भारतीय प्रौद्योगिकी संस्थान मुंबई

Indian Institute of Technology Bombay

CS 6001: Game Theory and Algorithmic Mechanism Design

Week 10

Swaprava Nath

Slide preparation acknowledgments: Onkar Borade

ज्ञानम् परमम् ध्येयम्

Knowledge is the supreme goal

Contents

The Vickrey-Clarke-Groves Mechanism (VCG)

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- Interpretation of the **payment**: **sum value of others (in absence of i – in presence of i)**

An Observation on VCG Mechanism

Utility under VCG mechanism:

$$v_i(f^{AE}(\theta_i, \theta_{-i}), \theta_i) - p_i^{VCG}(\theta_i, \theta_{-i}) = \sum_{j \in N} v_j(f^{AE}(\theta_i, \theta_{-i}), \theta_j) - \max_{a \in A} \sum_{j \neq i} v_j(a, \theta_j)$$

- Interpretation of the **utility** under VCG mechanism: marginal contribution of i in the social welfare

Examples

1. **Single Object Allocation**
   Type = value of the object if allocated, the agent get this value and zero otherwise

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   - This is **second price auction**

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   3 players having the following valuations :

	Football	Library	Museum
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- The agent whose presence *changes the outcome* is charged money
  They are the **pivotal** players

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3. Combinatorial Allocation: sale of multiple objects
3 players having the following valuations (value is the type itself $v_i(a, \theta_i) = \theta_i(a)$)

	$\emptyset$	$\{1\}$	$\{2\}$	$\{1\&2\}$
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- $p_2^{VCG}(\theta_1, \theta_2) = 12 - 6 = 6$, payoff $= 9 - 6 = 3$

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- An allocation in this case is a partition of the objects, i.e.,

$$a = \{a_0, a_1, a_2, \ldots, a_n\},\ a_i \in \Omega,\ a_i \cap a_j = \emptyset,\ \forall i \neq j$$

$$a_0 : \text{ set of unallocated objects}, \cup_{i=0}^{n} a_i = M$$

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- Also assume **selfish valuations**, i.e., $\theta_i(a) = \theta_i(a_i)$, agent i's valuation does *not* depend on the allocations of others

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- Note, $p_i^{VCG}(\theta) \geqslant 0$, also $p_i^{VCG}(\theta) = \sum_{j \neq i} \theta_j(a_{-i}^*) - \sum_{j \neq i} \theta_j(a^*)$

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- $p_i^{VCG}(\theta) = \sum_{j \in N} \theta_j(a_{-i}^*) - \sum_{j \in N} \theta_j(a^*) \leqslant 0 \implies p_i^{VCG}(\theta) = 0$

Individual Rationality in Mechanism with Transfers

Definition (Ex-post Individual Rationality)

A mechanism (f, p) is *ex-post individually rational* (ex-post IR) if
$v_i(f(\theta), \theta_i) - p_i(\theta) \geqslant 0, \forall \theta \in \Theta, \forall i \in N$

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Definition (No Negative Externality)

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efficient allocation without an agent yields non-negative value to that agent

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Corollary

VCG is ex-post IR for combinatorial allocations.

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- Small businesses can customize these ads via exchanges

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- This is the total expected payment, to convert this to the pay-per-click: $\frac{1}{p_i \hat{\rho}_i} p_i^{VCG}(b)$

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   - If 1 and 2 collude and bid higher, both of them reduce their payments $\implies$ utility increases

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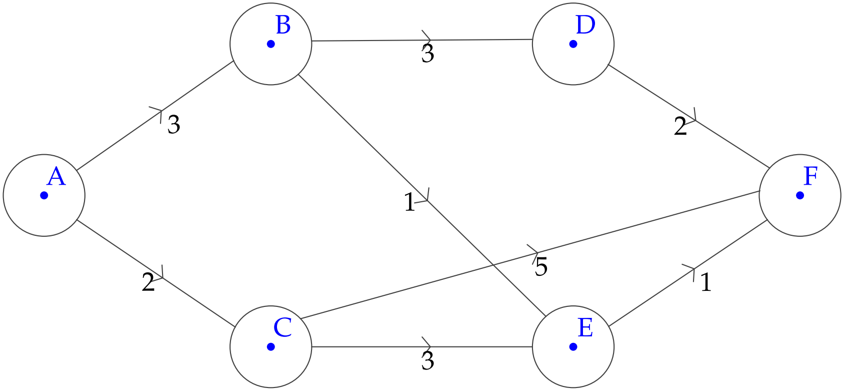

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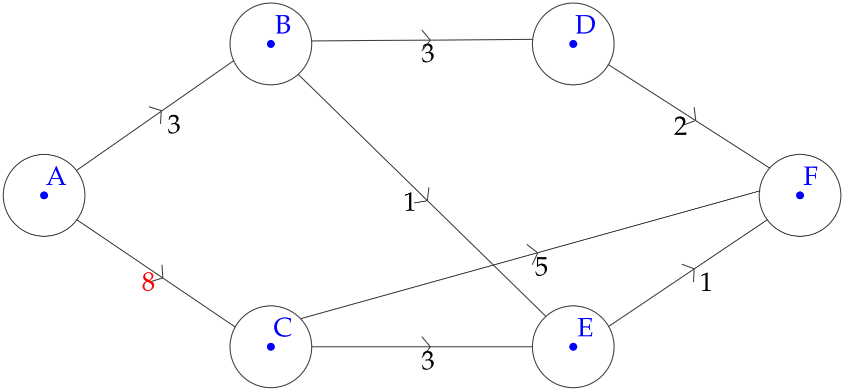

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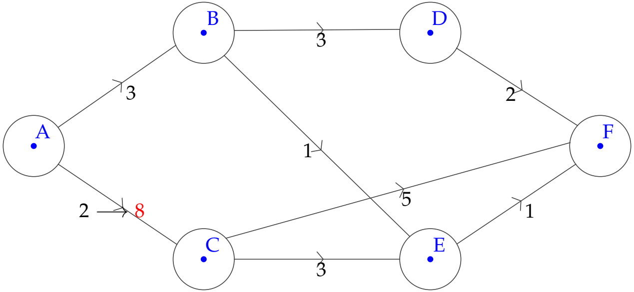

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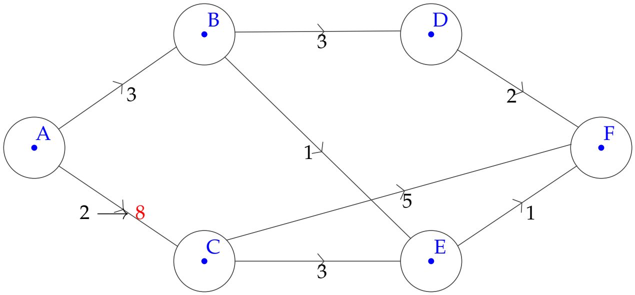

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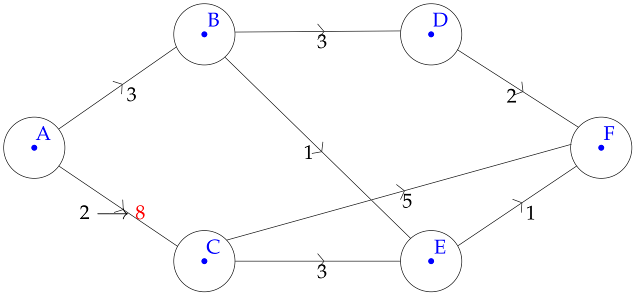

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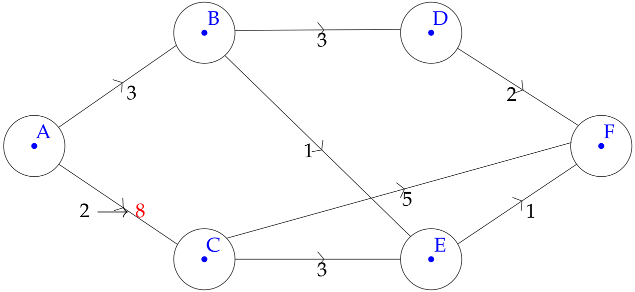

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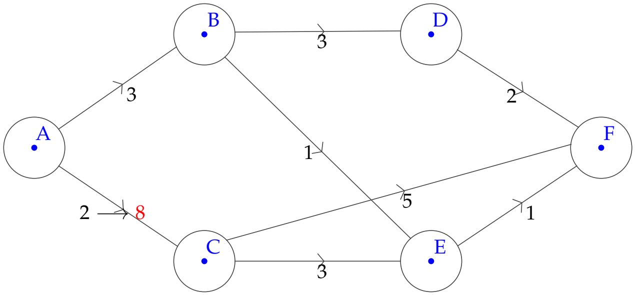

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   - This money cannot be redistributed among the same players, since that will change their payoffs and the resulting mechanism would not remain DSIC
   - If the players are partitioned into two groups and the surplus of one group is redistributed over the other group, then it is budget balanced, but the overall efficiency is compromised

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These are certain limitations that are good to know for effective use of VCG, however, it is the most widely used mechanism in the literature

भारतीय प्रौद्योगिकी संस्थान मुंबई

Indian Institute of Technology Bombay