Lexical Analysis

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September 2007
The input program – as you see it.

```c
main ()
{
    int i,sum;
    sum = 0;
    for (i=1; i<=10; i++);
    sum = sum + i;
    printf("%d\n",sum);
}
```
Recap

The same program – as the compiler sees it (initially).

```c
main() { int i, sum; sum = 0; for (i = 1; i <= 10; i++) sum = sum + i; printf("%d\n", sum); }
```

- The blank space character
- The return character
Recap

The same program – as the compiler sees it (initially).

```c
main() { int i, sum; sum = 0;
for (i = 1; i <= 10; i++)
    sum = sum + i;
printf("%d\n", sum);
}
```

How do you make the compiler see what you see?
Recap - Discovering the structure of the program

Step 1:

a. Break up this string into ‘words’—the smallest logical units.

```
main ( ) { int i, sum;
    sum = 0;
    for ( i = 1; i <= 10; i++ ) {
        sum = sum + i;
        printf ( "%d\n" , sum );
    }
```

We get a sequence of *lexemes* or *tokens*.
Recap - Discovering the structure of the program

Step 1:

b. Clean up – remove the [ ] and the ← characters.

```c
main ( ) { int i , sum ; sum = 0 ; for ( i = 1 ; i <= 10 ; i ++ ) ; sum = sum + i ; printf ( "%d\n" , sum ) ; }
```

Steps 1a. and 1b. are interleaved.
Recap - Discovering the structure of the program

Step 1:

b. Clean up – remove the ® and the ← characters.

```
main ( ) { int i , sum ; sum = 0 ; for ( i = 1 ; i <= 10 ; i ++ ) ; sum = sum + i
; printf ( "%d\n" , sum ) ; }
```

This is *lexical analysis* or *scanning*.
Recap - Discovering the structure of the program

Step 2:
Now group the lexemes to form larger structures.

```c
main ( ) { int i , sum ; sum = 0 ; for ( i = 1 ; i <= 10 ; i ++ ) ; sum = sum + i ; printf ( "%d\n" , sum ) ; }
```
Recap - Discovering the structure of the program

Step 2:
Now group the lexemes to form larger structures.

```c
main ( ) { int i , sum ; sum = 0 ; for ( 
    i = 1 ; i <= 10 ; i ++ ) ; sum = sum + i 
; printf ( "%d\n" , sum ) ; }
```

fundef
Step 2:
Now group the lexemes to form larger structures.

```c
main ( ) { int i , sum ; sum = 0 ;
for ( i = 1 ; i <= 10 ; i ++ ) ; sum = sum + i ;
printf ( "%d\n" , sum ) ; }
```
Recap - Discovering the structure of the program

Step 2:
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main ( ) { int i , sum ; sum = 0 ; for ( i = 1 ; i <= 10 ; i ++ ) ; sum = sum + i ; printf ( "%d\n" , sum ) ; }
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Now group the lexemes to form larger structures.

```c
main ( ) { int i , sum ; sum = 0 ; for ( i = 1 ; i <= 10 ; i ++ ) ; sum = sum + i ; printf ( "%d\n" , sum ) ; }
```

![Diagram of the program structure](image)
Recap - Discovering the structure of the program

Step 2:
Now group the lexemes to form larger structures.

```c
main ( ) { int i, sum ; sum = 0 ; for ( i = 1 ; i <= 10 ; i ++ ) ; sum = sum + i ; printf ( "%d\n" , sum ) ; }
```
Recap - Discovering the structure of the program

This is *syntax analysis* or *parsing*.
Lexemes, Tokens and Patterns

Definition: *Lexical analysis* is the operation of dividing the input program into a sequence of *lexemes* (*tokens*).
Lexemes, Tokens and Patterns

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Distinguish between
Lexemes, Tokens and Patterns

**Definition:** *Lexical analysis* is the operation of dividing the input program into a sequence of *lexemes* (*tokens*).

Distinguish between

- **lexemes** – smallest logical units (words) of a program.
  Examples – `i`, `sum`, `for`, `10`, `++`, `"%d\n"`, `<=`.
Lexemes, Tokens and Patterns

Definition: *Lexical analysis* is the operation of dividing the input program into a sequence of *lexemes* (*tokens*).

Distinguish between

- **lexemes** – smallest logical units (words) of a program.
  Examples – *i*, *sum*, *for*, *10*, *++*, "%d\n", <=.

- **tokens** – sets of similar lexemes.
  Examples –
  - *identifier* = {*i*, *sum*, *buffer*, …}
  - *int_constant* = {*1*, *10*, …}
  - *addop* = {*+, -, ...*
Lexemes, Tokens and Patterns

Things that are not counted as lexemes –
Lexemes, Tokens and Patterns

Things that are not counted as lexemes –

- white spaces – tab, blanks and newlines
Things that are not counted as lexemes –

- white spaces – tab, blanks and newlines
- comments
Lexemes, Tokens and Patterns

Things that are not counted as lexemes –

- white spaces – tab, blanks and newlines
- comments

These too have to be detected and ignored.
Lexemes, Tokens and Patterns

The lexical analyser:

- detects the next lexeme
- categorises it into the right token
- passes to the syntax analyser
  - the token name for further syntax analysis
  - the lexeme itself, in some form, for stages beyond syntax analysis
How does one describe the lexemes that make up the token *identifier*.

Variants in different languages.
Recap - Lexemes, Tokens and Patterns

How does one describe the lexemes that make up the token *identifier*.

Variants in different languages.

- **String of alphanumeric characters.** The first character is an alphabet.
Recap - Lexemes, Tokens and Patterns

How does one describe the lexemes that make up the token *identifier*.

Variants in different languages.

- String of alphanumeric characters. The first character is an alphabet.
- a string of alphanumeric characters in which the first character is an alphabet. *It has a length of at most 31.*
Recap - Lexemes, Tokens and Patterns

How does one describe the lexemes that make up the token *identifier*.

Variants in different languages.

- String of alphanumeric characters. The first character is an alphabet.
- A string of alphanumeric characters in which the first character is an alphabet. It has a length of at most 31.
- A string of alphabet or numeric or underline characters in which the first character is an alphabet or an underline. It has a length of at most 31. Any characters after the 31st character are ignored.
Recap - Lexemes, Tokens and Patterns

How does one describe the lexemes that make up the token *identifier*.

Variants in different languages.

- String of alphanumerical characters. The first character is an alphabet.
- A string of alphanumerical characters in which the the first character is an alphabet. *It has a length of at most 31.*
- A string of alphabet or numeric or *underline* characters in which the first character is an alphabet or an underline. *It has a length of at most 31.* *Any characters after the 31st character are ignored.*

Such descriptions are called *patterns*. The description may be informal or formal.
Recap - Basic concepts and issues

A pattern is used to

- specify tokens precisely
- build a recognizer from such specifications
Example – tokens in Java

1. **Identifier:** A *Javaletter* followed by zero or more *Javaletterordigits.*
   A *Javaletter* includes the characters a–z, A–Z, _ and $.

2. **Constants:**
   2.1 Integer Constants
      - Octal, Hex and Decimal
      - 4 byte and 8 byte representation
   2.2 Floating point constants
      - float - ends with f
      - double
   2.3 Boolean constants – true and false
   2.4 Character constants – ’a’, ’\u0034’, ’\t’
   2.5 String constants – "", "\\"", "A string".
   2.6 Null constant – null.

3. **Delimiters:** (, ), {, }, [, ] , ;, . and ,

4. **Operators:** =, >, < ... >>>= 

5. **Keywords:** abstract, boolean ... volatile, while.
Recap - Basic concepts and issues

*Where does a lexical analyser fit into the rest of the compiler?*

- The front end of most compilers is parser driven.
- When the parser needs the next token, it invokes the Lexical Analyser.
- Instead of analysing the entire input string, the lexical analyser sees enough of the input string to return a single token.
- The actions of the lexical analyser and parser are intertwined.
Recap - Token Attributes

Apart from the token itself, the lexical analyser also passes other informations regarding the token. These items of information are called *token attributes*

**EXAMPLE**

<table>
<thead>
<tr>
<th>lexeme</th>
<th>(&lt;\text{token, token attribute}&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>(&lt;\text{const, 3}&gt;)</td>
</tr>
<tr>
<td>A</td>
<td>(&lt;\text{identifier, A}&gt;)</td>
</tr>
<tr>
<td>if</td>
<td>(&lt;\text{if, –}&gt;)</td>
</tr>
<tr>
<td>(=)</td>
<td>(&lt;\text{assignop, –}&gt;)</td>
</tr>
<tr>
<td>(&gt;)</td>
<td>(&lt;\text{gt, –}&gt;)</td>
</tr>
<tr>
<td>;</td>
<td>(&lt;\text{semicolon, –}&gt;)</td>
</tr>
</tbody>
</table>
Creating a Lexical Analyzer

Two approaches:
Creating a Lexical Analyzer

Two approaches:

1. *Hand code* – This is only of historical interest now.
   - Possibly more efficient.
Two approaches:

1. *Hand code* – This is only of historical interest now.
   - Possibly more efficient.

2. *Use a generator* – To generate the lexical analyser from a formal description.
   - The generation process is faster.
   - Less prone to errors.
Automatic Generation of Lexical Analysers

Inputs to the lexical analyser generator:

- A specification of the tokens of the source language, consisting of:
  - a regular expression describing each token, and
  - a code fragment describing the action to be performed, on identifying each token.

The generated lexical analyser consists of:

- A determinstic finite automaton (DFA) constructed from the token specification.
- A code fragment (a driver routine) which can traverse any DFA.
- Code for the action specifications.
Automatic Generation of Lexical Analysers

specification

regular expression  action routines
processed  copied

dfa  action routines  Driver routine

Input program  tokens

generated lexical analyser
Example of Lexical Analyser Generation

Suppose a language has two tokens

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>a*b</td>
<td>{ printf( &quot;Token 1 found&quot;); }</td>
</tr>
<tr>
<td>c+</td>
<td>{ printf( &quot;Token 2 found&quot;); }</td>
</tr>
</tbody>
</table>

From the description, construct a structure called a deterministic finite automaton (DFA).
Now consider the following together:

The DFA

```
0 ----> a ----> 1
     |           |           |
     v           v           v
     3           2           c
     |           |           |
     b ----> b ----> b
```

The driver routine

```c
void nexttoken()
{
    state = 0; c = nextchar();
    while (valid(nextstate[state,c]))
    {
        state = nextstate[state,c];
        c = nextchar();
    }
    if (!final(state))
    {
        error; return;
    }
    else
    {
        unput(c); action(); return;
    }
}
```

The actions

```c
void action();
{
    switch(state)
    {
        case 2:{printf("Token 1 found"); break;}
        case 3:{printf("Token 2 found"); break;}
    }
}
```

The input and output

Input: aabadbcc
Output:
Now consider the following together:

**The DFA**

- **State 0**: Initial state.
- **State 1**: Transition on 'a' to State 1 and 'b' to State 2.
- **State 2**: Transition on 'a' to State 1 and 'c' to State 3.
- **State 3**: Transition on 'c' to State 3 and 'b' to State 2.

**The driver routine**

```c
void nexttoken()
{
    state = 0; c = nextchar();
    while (valid(nextstate[state,c]))
    {
        state = nextstate[state,c];
        c = nextchar();
    }
    if (!final(state))
    {
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    }
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    {
        unput(c); action(); return;
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}
```

**The actions**

```c
void action();
{
    switch(state)
    {
        2:{printf("Token 1 found");
            break;}
        3:{printf("Token 2 found");
            break;}
    }
}
```

**The input and output**

- **Input**: `aaababdbcc`
- **Output**: 

---

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Example of Lexical Analyser Generation

Now consider the following together:

The DFA

```
0 --a-- 1
   |   |   |   |
   a   b   c   
   |   |   |   |
   0 --c-- 3
```

The driver routine

```c
void nexttoken()
{
    state = 0; c = nextchar();
    while (valid(nextstate[state, c]))
    {
        state = nextstate[state, c];
        c = nextchar();
    }
    if (!final(state))
    {
        error; return;
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    else
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}
```

The actions

```c
void action()
{
    switch (state)
    {
        case 2:
            printf("Token 1 found");
            break;
        case 3:
            printf("Token 2 found");
            break;
    }
}
```

The input and output

Input: `aabadbcc`

Output:
Now consider the following together:

### The DFA

```
0 -----> 1
<p>| |
|       |</p>
<table>
<thead>
<tr>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>b</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>a</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>b</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>c</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
```

### The driver routine

```c
void nexttoken()
{
    state = 0; c = nextchar();
    while (valid(nextstate[state,c]))
    {
        state = nextstate[state,c];
        c = nextchar();
    }
    if (!final(state))
    {
        error; return;
    }
    else
    {
        unput(c); action(); return;
    }
}
```

### The actions

```c
void action();
{
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    {
        2:{printf("Token 1 found");
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    }
}
```

### The input and output

Input: `aabadbcc`

Output:
Now consider the following together:

The DFA

The driver routine

void nexttoken ()
{
    state = 0; c = nextchar();
    while (valid(nextstate[state,c]))
    {
        state = nextstate[state,c];
        c = nextchar();
    }
    if (!final(state))
    {
        error; return;
    }
    else
    {
        unput(c); action(); return;
    }
}

The actions

void action();
{
    switch(state)
    {
        2:{printf("Token 1 found");
            break;}
        3:{printf("Token 2 found");
            break;}
    }

The input and output

Input: aabadbcc
Output: Token 1 found
Example of Lexical Analyser Generation

Now consider the following together:

The DFA

The driver routine

```c
void nexttoken()
{
    state = 0; c = nextchar();
    while (valid(nextstate[state,c]))
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void action();
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```

The input and output

Input: aabadbcc
Output: Token 1 found
Now consider the following together:

**The DFA**

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        unput(c); action(); return;
    }
}
```

**The actions**

```c
void action();
{
    switch(state)
    2:{printf("Token 1 found");
        break;}
    3:{printf("Token 2 found");
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}
```

**The input and output**

**Input:** `aabadbcc`

**Output:** `Token 1 found`
Example of Lexical Analyser Generation

Now consider the following together:

The DFA

```
0 1 2 3
```

The actions

```
void action();
{
    switch(state)
    2:{printf("Token 1 found");
        break;}
    3:{printf("Token 2 found");
        break;}
}
```

The input and output

```
Input: aabadbcc←
Output: Token 1 found
```

The driver routine

```
void nexttoken()
{
    state = 0; c = nextchar();
    while (valid(nextstate[state,c]))
    {
        state = nextstate[state,c];
        c = nextchar();
    }
    if (!final(state))
    {
        error; return;
    }
    else
    {
        unput(c); action(); return;
    }
```

The input and output

```
Input: aabadbcc←
Output: Token 1 found
```

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Now consider the following together:

### The DFA

```
0 -> 1 (a, b, c) | 1 -> 2 (a) | 2 -> 3 (c) | 3 -> 0 (c)
```

### The driver routine

```c
void nexttoken ()
{
    state = 0; c = nextchar();
    while (valid(nextstate[state,c]))
    {
        state = nextstate[state,c];
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}
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### The actions

```c
void action();
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    switch(state)
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        2:{printf("Token 1 found");
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        3:{printf("Token 2 found");
            break;}
    }
}
```

### The input and output

Input: `aaba\nbc\nc`

Output: `Token 1 found`
Now consider the following together:

### The DFA

```
0 --a-> 1
    ^
    |   a
    |   v

1 --b-> 2

1 --c-> 3
```

### The driver routine

```c
void nexttoken()
{
    state = 0; c = nextchar();
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    {
        state = nextstate[state, c];
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        unput(c); action(); return;
    }
}
```

### The actions

```c
void action();
{
    switch(state)
    {
        2:{printf("Token 1 found");
            break;}
        3:{printf("Token 2 found");
            break;}
    }
}
```

### The input and output

**Input:** `aababdbcc`

**Output:**
- Token 1 found
- Token 1 found

---

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Example of Lexical Analyser Generation

Now consider the following together:

The DFA

```
0       1       2
 a      a      b
 b      c      c
 c
```

The driver routine

```c
void nexttoken ()
{
    state = 0; c = nextchar();
    while (valid(nextstate[state,c]))
        {state = nextstate[state,c];
         c = nextchar();}
    if (!final(state))
        {error; return;}
    else
        {unput(c); action(); return;}
}
```

The actions

```c
void action();
{
    switch(state)
    2:{printf("Token 1 found");
        break;}
    3:{printf("Token 2 found");
        break;}
}
```

The input and output

Input: `aabadbcc`
Output: Token 1 found
        Token 1 found
Example of Lexical Analyser Generation

Now consider the following together:

The DFA

```
0 -> 1
   ↓
   ↑
3
```

The driver routine

```c
void nexttoken ()
{
    state = 0; c = nextchar();
    while (valid(nextstate[state,c]))
    {
        state = nextstate[state,c];
        c = nextchar();
    }
    if (!final(state))
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        error; return;
    }
    else
    {
        unput(c); action(); return;
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}
```

The actions

```c
void action();
{
    switch(state)
    2:{printf("Token 1 found");
        break;}
    3:{printf("Token 2 found");
        break;}
}
```

The input and output

Input: `aabadbcc`
Output: Token 1 found
         Token 1 found
Now consider the following together:

The DFA

- States: 0, 1, 2, 3
- Transitions:
  - 0 → 1 on 'a'
  - 0 → 2 on 'b'
  - 0 → 3 on 'c'
  - 1 → 0 on 'c'
  - 2 → 3 on 'b'
  - 3 → 1 on 'c'

The driver routine

```c
void nexttoken()
{
    state = 0;
    c = nextchar();
    while (valid(nextstate[state, c]))
    {
        state = nextstate[state, c];
        c = nextchar();
    }
    if (!final(state))
    {
        error; return;
    }
    else
    {
        unput(c); action(); return;
    }
}
```

The actions

```c
void action()
{
    switch(state)
    {
        2:{printf("Token 1 found");
            break;}
        3:{printf("Token 2 found");
            break;}
    }
}
```

The input and output

- Input: `aabaabc`
- Output: Token 1 found
          Token 2 found
Example of Lexical Analyser Generation

In summary:

- The DFA, the driver routine and the action routines taken together, constitute the lexical analyser.
  - actions are supplied as part of specification.
  - driver routine is common to all generated lexical analyzers

The only issue – how are the patterns, specified by regular expressions, converted to a DFA.

In two steps:
  - Convert regular expression into NFA.
  - Convert NFA to DFA.
Consider a language with the following tokens:

- **begin** – representing the lexeme `begin`
- **integer** – Examples: 0, -5, 250
- **identifier** – Examples: a, A1, max
Converting Regular Expressions to NFA

In two parts;

- First convert the regular expression corresponding to each token into a NFA.
  - Invariant: A single final state corresponding to each token.
- Join the NFAs obtained for all the tokens.
Converting Regular Expressions to DFA

**RE for ε**

- Initial state: ε
- Final state: ε

**RE for a**

- Initial state: a
- Final state: a
Converting Regular Expressions to NFA

$\text{RE for } r_1 \cdot r_2$
Converting Regular Expressions to NFA

RE for \( r_1 \mid r_2 \)
Converting Regular Expressions to NFA

RE for $r$

\[ r^* \]
Converting NFA to DFA
Converting NFA to DFA

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Converting NFA to DFA

![Diagram of NFA to DFA conversion](image-url)
Converting NFA to DFA
Converting NFA to DFA
Converting NFA to DFA

0, 1, 7, 10

0
Converting NFA to DFA
Converting NFA to DFA
Converting NFA to DFA
Converting NFA to DFA

0 \xrightarrow{\epsilon} 7 \xrightarrow{-} 8 \xrightarrow{\text{digit}} 9 \xrightarrow{\text{digit}} 11 \xrightarrow{\text{letter|digit}} 10 \xrightarrow{\epsilon} 0

1 \xrightarrow{b} 2 \xrightarrow{e} 3 \xrightarrow{g} 4 \xrightarrow{i} 5 \xrightarrow{n} 6

0,1,7,10 \xrightarrow{b} 1

\{b\} \xrightarrow{-} 8 \xrightarrow{\text{digit}} 6 \xrightarrow{-} 0 \xrightarrow{0,1,7,10} 8

2,11
Converting NFA to DFA
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LEXICAL ERRORS

Primarily of two kinds:

1. Lexemes whose length exceed the bound specified by the language.
   - In Fortran, an identifier more than 7 characters long is a lexical error.
   - Most languages have a bound on the precision of numeric constants. A constant whose length exceeds this bound is a lexical error.

2. Illegal characters in the program.
   - The characters ~, &, and @ occurring in a Pascal program (but not within a string or a comment) are lexical errors.

3. Unterminated strings or comments.
issuing an error message, the action taken on detection of an error are:

1. Issue an appropriate error message.
2. Error of the first type – the entire lexeme is read and then truncated to the specified length.
   - Error of the second type –
     - skip illegal character.
     - pass the character to the parser which has better knowledge of the context in which error has occurred. more possibilities of recovery - replacement instead of deletion.
   - Error of the third type – wait till end of file an issue error message.
MINIMIZING THE NUMBER OF STATES

- The DFA constructed for \((b|\epsilon)(a|b)^*abb\).
The DFA constructed for \((b|\epsilon)(a|b)^{*}abb\).

There is another DFA for the same regular expression with lesser number of states.
MINIMIZING THE NUMBER OF STATES

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MINIMIZING THE NUMBER OF STATES

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- There is another DFA for the same regular expression with lesser number of states.

- For a typical language, the number of states of the DFA is in order of hundreds.
- Therefore we should try to minimize the number of states.
The second DFA has been obtained by merging states 1 and 3 of the first DFA.
The second DFA has been obtained by merging states 1 and 3 of the first DFA.

Under what conditions can this merging take place?
MINIMIZING THE NUMBER OF STATES

- The string $bb$ takes both states 1 and 3 to a final state.
MINIMIZING THE NUMBER OF STATES

- The string $\text{bb}$ takes both states 1 and 3 to a final state.
- The string $\text{aba}$ takes both states 1 and 3 to a non-final state.
MINIMIZING THE NUMBER OF STATES

- The string $bb$ takes both states 1 and 3 to a final state.
- The string $aba$ takes both states 1 and 3 to a non-final state.
- The string $\epsilon$ takes both states 1 and 3 to a non-final state.
MINIMIZING THE NUMBER OF STATES

- The string $bb$ takes both states 1 and 3 to a final state.
- The string $aba$ takes both states 1 and 3 to a non-final state.
- The string $\epsilon$ takes both states 1 and 3 to a non-final state.
- The string $bbabb$ takes both states 1 and 3 to a final state.
The string \texttt{bb} takes both states 1 and 3 to a final state.

The string \texttt{aba} takes both states 1 and 3 to a non-final state.

The string \( \varepsilon \) takes both states 1 and 3 to a non-final state.

The string \texttt{bbabb} takes both states 1 and 3 to a final state.

**Conclusion:**

Any string that takes state 1 to a final state also takes 3 to a final state. Conversely, any string that takes state 1 to a non-final state also takes 3 to a non-final state.
MINIMIZING THE NUMBER OF STATES

- States 1 and 3 are said to be indistinguishable.
- Minimization strategy:
  - Find indistinguishable states.
  - Merge them.
- Question: How does one find indistinguishable states?
MINIMIZING THE NUMBER OF STATES

Key idea:
MINIMIZING THE NUMBER OF STATES

Key idea:

- Initially assume all states to be indistinguishable. Put them in a single set.
Key idea:

- The string $\epsilon$ distinguishes between final states and non-final states. Create two partitions.
MINIMIZING THE NUMBER OF STATES

Key idea:

- $b$ takes 4 to a red partition and retains other blue states in blue partition. Put 4 in a separate partition.
MINIMIZING THE NUMBER OF STATES

Key idea:

- The string $b$ distinguishes 3 from other states in the blue partition.
MINIMIZING THE NUMBER OF STATES

Key idea:

- No other partition possible. Merge all states in the same partition.
Summary of the Method

1. Construct an initial partition $\pi = \{S - F, F_1, \ldots, F_n\}$, where $F = F_1 \cup F_2 \cup \ldots F_n$, and each $F_i$ is the set of final states for some token $i$.

2. for each set $G$ in $\pi$ do
   partition $G$ into subsets such that two states $s$ and $t$ of $G$ are in the same subset if and only if for all input symbols $a$, states $s$ and $t$ have transitions onto states in the same set of $\pi$;
   replace $G$ in $\pi_{new}$ by the set of all subsets formed

3. If $\pi_{new} = \pi$, let $\pi_{final} := \pi$ and continue with step 4. Otherwise repeat step 2 with $\pi := \pi_{new}$.

4. Merge states in the same set of the partition.

5. Remove any dead states.
EFFICIENT REPRESENTATION OF DFA

A naive method to represent a DFA uses a two dimensional array.

For a typical language:
- the number of DFA states is in the order of hundreds (sometimes 1000),
- the number of input symbols is greater than 100.

It is desirable to find a space-efficient representation of the DFA.
Key Observation For a DFA that we have seen earlier, the states marked with # behave like state 8 on all symbols except for one symbol.

Therefore information about state 8 can also be used for these states.
Four Arrays Representation of DFA

**Symbols and their numbering**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Numbering</th>
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<tbody>
<tr>
<td>a-z</td>
<td>0–25</td>
</tr>
<tr>
<td>0–9</td>
<td>26–35</td>
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<td>-</td>
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</table>
### Four Arrays Representation of DFA

**Symbols and their numbering**

- **a–z** 0–25
- **0–9** 26–35
- **-** 36

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<tr>
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Amitabha Sanyal

IIT Bombay
Symbols and their numbering

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### Symbols and their numbering

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Four Arrays Representation of DFA

If $s$ is a state and $a$ is the numeric representation of a symbol, then

1. $BASE[s]$ gives the base location for the information stored about state $s$.

2. $NEXT[BASE[s]+a]$ gives the next state for $s$ and symbol $a$, only if $CHECK[BASE[s]+a] = s$.

3. If $CHECK[BASE[s]+a] \neq s$, then the next state information is associated with $DEFAULT[s]$.

```plaintext
function nextstate(s,a);
begin
    if $CHECK[BASE[s] + a] = s$ then $NEXT[BASE[s]+a]$ 
    else return(nextstate($DEFAULT[s],a$))
end
```
Four Arrays Representation of DFA

- All the entries for state 8 have been stored in the array NEXT. The CHECK array shows that the entries are valid for state 8.
- State 1 has a transition on e(4), which is different from the corresponding transition on state 8. This differing entry is stored in NEXT[37]. Therefore BASE[1] is set to $37 - 4 = 33$.
- By a similar reasoning BASE[0] is set to 36.
- To find nextstate[1, 0], we first refer to NEXT[33 + 0], But since CHECK[33 + 0] is not 1 we have to refer to DEFAULT[1] which is 8. So the correct next state is found from NEXT[BASE[8] + 0] = 8.
- To fill up the four arrays, we have to use a heuristic method. One possibility, which works well in practice, is to find for a given state, the lowest BASE, so that the special entries can be filled without conflicting with existing entries.