CS 6002: Selected Areas of Mechanism Design **Jan-Apr 2025**

Lecture 3: Kidney Exchanges

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3.1 Applications of One-Sided Matching: Kidney Exchanges

3.1.1 The Real-World Problem

Many people suffer from organ diseases such as liver, pancreas, or kidney ailments. To save these patients, modern medical science enables transplantation of organs from living or deceased donors. Among these, kidney transplants have garnered significant attention due to the high prevalence of kidney diseases.

Prevalence of Kidney Ailments

Hundreds of thousands of people suffer from kidney-related diseases. While precise data for India is limited, it is estimated that the numbers are significantly larger than in the US, where the United Network for Organ Sharing (UNOS) maintains a comprehensive database of patients and donors. In India, the National Organ and Tissue Transplant Organization (NOTTO) serves a similar role.

Treatment Options

Patients with kidney ailments typically have two primary options:

1. **Dialysis:** A regular and life-sustaining process, but it significantly impacts quality of life.
2. **Kidney Transplant:** This option offers a better quality of life and is less costly in the long run. However, it requires compatibility between the donor and the recipient.

Compatibility Factors

Successful transplants depend on several factors, including:

- Blood-type compatibility,
- Tissue-type compatibility,
- Other medical considerations.

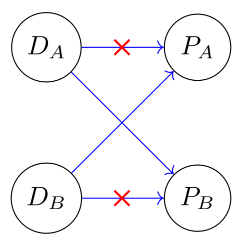
3.1.2 Historical Milestones

The first successful kidney transplant was performed by Dr. Joseph Murray in 1954. The patient survived for 8 years, and Dr. Murray was awarded the Nobel Prize in Medicine in 1990 for his pioneering work.

In India, the first successful kidney transplant was conducted in 1971 at Christian Medical College (CMC), Vellore, by Dr. Mohan Rao. As of 2021, CMC alone has conducted 3,755 transplants.

3.1.3 Donor-Pair Exchange Mechanism

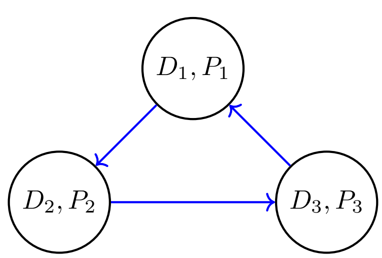
In cases where the donor is incompatible with the intended recipient, a donor-pair exchange can be arranged. For example:



In this example, donors D_a and D_b are initially incompatible with their respective patients P_a and P_b. However, through an exchange, D_a donates to P_b, and D_b donates to P_a, enabling successful transplants for both patients.

3.2 Kidney-Pair Donation

Kidney-Pair Donation (KPD) started in 1986. It enables exchanges between incompatible donor-patient pairs to maximize the number of successful transplants. Here is an example scenario:
Consider a graph, each node is a donor-patient pair where the donor is willing to donate their kidney as long as the patient receives a kidney. Consider a directed edge e = (u,v) if the donor in u is compatible with the donor in v. Whenever this graph has a cycle, it seems like we can perform the donations along the cycle and everyone in the cycle will agree. This kind of problem is easily solvable by TTC.



3.2.1 Proposal 1: Top Trading Cycle (TTC)

Top Trading Cycle can be adapted for Kidney-Pair Donation by treating kidneys as equivalent to houses, preferences as medical compatibilities, and patients as house owners. The standard framework of house allocation is applied here.

Advantages of TTC

- Truthful: Patients have no incentive to misreport their preferences.
- Pareto efficient: Ensures that no other allocation can make one patient better off without making another worse off.
- TTC is in the core and hence stable: No group of kidney-pair donors can deviate and achieve a better allocation.
- Individual Rationality: Every patient is weakly better off by participating.
- Polynomial-time algorithm: Efficient computation of the allocations.

Disadvantages of TTC

- Only works with initial endowments: Each patient must have a donor in the original formulation.
- Limitation with unpaired patients and altruistic donors: Real-world cases may involve patients without donors and deceased/altruistic donors. A modified version of TTC can handle houses without owners and owners without houses, which is useful in cases involving deceased or altruistic donors.

3.2.2 Proposal 2: "You request my house, I get your turn"

The mechanism proposed by Abdulkadiroglu and Sonmez (1999) is a variant of TTC and satisfies Pareto Efficiency (PE), strategy-proofness, and several other desirable properties.

In this approach, an order is decided among the players (in this case, the owners) based on a predefined metric, such as life expectancy, which determines their priority in the process. This order dictates the sequence in which the owners make their house requests.

Step-by-Step Process:

1. **Initial Request:** The owner with the highest priority (based on the chosen metric, like life expectancy) starts by requesting their most preferred house.
2. **House Availability Check:**
   - If the house is *unallocated* (i.e., no one currently owns it), the requesting owner is granted that house immediately.
   - If the house is *allocated* to someone else, the requesting owner does not get the house. Instead, they are assigned the preference number of the person who currently holds that house.
3. **Cycle Detection:** The process continues until all owners have made their requests. If there is a cycle (i.e., a situation where owners are caught in a loop of requests), the owners are assigned their houses according to the cycle's resolution.

Example:

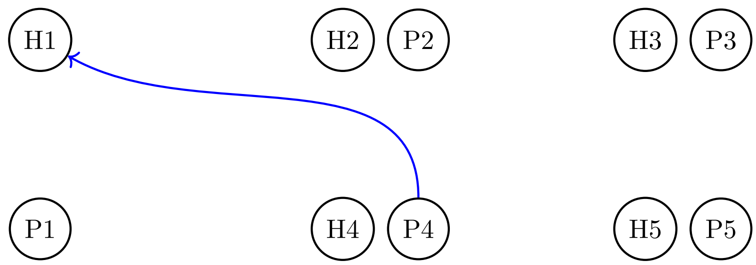


Figure 3.1: Current priority order: $P_4 > P_1 > P_2 > P_3 > P_5$. P_4 prefers H_1. Since H_1 is unallocated, it is matched with P_4

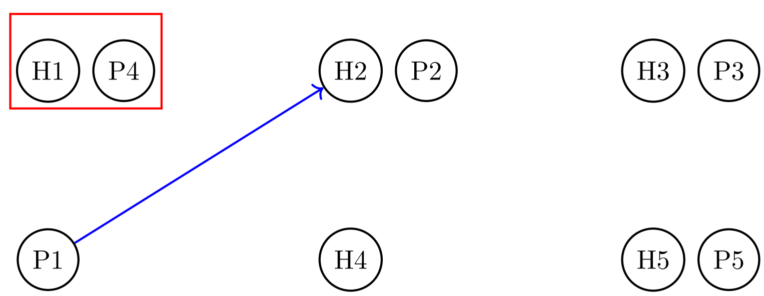


Figure 3.2: Current priority order: $P_1 > P_2 > P_3 > P_5$. P_1 prefers H_2. Since H_2 is allocated to P_2, P_2 gets P_1's priority.

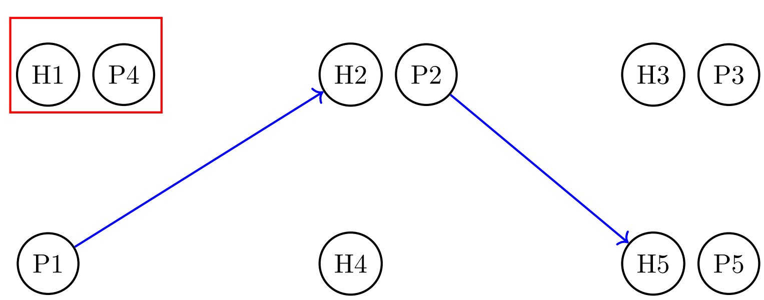


Figure 3.3: Current priority order: $P_2 > P_1 > P_3 > P_5$. P_2 prefers H_5. Since H_5 is allocated to P_5, P_5 gets P_2's priority.

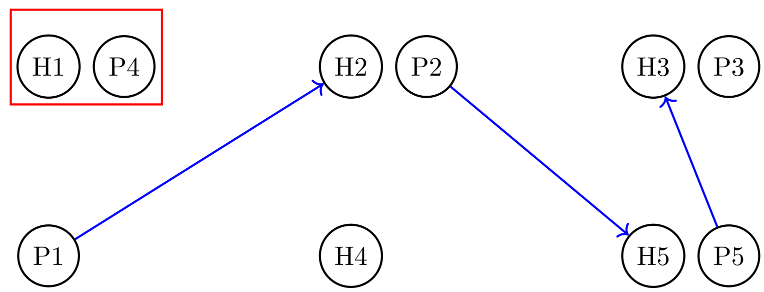


Figure 3.4: Current priority order: $P_5 > P_2 > P_1 > P_3$. P_5 prefers H_3. Since H_3 is allocated to P_3, P_3 gets P_5's priority.

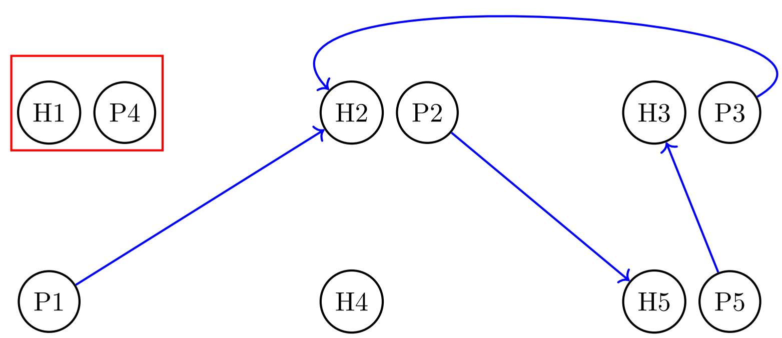


Figure 3.5: Current priority order: $P_3 > P_5 > P_2 > P_1$. P_3 prefers H_2. Since H_2 is allocated to P_2, P_2 gets P_3's priority. Since we get a cycle, P_3 is matched with H_2, P_2 with H_5 and P_5 with H_3.

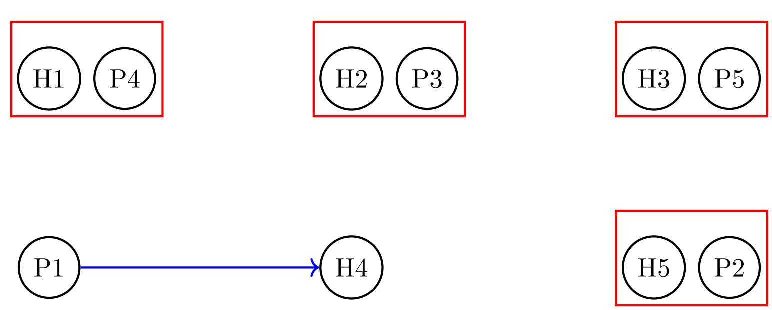


Figure 3.6: Current priority order: P_1. P_1 prefers H_4. Since H_4 is unallocated, P_1 is matched with H_4.

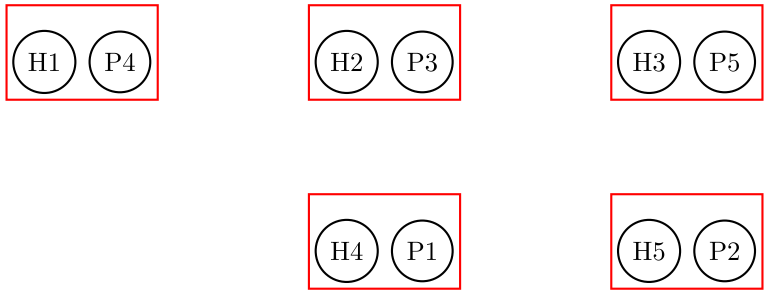
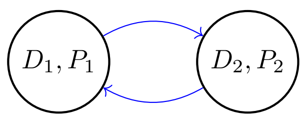


Figure 3.7: Final allocation

Disadvantages

- **Long cycles of exchanges are difficult to execute in a medical context.**

  For example, consider the following two pairs:

  This example requires 4 surgeries: one for each donor-patient pair. A larger cycle involving k pairs would require $2k$ surgeries. These surgeries must be performed simultaneously.

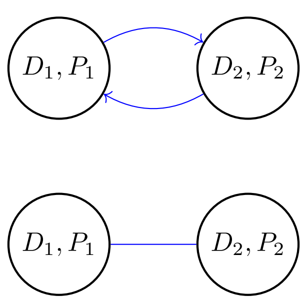
  If the surgeries are not performed simultaneously, the donor of the patient receiving an organ may withdraw before the transplant is completed. This makes simultaneous surgeries necessary to avoid the risk of a failed cycle.

- **Strict preference ordering is not required.** Instead, binary preferences are sufficient. In this setup, all compatible pairs shoule be considered equivalent, and no further preference ranking is needed.

3.2.3 Proposal 3: Priority Matching Algorithm

In this proposal, we only consider cycles of two lengths, as larger cycles are logistically impossible in practice. The two-length cycle is represented with an undirected edge.

The following figures are equivalent.



3.2.3.1 Kidney Exchange using Matchings (Graph)

Given an undirected graph of patient-donor pairs, the goal is to find a maximum cardinality matching. However, this can lead to strategic manipulations as shown in the diagram below.

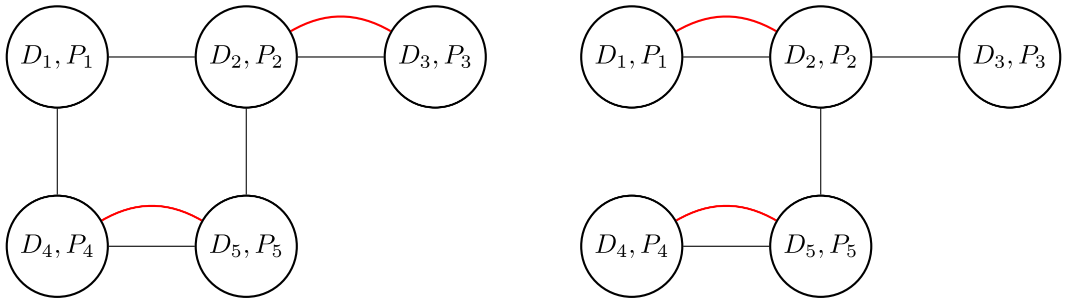


Figure 3.8: Patient P_1 can mis-report his compatibilities in order to get a matching.

3.2.3.2 Priority Matching Algorithm by Roth, Sonmez, Ãœnver (JET 2005)

The priority matching algorithm addresses strategic manipulation in kidney exchange matching.

Algorithm 1 Priority Matching Algorithm

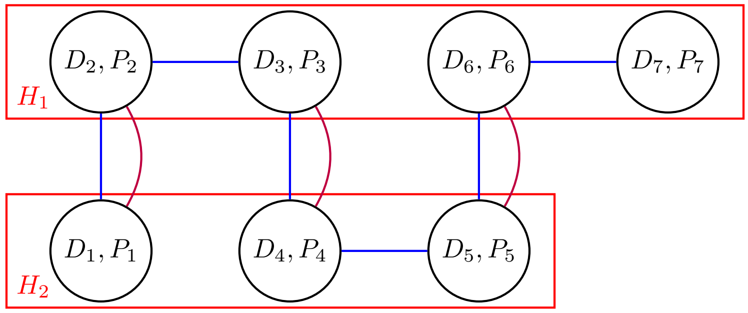
Consider a priority order $1, 2, \ldots, n$ over the vertices.
$M_0 \leftarrow$ set of maximum cardinality matching over the given graph.
for each $i = 1, 2, \ldots, n$ **do**
    Let S_i be the set of matchings in M_{i-1} that matched vertex i.
    if $S_i \neq \emptyset$ **then**
        $M_i \leftarrow S_i$
    else
        $M_i \leftarrow M_{i-1}$
    end if
end for
return M_n

Advantages:

- Each matching in M_n matches the same set of vertices.
- No agent under-reports the set of its compatible edges.
- Pareto optimal: No other matching can match a superset of the matched agents.
- Polynomial-time algorithm: uses Edmonds-Gallai decomposition from graph theory.

Disadvantages:

- Kidney exchanges are conducted via hospitals, which are the new players in the system. Hospitals share data with a centralized exchange to run the matching algorithm. In some cases, patients may not get matched due to data manipulation, leading to strategic under-reporting.



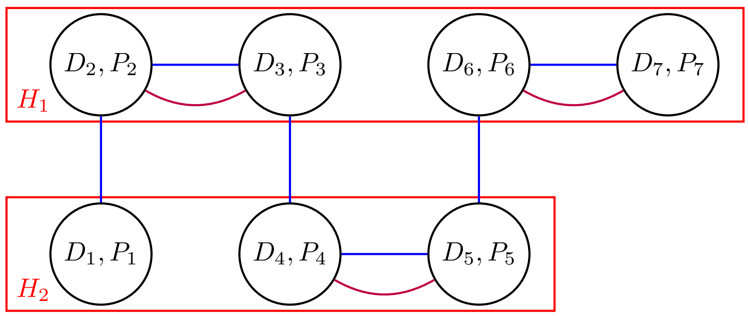


Figure 3.9: After matching across hospitals H_1 and H_2, H_1 may not get one of its patients matched. Therefore it can stop reporting data of (D_2, P_2) and (D_3, P_3) and be better off.

3.3 Voluntary Donation (Altruistic Donor)

A voluntary donor is someone who donates an organ without having a patient. This category of donation can greatly impact kidney exchanges.

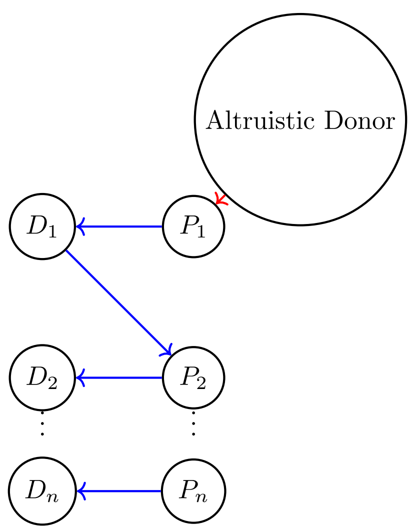


Figure 3.10: Unlike closed cycles where all surgeries must happen simultaneously, these chains can be performed sequentially, making them logistically easier to manage.

Advantages:

- Reduces the waiting list for kidney transplants.
- Activates a chain of organ donations that benefits multiple recipients.

In 2012, Alvin Roth and Lloyd Shapley were awarded the Nobel Prize in Economics for their work on organ donation matching and stable matching theory for two-sided markets.