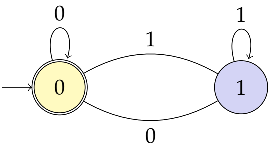
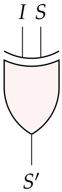
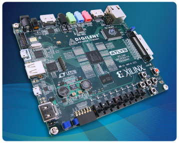
CS 226: Digital Logic Design

Lecture 3: Binary Numbers (Contd.)

Ashutosh Trivedi







Department of Computer Science and **Engineering**,
Indian Institute of Technology Bombay.

Recap: Number-Base Conversions

Binary Arithmetic

Surprise Quiz!

1. Enumerate the first 16 binary numbers.
2. Enumerate the first 20 base-4 numbers.
3. Convert the following numbers with the indicated bases to decimal:
   3.1 $(4310)_5$
   3.2 $(123)_8$
4. Convert $(243)_{10}$ to binary.
5. Convert $(1010101.11)_2$ to octal and hexadecimals.
6. Convert $(.56)_{10}$ to octal up to five significant digits.

Recap: Number-Base Conversions

Binary Arithmetic

Let's generalize Decimal Arithmetic

- Addition
  - What do you need to remember?
  - What is the algorithm?
  - How to extend that in Binary?
- Subtraction
  - What do you need to remember?
  - What is the algorithm?
  - How to extend that in Binary?
- Multiplication
  - What do you need to remember?
  - What is the algorithm?
  - How to extend that in Binary?
- Division
  - What do you need to remember?
  - What is the algorithm?
  - How to extend that in Binary?

Binary Addition

- Binary Addition
  - $0 + 0 = 0$ — sum is 0 and carry is 0;
  - $0 + 1 = 1$ — sum is 1 and carry is 0;
  - $1 + 0 = 1$ — sum is 1 and carry is 0;
  - $1 + 1 = 10$ — sum is 0 and carry is 1.

Binary Addition

- Binary Addition
  - $0 + 0 = 0$ — sum is 0 and carry is 0;
  - $0 + 1 = 1$ — sum is 1 and carry is 0;
  - $1 + 0 = 1$ — sum is 1 and carry is 0;
  - $1 + 1 = 10$ — sum is 0 and carry is 1.
- Binary Addition with Carry (Blue bit is carry).
  - $1 + 0 + 0 = 0$ — sum is 1 and carry is 0;
  - $1 + 0 + 1 = 1$ — sum is 0 and carry is 1;
  - $1 + 1 + 0 = 1$ — sum is 0 and carry is 1;
  - $1 + 1 + 1 = 11$ — sum is 1 and carry is 1.

Binary Addition

- Binary Addition
  - $0 + 0 = 0$ — sum is 0 and carry is 0;
  - $0 + 1 = 1$ — sum is 1 and carry is 0;
  - $1 + 0 = 1$ — sum is 1 and carry is 0;
  - $1 + 1 = 10$ — sum is 0 and carry is 1.
- Binary Addition with Carry (Blue bit is carry).
  - $1 + 0 + 0 = 0$ — sum is 1 and carry is 0;
  - $1 + 0 + 1 = 1$ — sum is 0 and carry is 1;
  - $1 + 1 + 0 = 1$ — sum is 0 and carry is 1;
  - $1 + 1 + 1 = 11$ — sum is 1 and carry is 1.
- Examples.

$$
\begin{array}{rl}
1\_\_\_\_1\_ & \text{carry} \\
11101 & \text{augend} \\
+10001 & \text{addend} \\
\hline
101110 & \text{sum}
\end{array}
$$

Binary Subtraction

- Binary Subtraction
  - $0 - 0 = 0$
  - $1 - 1 = 0$
  - $1 - 0 = 1$
  - $10 - 1 = 1$ (borrow 1 from a higher bit).

Binary Subtraction

- Binary Subtraction
  - $0 - 0 = 0$
  - $1 - 1 = 0$
  - $1 - 0 = 1$
  - $10 - 1 = 1$ (borrow 1 from a higher bit).
- Examples.

$_{-}111$	borrow
$_{-}111$	borrow
1000	minuend
-0011	subtrahend
0101	difference

Binary Subtraction

- Binary Subtraction
  - $0 - 0 = 0$
  - $1 - 1 = 0$
  - $1 - 0 = 1$
  - $10 - 1 = 1$ (borrow 1 from a higher bit).
- Examples.

$_{-}111$	borrow
$_{-}111$	borrow
1000	minuend
-0011	subtrahend
0101	difference

- $1000 - 0011 =?$
- $1001.10 - 0101.1 =?$
- $45 - 39 =?$

Binary Multiplication

- Binary Multiplication
  - $0 \times 0 = 0$
  - $0 \times 1 = 0$
  - $1 \times 0 = 0$
  - $1 \times 1 = 1$

Binary Multiplication

- Binary Multiplication
  - $0 \times 0 = 0$
  - $0 \times 1 = 0$
  - $1 \times 0 = 0$
  - $1 \times 1 = 1$
- Multiplying a binary number by 2 (i.e. $(10)_2$).

Binary Multiplication

- Binary Multiplication
  - $0 \times 0 = 0$
  - $0 \times 1 = 0$
  - $1 \times 0 = 0$
  - $1 \times 1 = 1$
- Multiplying a binary number by 2 (i.e. $(10)_2$).
- Examples.

$$
\begin{array}{rrrrrrrrl}
 & & & & 1 & 1 & 0 & 0 & \quad \text{multiplicand} \\
 & & & \times & 1 & 0 & 1 & 1 & \quad \text{multiplier} \\
\hline
 & & & & 1 & 1 & 0 & 0 & \\
 & & & 1 & 1 & 0 & 0 & \times & \\
 & & 0 & 0 & 0 & 0 & \times & \times & \\
 & 1 & 1 & 0 & 0 & \times & \times & \times & \\
\hline
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \quad \text{product} \\
\hline
\end{array}
$$

Binary Division

- Recall Long Division Algorithm for Decimal numbers
- Let's divide $(24158)_{10}$ by $(6)_{10}$.

Binary Division

- Recall Long Division Algorithm for Decimal numbers
- Let's divide $(24158)_{10}$ by $(6)_{10}$.
- Generalize it to divide $(1011110)_2$ by $(101)_2$.

$$
\begin{array}{rr}
 & 10010 \\
101 & \overline{)1011110} \\
 & \underline{101} \\
 & 111 \\
 & \underline{101} \\
 & 100
\end{array}
$$