid, is the blue curve. Myerson's lemma says the curve alone fixes what you pay. The provider's only remaining freedom is the constant $p_i(0, t_{-i})$.*

Remark 22.6 (Difference with Roberts' theorem) *Roberts' result gives a functional form (affine maximizer), while Myerson's result is a more implicit property: it characterizes mechanisms through monotonicity and a payment structure. Sometimes functional forms help answer questions more directly.*

22.3.1 Proof: forward direction

Assume (f, p) is DSIC. Fix t_{-i}, and write $u_i(t_i, t_{-i}) = t_i f_i(t_i, t_{-i}) - p_i(t_i, t_{-i})$ for the truthful utility at type t_i. DSIC says the agent with type t_i does not gain by reporting s_i:

$$\begin{aligned} u_i(t_i, t_{-i}) = t_i f_i(t_i, t_{-i}) - p_i(t_i, t_{-i}) &\geqslant t_i f_i(s_i, t_{-i}) - p_i(s_i, t_{-i}) \\ &= s_i f_i(s_i, t_{-i}) + (t_i - s_i) f_i(s_i, t_{-i}) - p_i(s_i, t_{-i}) \\ &= u_i(s_i, t_{-i}) + (t_i - s_i) f_i(s_i, t_{-i}). \end{aligned}$$

Define $g(t_i) = u_i(t_i, t_{-i})$ and $\phi(t_i) = f_i(t_i, t_{-i})$. The inequality reads

$$g(t_i) \geqslant g(s_i) + \phi(s_i)(t_i - s_i), \quad \forall s_i, t_i \in T_i. \tag{22.2}$$

This says that $\phi(s_i)$ is a subgradient of g at s_i, provided g is convex. So it remains to show that g is convex.

Pick $x_i, z_i \in T_i$, and let $y_i = \lambda x_i + (1-\lambda) z_i$ with $\lambda \in [0,1]$. Applying (22.2) at $s_i = y_i$,

$$g(x_i) \geqslant g(y_i) + \phi(y_i)(x_i - y_i), \qquad g(z_i) \geqslant g(y_i) + \phi(y_i)(z_i - y_i).$$

Multiply the first by λ, the second by $1-\lambda$, and add:

$$\lambda g(x_i) + (1-\lambda) g(z_i) \geqslant g(y_i) + \phi(y_i)\big(\lambda x_i + (1-\lambda) z_i - y_i\big) = g(\lambda x_i + (1-\lambda) z_i).$$

So g is convex, and $\phi(t_i) \in \partial g(t_i)$ for all t_i. Now apply the two lemmas.

- By Lemma 22.1, $\phi = f_i(\cdot, t_{-i})$ is non-decreasing. This proves part 1.
- By Lemma 22.2,

$$g(t_i) = g(0) + \int_0^{t_i} \phi(x)\,dx \implies t_i f_i(t_i, t_{-i}) - p_i(t_i, t_{-i}) = -p_i(0, t_{-i}) + \int_0^{t_i} f_i(x, t_{-i})\,dx$$
$$\implies p_i(t_i, t_{-i}) = p_i(0, t_{-i}) + t_i f_i(t_i, t_{-i}) - \int_0^{t_i} f_i(x, t_{-i})\,dx.$$

  This proves part 2.

22.3.2 Proof: reverse direction

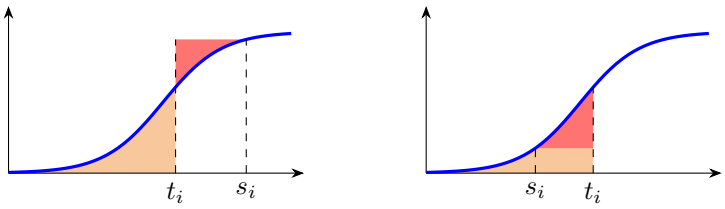
Assume f is non-decreasing and the payment satisfies (22.1). The utility of agent i with true type t_i who reports s_i is

$$t_i f_i(s_i, t_{-i}) - p_i(s_i, t_{-i}) = (t_i - s_i) f_i(s_i, t_{-i}) + \int_0^{s_i} f_i(x, t_{-i})\,dx - p_i(0, t_{-i}).$$

At $s_i = t_i$ this is $\int_0^{t_i} f_i(x, t_{-i})\,dx - p_i(0, t_{-i})$. Subtracting, the gain from telling the truth is

$$[t_i f_i(t_i, t_{-i}) - p_i(t_i, t_{-i})] - [t_i f_i(s_i, t_{-i}) - p_i(s_i, t_{-i})] = (s_i - t_i) f_i(s_i, t_{-i}) + \int_{s_i}^{t_i} f_i(x, t_{-i})\,dx. \tag{22.3}$$

If $s_i < t_i$, the integral is over $[s_i, t_i]$ where $f_i(x, t_{-i}) \geqslant f_i(s_i, t_{-i})$, so the integral is at least $(t_i - s_i) f_i(s_i, t_{-i})$, and (22.3) is non-negative. If $s_i > t_i$, (22.3) equals $\int_{t_i}^{s_i} \big(f_i(s_i, t_{-i}) - f_i(x, t_{-i})\big)\,dx$, which is non-negative since $f_i(x, t_{-i}) \leqslant f_i(s_i, t_{-i})$ on $[t_i, s_i]$. So truthful reporting is a dominant strategy.



Overbid ($s_i > t_i$) Underbid ($s_i < t_i$)

Figure 22.2: Proof by picture, with $p_i(0, t_{-i}) = 0$. Orange: the utility from the misreport. Red: the utility lost relative to truth-telling, which is (22.3). The red area is non-negative because the curve is non-decreasing.

Figure 22.2 gives the same argument as a picture. An overbidder pays for extra win chance that she values less than its price. An underbidder gives up win chance that was cheap for her. Either way she loses the red area.

Corollary 22.7 *An allocation rule in the single object setting is implementable in dominant strategies if and only if it is non-decreasing.*

Remark 22.8 *Suppose we train an agent to bid against a Myerson mechanism. Its optimal policy is the identity map, bid your value, whatever the opponents do. No model of the opponents is needed, and the cognitive load is minimal.*

22.4 Examples of Single Object Allocation

We now check several allocation rules against Corollary 22.7.

1. **Constant allocation rule.** The object is allocated with fixed probabilities, independent of bids (for example, a hackathon gives a GPU to a uniformly random team). Then $f_i(t_i, t_{-i})$ is constant, hence non-decreasing, and the payment is a constant (for example, 0).

2. **Dictatorial allocation.** The object always goes to a designated dictator (the founder's pet project always gets the GPU): $f_i(t) = 1$ if i is the dictator and 0 otherwise. The allocation does not depend on t_i, so it is non-decreasing, and the payment is a constant (for example, 0).

3. **Second price (Vickrey) auction.** The highest bidder wins and pays the second-highest bid. With $p_i(0, t_{-i}) = 0$, the allocation curve of agent i is a step from 0 to 1 at $t_{-i}^{(2)}$. If $t_i > t_{-i}^{(2)}$,

$$p_i(t_i, t_{-i}) = 0 + t_i \cdot 1 - \int_0^{t_i} f_i(x, t_{-i})\, dx = t_i - (t_i - t_{-i}^{(2)}) = t_{-i}^{(2)},$$

   and the payment is 0 otherwise. Myerson's formula derives Vickrey's price.

4. **Efficient allocation with a reserve price.** Let r be the reserve price (an auction with a floor price). If the highest value is below r, nobody gets the object. Otherwise it goes to the highest bidder. So i wins if $t_i > \max\{t_{-i}^{(2)}, r\}$, and pays $\max\{t_{-i}^{(2)}, r\}$. The rule is non-decreasing in each t_i.

5. **A not so common allocation rule.** Let $N = \{1, 2\}$ and $A = \{a_0, a_1, a_2\}$. Given $t = (t_1, t_2)$, the seller computes

$$m(t) = \max\{2, t_1^2, t_2^3\},$$

   and selects a_0, a_1 or a_2 according to which of the three terms is the maximum, breaking ties in favor of $0 > 1 > 2$. Player 1 gets the object if $t_1 > \sqrt{\max\{2, t_2^3\}}$, and player 2 gets it if $t_2 > \sqrt[3]{\max\{2, t_1^2\}}$. Fixing t_2, the allocation of player 1 is a step at the threshold $\sqrt{\max\{2, t_2^3\}}$, so it is non-decreasing; similarly for player 2. The three regions are separated by the curves where two terms of the maximum are equal: $t_1 = \sqrt{2}$ (between a_0 and a_1), $t_2 = \sqrt[3]{2}$ (between a_0 and a_2), and $t_2^3 = t_1^2$, i.e., $t_2 = t_1^{2/3}$ (between a_1 and a_2). The first two are straight lines; the third is a curve (Figure 22.3). By Corollary 22.7, this rule is DSIC with a suitable payment. With $p_i(0, t_{-i}) = 0$, the winner pays her threshold, exactly as in the Vickrey case.

22.5 Individual Rationality (Ex-post)

Definition 22.9 *A mechanism (f, p) is **ex-post individually rational (IR)** if*

$$t_i f_i(t_i, t_{-i}) - p_i(t_i, t_{-i}) \geqslant 0, \quad \forall t_i \in T_i,\ t_{-i} \in T_{-i},\ \forall i \in N.$$

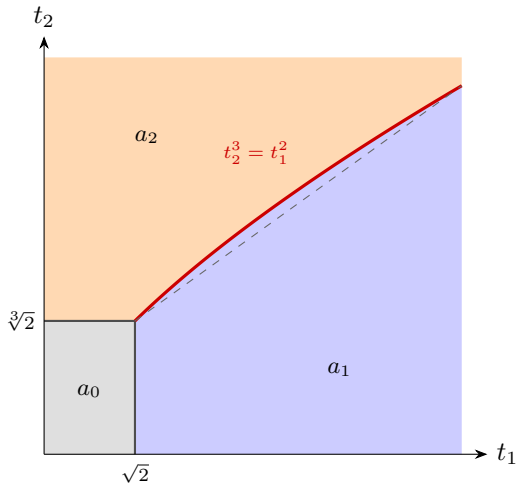


Figure 22.3: Regions of the rule in Example 5. The boundary $t_2 = t_1^{2/3}$ between a_1 and a_2 is curved; the dashed chord shows the bend. Moving right (raising t_1) never leaves region a_1 once inside it; moving up (raising t_2) never leaves region a_2.

Ex-post means the condition holds for every realization of all the types: even after all agents have revealed their types, participating is weakly preferred. Nobody regrets having walked into the auction.

Lemma 22.10 *In the single object allocation setting, consider a DSIC mechanism (f, p).*

1. *It is IR if and only if $p_i(0, t_{-i}) \leqslant 0$ for all $i \in N$ and $t_{-i} \in T_{-i}$.*
2. *It is IR and satisfies **no subsidy**, i.e., $p_i(t_i, t_{-i}) \geqslant 0$ for all t_i, t_{-i}, i, if and only if $p_i(0, t_{-i}) = 0$ for all $i \in N$ and $t_{-i} \in T_{-i}$.*

Proof: Part 1. If (f, p) is IR, then at $t_i = 0$ we get $0 - p_i(0, t_{-i}) \geqslant 0$, so $p_i(0, t_{-i}) \leqslant 0$. Conversely, if $p_i(0, t_{-i}) \leqslant 0$, then by Myerson's payment formula the payoff of i is

$$t_i f_i(t_i, t_{-i}) - p_i(t_i, t_{-i}) = -p_i(0, t_{-i}) + \int_0^{t_i} f_i(x, t_{-i})\, dx \geqslant 0.$$

Part 2. IR gives $p_i(0, t_{-i}) \leqslant 0$, and no subsidy at $t_i = 0$ gives $p_i(0, t_{-i}) \geqslant 0$. So $p_i(0, t_{-i}) = 0$. Conversely, if $p_i(0, t_{-i}) = 0$, Part 1 gives IR, and the payment formula gives $p_i(t_i, t_{-i}) = t_i f_i(t_i, t_{-i}) - \int_0^{t_i} f_i(x, t_{-i})\, dx \geqslant 0$, since f_i is non-decreasing. So there is no subsidy. ■

So $p_i(0, t_{-i})$ is a lever. A negative value is a gift to the bidder, and keeps IR. A positive value is an entry fee, and breaks IR. The two auctions below use this gift to send money back to the bidders.

22.6 Non-Vickrey Mechanisms

Both examples allocate efficiently but are not the Vickrey auction. Both are DSIC, since they fit Myerson's lemma. Assume without loss of generality that $t_1 > t_2 > \cdots > t_n$.

Example 1: a deterministic redistribution mechanism. (A hackathon gives one GPU cluster to one of n teams, and wants the money to stay with the teams.) The object goes to the highest bidder. Every agent receives $\frac{1}{n}$ of the second highest bid among the others:

$$p_i(0, t_{-i}) = -\frac{1}{n} \times \big(\text{second highest bid in } \{t_j : j \neq i\}\big).$$

For agents 1 and 2 this is $-\frac{1}{n}t_3$, and for every other agent it is $-\frac{1}{n}t_2$. By Myerson's formula,

- agent 1 pays $-\frac{1}{n}t_3 + t_1 - \int_0^{t_1} f_1(x, t_{-1})\,dx = -\frac{1}{n}t_3 + t_1 - (t_1 - t_2) = t_2 - \frac{1}{n}t_3$;
- agent 2 pays $-\frac{1}{n}t_3$, and every other agent pays $-\frac{1}{n}t_2$.

The total payment is

$$t_2 - \frac{1}{n}t_3 - \frac{1}{n}t_3 - \frac{n-2}{n}t_2 = \frac{2}{n}(t_2 - t_3),$$

which tends to 0 for large n. Each rebate depends only on the others' bids, so it is a $p_i(0, t_{-i})$ term and does not disturb truthfulness. This is a deterministic mechanism that redistributes the money.

Example 2: a randomized redistribution mechanism. (The same hackathon, but the runner-up also gets a chance.) The object goes to the highest bidder with probability $1 - \frac{1}{n}$ and to the second highest with probability $\frac{1}{n}$. As before, $p_i(0, t_{-i}) = -\frac{1}{n}\times$ (second highest bid in $\{t_j : j \neq i\}$).

Fix the others and slide agent 1's bid x upwards. For $x < t_3$, agent 1 is third or lower and never wins. For $t_3 < x < t_2$, she is the runner-up and wins with probability $\frac{1}{n}$. For $x > t_2$, she is the highest and wins with probability $1 - \frac{1}{n}$. So

- agent 1 pays $-\frac{1}{n}t_3 + \big(1 - \frac{1}{n}\big)t_1 - \frac{1}{n}(t_2 - t_3) - \big(1 - \frac{1}{n}\big)(t_1 - t_2) = \big(1 - \frac{2}{n}\big)t_2$;
- agent 2 pays $-\frac{1}{n}t_3 + \frac{1}{n}t_2 - \frac{1}{n}(t_2 - t_3) = 0$;
- every other agent pays $-\frac{1}{n}t_2$.

Together, $\big(1 - \frac{2}{n}\big)t_2 - \frac{n-2}{n}t_2 = 0$. This is a randomized mechanism that redistributes all the money.

22.7 Revenue Maximization

So far we have asked for truthfulness and individual rationality. A natural next question: among such mechanisms, which one maximizes the auctioneer's expected revenue? And maximize with respect to what knowledge of the auctioneer? The answer is the **common prior** distribution over types. Accordingly, incentive compatibility and individual rationality are replaced by their Bayesian analogues.

For example, a GPU startup that has run hundreds of past auctions cannot see today's bids, but its logs tell it how bids are distributed. That distribution is the prior.

22.8 Bayesian Setup

Let $T_i = [0, b_i]$, and let G be the common prior over $T = \times_{i=1}^{n} T_i$, with density g. Let $G_{-i}(s_{-i} \mid s_i)$ be the conditional distribution of the others' types given that i's type is s_i, and $g_{-i}(s_{-i} \mid s_i)$ the corresponding density, derived from g by Bayes' rule.

Every mechanism $(f, p_1, \ldots, p_n)$ induces an **expected allocation** and an **expected payment** rule (α, π):

$$\alpha_i(s_i; t_i) = \int_{s_{-i} \in T_{-i}} f_i(s_i, s_{-i})\, g_{-i}(s_{-i} \mid t_i)\, ds_{-i}, \qquad \pi_i(s_i; t_i) = \int_{s_{-i} \in T_{-i}} p_i(s_i, s_{-i})\, g_{-i}(s_{-i} \mid t_i)\, ds_{-i},$$

where s_i is the reported type and t_i is the true type. The expected utility of agent i is

$$u_i(s_i; t_i) = t_i \alpha_i(s_i; t_i) - \pi_i(s_i; t_i).$$

Definition 22.11 (Bayesian Incentive Compatibility) *A mechanism (f, p) is **Bayesian incentive compatible (BIC)** if for all $i \in N$ and all $s_i, t_i \in T_i$,*

$$t_i \alpha_i(t_i; t_i) - \pi_i(t_i; t_i) \geqslant t_i \alpha_i(s_i; t_i) - \pi_i(s_i; t_i).$$

Truthful reporting maximizes each agent's *expected* utility, given her belief about the others' types. Every DSIC mechanism is BIC, but not conversely.

Definition 22.12 (Bayesian implementability) *An allocation rule f is **Bayesian implementable** if there is a payment rule p such that (f, p) is BIC.*

22.9 Independent Priors and the Characterization of BIC Mechanisms

Assume the priors are **independent**: agent i's type is drawn from G_i (density g_i) independently of the others, so

$$G(s_1, \ldots, s_n) = \prod_{i \in N} G_i(s_i), \qquad G_{-i}(s_{-i} \mid t_i) = \prod_{j \neq i} G_j(s_j).$$

Then $\alpha_i(s_i; t_i)$ and $\pi_i(s_i; t_i)$ do not depend on t_i, and we use the shorthand $\alpha_i(t_i) = \alpha_i(t_i; t_i)$ and $\pi_i(t_i) = \pi_i(t_i; t_i)$.

Definition 22.13 (Non-decreasing in expectation) *An allocation rule f is **non-decreasing in expectation (NDE)** if for every $i \in N$ and all $s_i, t_i \in T_i$ with $s_i < t_i$, we have $\alpha_i(s_i) \leqslant \alpha_i(t_i)$.*

Every non-decreasing (ND) rule is NDE, since averaging a non-decreasing function keeps it non-decreasing. But some NDE rules are not ND.

Example 22.14 (NDE but not ND) *There are two agents, and t_1 and t_2 each take five equally likely values. In Figure 22.4, a cell marked 1 means agent 1 gets the object; in every other cell agent 2 gets it.*

Averaging each column over t_2,

$$\alpha_1(t_1) = 0,\ 0,\ \tfrac{1}{5},\ \tfrac{2}{5},\ \tfrac{3}{5} \quad \text{for } t_1 = 1, \ldots, 5,$$

which is non-decreasing. Averaging each row over t_1, agent 2 wins in $3, 3, 4, 4, 5$ of the five cells, so

$$\alpha_2(t_2) = \tfrac{3}{5},\ \tfrac{3}{5},\ \tfrac{4}{5},\ \tfrac{4}{5},\ 1 \quad \text{for } t_2 = 1, \ldots, 5,$$

which is also non-decreasing. So the rule is NDE. But it is not ND: in row $t_2 = 3$, agent 1 wins at $t_1 = 4$ and loses at $t_1 = 5$. Her allocation drops as her type rises.

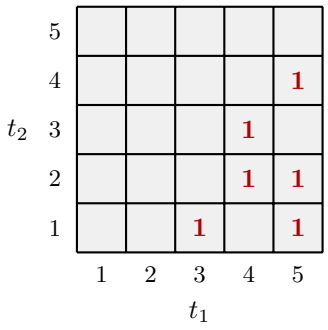


Figure 22.4: An allocation rule that is NDE but not ND.

Example 22.14 also shows that a BIC mechanism need not be DSIC. The following is the Bayesian version of Myerson's lemma.

Theorem 22.15 (Myerson 1981) *A mechanism (f, p) in the independent prior setting is BIC if and only if*

1. *f is NDE, and*
2. *the expected payments satisfy*

$$\pi_i(t_i) = \pi_i(0) + t_i\alpha_i(t_i) - \int_0^{t_i} \alpha_i(x)\,dx, \quad \forall t_i \in T_i,\ \forall i \in N.$$

The proof follows the same lines as Theorem 22.4, with $f_i(\cdot, t_{-i})$ replaced by $\alpha_i(\cdot)$ and $p_i(\cdot, t_{-i})$ by $\pi_i(\cdot)$ (exercise). The pictures in Figures 22.1 and 22.2 carry over with the expected allocation curve.

22.10 Interim Individual Rationality

In the Bayesian setting we modify individual rationality as follows.

Definition 22.16 (Interim individual rationality) *A mechanism (f, p) is **interim individually rational (IIR)** if for every agent $i \in N$,*

$$t_i\alpha_i(t_i) - \pi_i(t_i) \geqslant 0, \quad \forall t_i \in T_i.$$

Interim means the agent knows her own type but not the others'. On average over the others' types, participating is weakly better.

Lemma 22.17 *A mechanism (f, p) is BIC and IIR if and only if*

1. *f is NDE,*
2. *$\pi_i(t_i) = \pi_i(0) + t_i\alpha_i(t_i) - \int_0^{t_i} \alpha_i(s_i)\,ds_i$ for all $t_i \in T_i$ and $i \in N$, and*
3. *$\pi_i(0) \leqslant 0$ for all $i \in N$.*

Proof: By Theorem 22.15, conditions 1 and 2 are equivalent to BIC. So we must show that, given 1 and 2, IIR is equivalent to condition 3.

Forward direction. Apply condition 2 at $t_i = 0$: the expected utility is $u_i(0) = -\pi_i(0)$. IIR requires this to be non-negative, so $\pi_i(0) \leqslant 0$.

Reverse direction. By condition 2,

$$t_i \alpha_i(t_i) - \pi_i(t_i) = -\pi_i(0) + \int_0^{t_i} \alpha_i(s_i)\, ds_i.$$

The integral is non-negative since $\alpha_i \geqslant 0$, and $-\pi_i(0) \geqslant 0$ by condition 3. So the expected utility is non-negative for all t_i, and the mechanism is IIR. ■

Example 22.18 (The mining-pool entry fee) *A BIC auction for a block-building slot charges every bidder a fee upfront, even one who values the slot at* 0*. That is* $\pi_i(0) > 0$*. A bidder with value* 0 *gets expected utility* $-\pi_i(0) < 0$*, so the auction is not IIR.*

22.11 Summary

- Myerson's lemma: in the single parameter domain, (f, p) is DSIC if and only if f is non-decreasing and payments follow (22.1).
- Forward direction: DSIC makes the utility convex, with the win probability as a subgradient. Reverse direction: with a monotone curve, every misreport loses a non-negative area.
- Constant, dictatorial, Vickrey, reserve price and the threshold rule of Example 5 are all non-decreasing, hence DSIC.
- A DSIC mechanism is ex-post IR if and only if $p_i(0, t_{-i}) \leqslant 0$; adding no subsidy forces $p_i(0, t_{-i}) = 0$.
- Redistribution mechanisms use $p_i(0, t_{-i}) < 0$ to return money, while staying DSIC and efficient.
- With independent priors, BIC holds if and only if f is NDE and the expected payments follow the Bayesian payment formula. IIR additionally requires $\pi_i(0) \leqslant 0$.
- Next: the revenue-optimal mechanism.

References

[Mye81] R. B. Myerson, "Optimal auction design," *Mathematics of Operations Research*, 6(1), 1981, pp. 58–73.