Lexical Analysis

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Recap

The input program – as you see it.

```
main ()
{
    int i,sum;
    sum = 0;
    for (i=1; i<=10; i++);
    sum = sum + i;
    printf("%d\n",sum);
}
```

Recap

The same program – as the compiler sees it (initially).

```
main␣()↩{↩␣␣␣␣int␣i,sum;↩␣␣␣␣sum␣=␣0;↩␣␣␣␣
for␣(i=1;␣i<=10;␣i++);␣␣␣␣sum␣=␣sum␣+␣i;↩␣␣␣␣
printf("%d\n",sum);↩}
```

␣ – The blank space character

↩ – The return character

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printf("%d\n",sum);↩}
```

How do you make the compiler see what you see?

Recap - Discovering the structure of the program

Step 1:

a. Break up this string into 'words'–the smallest logical units.

```
main | ␣ | ( | ) | ↩ | { | ↩ | ␣ | ␣ | ␣ | ␣ | int | ␣ | i | , | sum
; | ↩ | ␣ | ␣ | ␣ | ␣ | sum | ␣ | = | ␣ | 0 | ; | ↩ | ␣ | ␣ | ␣ | ␣
for | ␣ | ( | i | = | 1 | ; | ␣ | i | <= | 10 | ; | ␣ | i | ++ | ) | ; | ␣
␣ | ␣ | ␣ | sum | ␣ | = | ␣ | sum | ␣ | + | ␣ | i; | ↩ | ␣ | ␣ | ␣
␣ | printf | ( | "%d\n" | , | sum | ) | ; | ↩ | }
```

We get a sequence of *lexemes* or *tokens*.

Recap - Discovering the structure of the program

Step 1:

b. Clean up – remove the ␣ and the ↩ characters.

```
main ( ) { int i , sum ; sum = 0 ; for (
i = 1 ; i <= 10 ; i ++ ) ; sum = sum + i
; printf ( "%d\n" , sum ) ; }
```

Steps 1a. and 1b. are interleaved.

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;	printf	(	"%d\n"	,	sum	)	;	}											

This is *lexical analysis* or *scanning*.

Recap - Discovering the structure of the program

Step 2:

Now group the lexemes to form larger structures.

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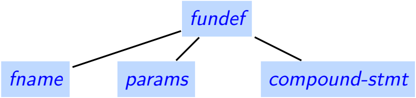

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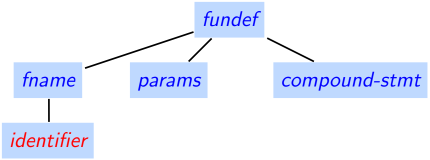

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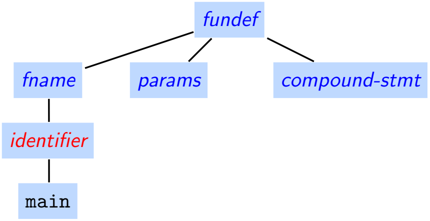

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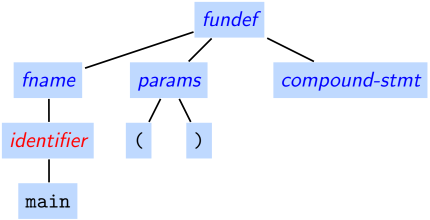

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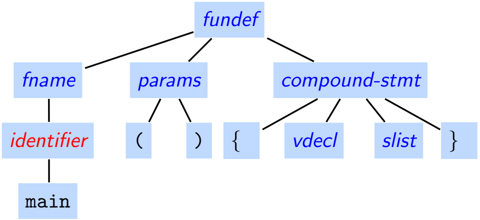

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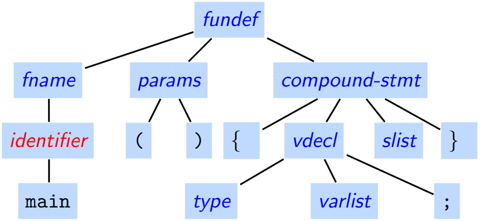

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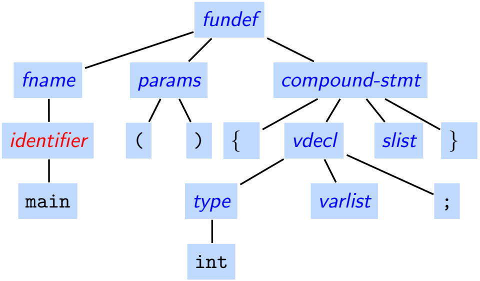

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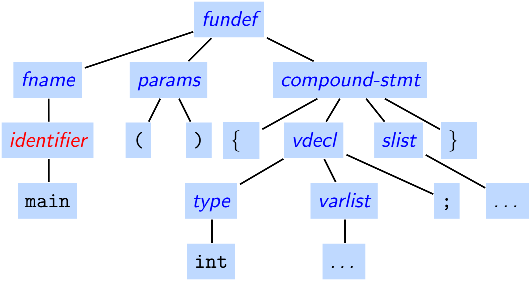

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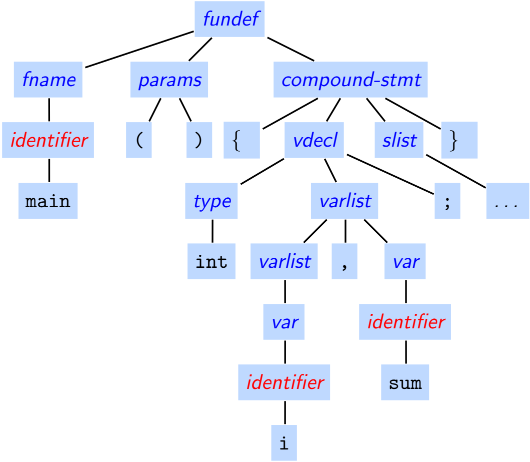

This is *syntax analysis* or *parsing*.

Lexemes, Tokens and Patterns

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- *lexemes* – smallest logical units (words) of a program.
  Examples – `i, sum, for, 10, ++, "%d\n", <=`.

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Distinguish between

- *lexemes* – smallest logical units (words) of a program.
  Examples – `i, sum, for, 10, ++, "%d\n", <=`.
- *tokens* – sets of similar lexemes.
  Examples –
  identifier = {`i, sum, buffer`, ...}
  int_constant = {`1, 10, ...`}
  addop = {`+, -`}

Lexemes, Tokens and Patterns

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These too have to be detected and ignored.

Lexemes, Tokens and Patterns

The lexical analyser:

- detects the next lexeme
- categorises it into the right token
- passes to the syntax analyser
  - the token name for further syntax analysis
  - the lexeme itself, in some form, for stages beyond syntax analysis

Recap - Lexemes, Tokens and Patterns

How does one describe the lexemes that make up the token *identifier*.

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Such descriptions are called *patterns.* The description may be informal or formal.

Recap - Basic concepts and issues

A pattern is used to

- *specify tokens* precisely
- *build a recognizer* from such specifications

Example – tokens in Java

1. **Identifier:** A *Javaletter* followed by zero or more *Javaletterordigits*. A *Javaletter* includes the characters `a-z`, `A-Z`, `_` and `$`.
2. **Constants:**
   2.1 Integer Constants
   - Octal, Hex and Decimal
   - 4 byte and 8 byte representation

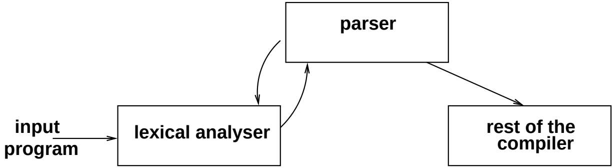
   2.2 Floating point constants
   - float - ends with `f`
   - double

   2.3 Boolean constants – `true` and `false`
   2.4 Character constants – `'a'`, `'\u0034'`, `'\t'`
   2.5 String constants – `""`, `"\""`, `"A string"`.
   2.6 Null constant – `null`.
3. **Delimiters:** `(`, `)`, `{`, `}`, `[`, `]` `,` `;`, `.` and `,`
4. **Operators:** =, >, < . . . >>>=
5. **Keywords:** `abstract`, `boolean` . . . `volatile`, `while`.

Recap - Basic concepts and issues

Where does a lexical analyser fit into the rest of the compiler?

- The front end of most compilers is parser driven.
- When the parser needs the next token, it invokes the Lexical Analyser.
- Instead of analysing the entire input string, the lexical analyser sees enough of the input string to return a single token.
- The actions of the lexical analyser and parser are intertwined.



Recap - Token Attributes

Apart from the token itself, the lexical analyser also passes other informations regarding the token. These items of information are called *token attributes*

EXAMPLE

lexeme	**<token, token attribute>**
`3`	< const, 3>
`A`	<identifier, `A`>
`if`	<if, –>
`=`	<assignop, –>
`>`	<gt, –>
`;`	<semicolon, –>

Creating a Lexical Analyzer

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Two approaches:

1. *Hand code* – This is only of historical interest now.
   - Possibly more efficient.
2. *Use a generator* – To generate the lexical analyser from a formal description.
   - The generation process is faster.
   - Less prone to errors.

Automatic Generation of Lexical Analysers

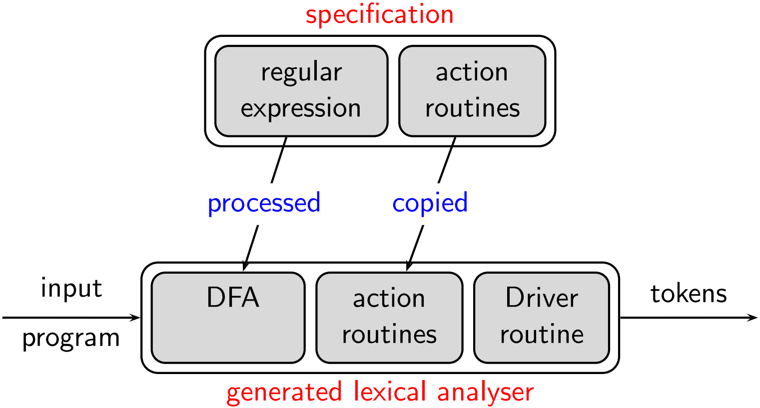
Inputs to the lexical analyser generator:

- A specification of the tokens of the source language, consisting of:
  - a regular expression describing each token, and
  - a code fragment describing the action to be performed, on identifying each token.

The generated lexical analyser consists of:

- A *deterministic finite automaton (DFA)* constructed from the token specification.
- A *code fragment* (a driver routine) which can traverse *any DFA*.
- Code for the *action specifications*.

Automatic Generation of Lexical Analysers

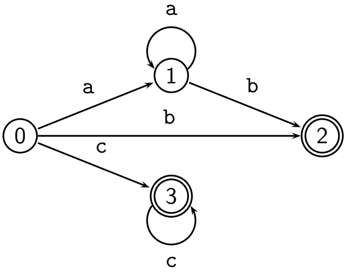


Example of Lexical Analyser Generation

Suppose a language has two tokens

Pattern	Action
`a*b`	`{ printf( "Token 1 found");}`
`c+`	`{ printf( "Token 2 found");}`

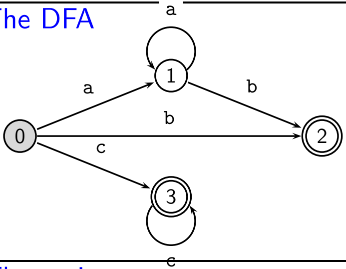
From the description, construct a structure called a deterministic finite automaton (DFA).



Example of Lexical Analyser Generation

Now consider the following together:

The DFA



The driver routine

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void nexttoken ()
  {state = 0; c = nextchar();
   while (valid(nextstate[state,c]))
    {state = nextstate[state,c];
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   if (!final(state))
     {error; return;}
   else
     {unput(c);action();return;}}
```

The actions

```
void action();
{
 switch(state)
  2:{printf("Token 1 found");
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The input and output

Input: aabadbcc⟵

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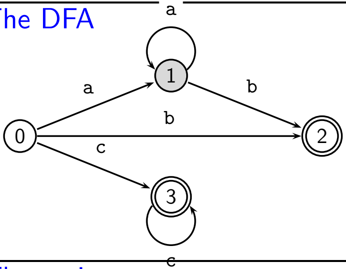

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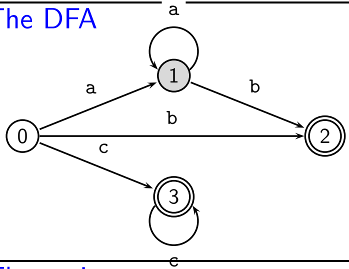

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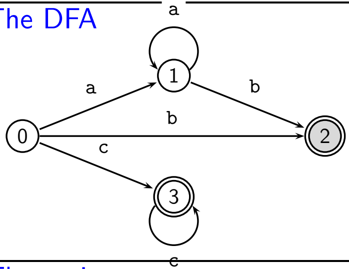

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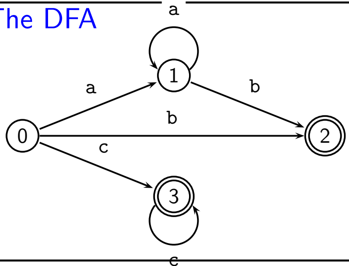

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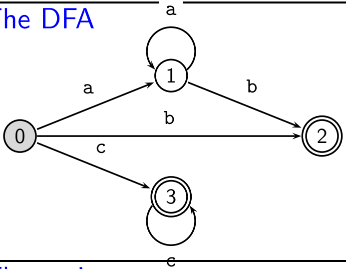

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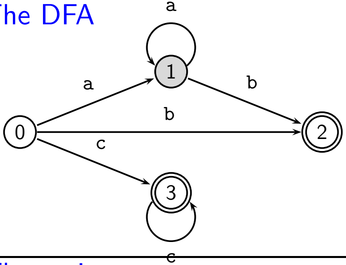

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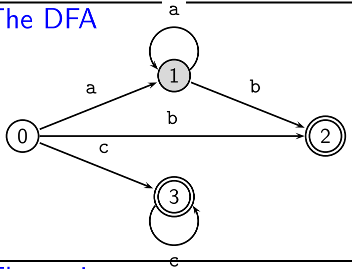

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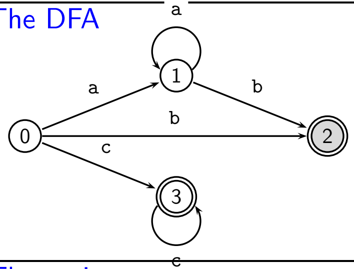

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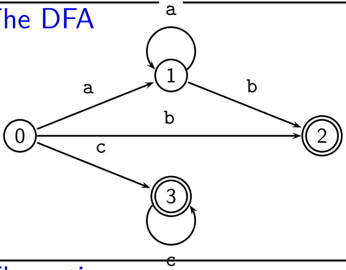

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The actions

```
void action();
{
 switch(state)
  2:{printf("Token 1 found");
     break;}
  3:{printf("Token 2 found");
     break;}
}
```

The input and output

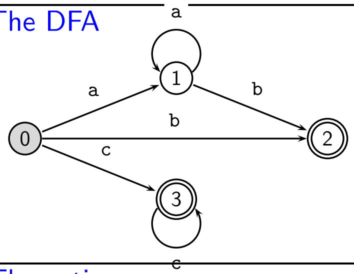
Input: aabadbcc⟵

Output: Token 1 found
Token 1 found

Example of Lexical Analyser Generation

Now consider the following together:

The DFA



The driver routine

```
void nexttoken ()
  {state = 0; c = nextchar();
   while (valid(nextstate[state,c]))
    {state = nextstate[state,c];
     c = nextchar();}
   if (!final(state))
     {error; return;}
   else
     {unput(c);action();return;}}
```

The actions

```
void action();
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     break;}
  3:{printf("Token 2 found");
     break;}
}
```

The input and output

Input: aabadbcc↩

Output: Token 1 found

Token 1 found

Example of Lexical Analyser Generation

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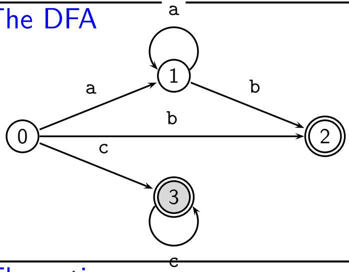

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Example of Lexical Analyser Generation

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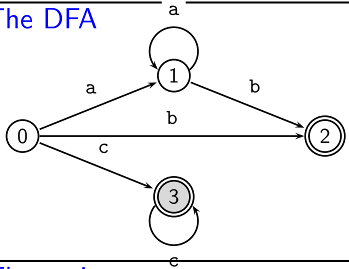

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}
```

The input and output

Input: aabadbcc⟵

Output: Token 1 found
Token 1 found
Token 2 found

Example of Lexical Analyser Generation

In summary:

- The DFA, the driver routine and the action routines taken together, constitute the lexical analyser.
-
  - actions are supplied as part of specification.
  - driver routine is common to all generated lexical analyzers

  The only issue – how are the patterns, specified by regular expressions, converted to a DFA.

  In two steps:

  - Convert regular expression into NFA.
  - Convert NFA to DFA.

Example of Lexical Analyser Generation

Consider a language with the following tokens:

- *begin* – representing the lexeme `begin`
- *integer* – Examples: `0`, `-5`, `250`
- *identifier* – Examples: `a`, `A1`, `max`

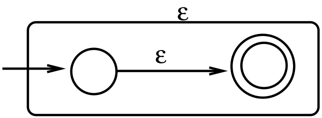
Converting Regular Expressions to NFA

In two parts;

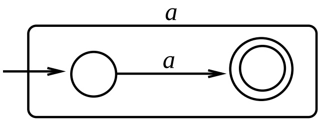
- First convert the regular expression corresponding to each token into a NFA.
  - Invariant: A single final state corresponding to each token.
- Join the NFAs obtained for all the tokens.

Converting Regular Expressions to DFA

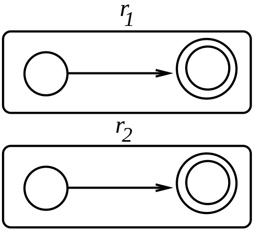
RE for ϵ



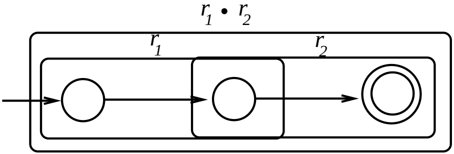
RE for a



Converting Regular Expressions to NFA

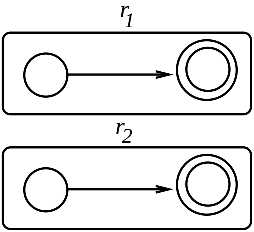


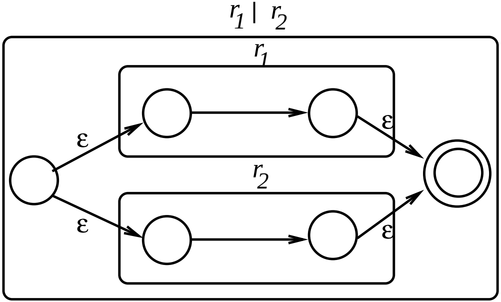
RE for $r_1 \bullet r_2$



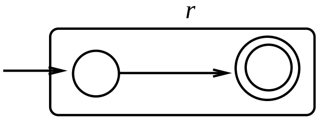
Converting Regular Expressions to NFA

RE for $r_1|r_2$

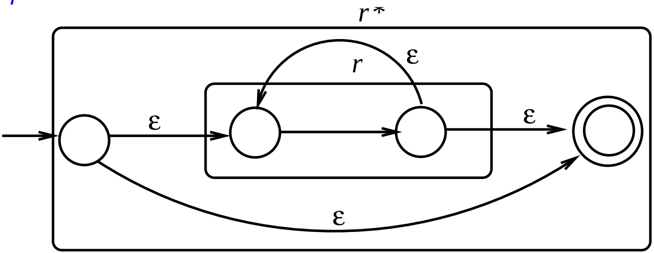




Converting Regular Expressions to NFA

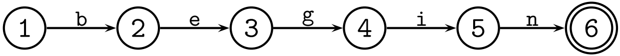


RE for r



Converting NFA to DFA

Converting NFA to DFA



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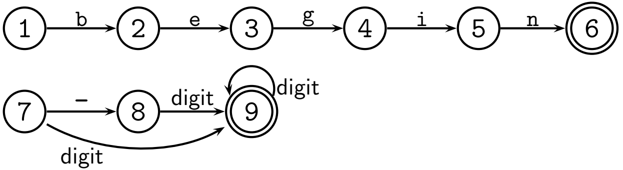

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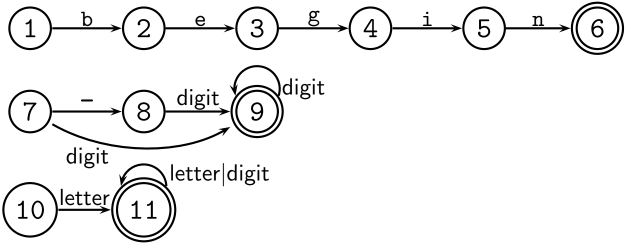

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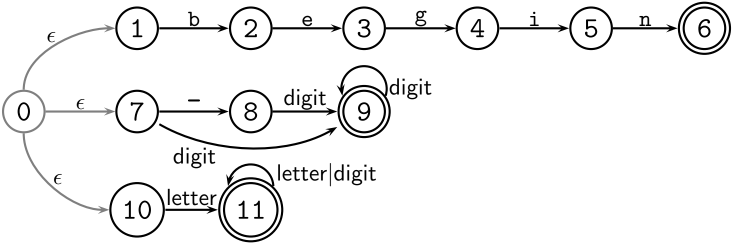

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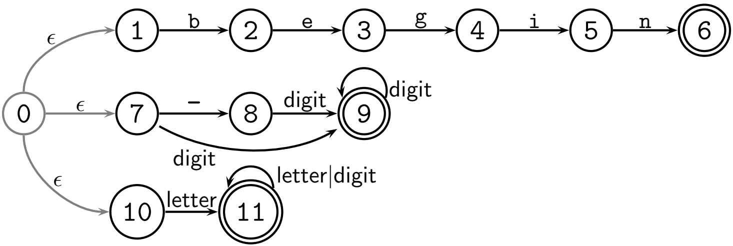
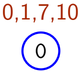
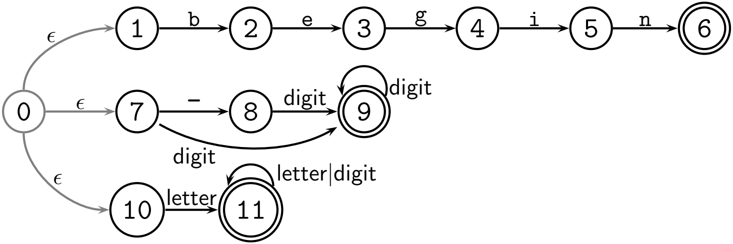
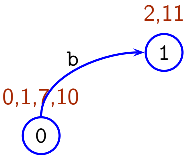




Converting NFA to DFA





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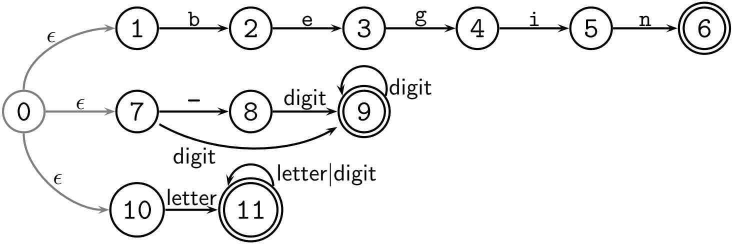

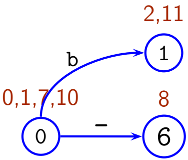

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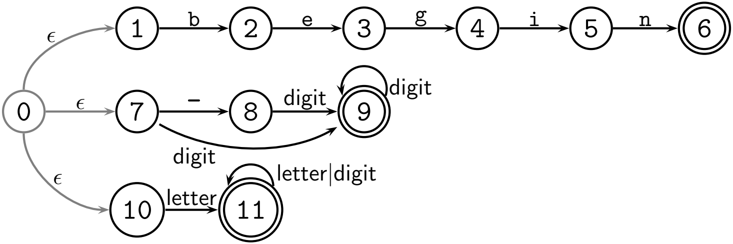

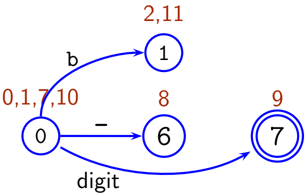

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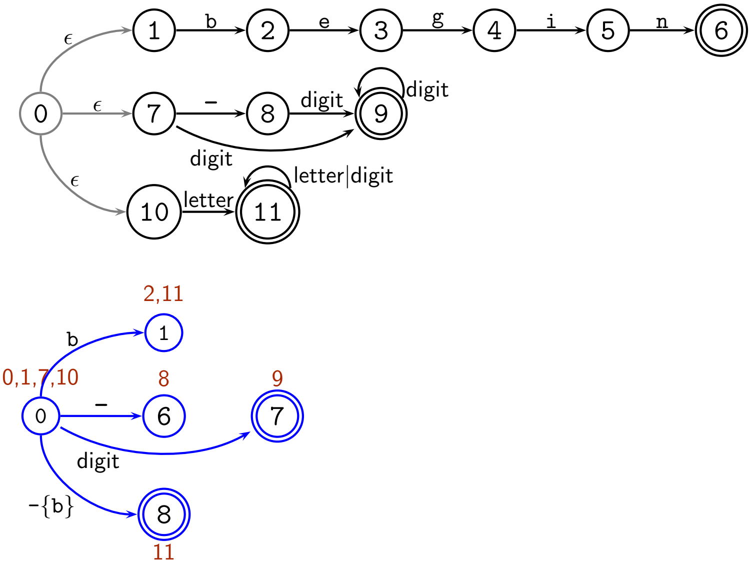

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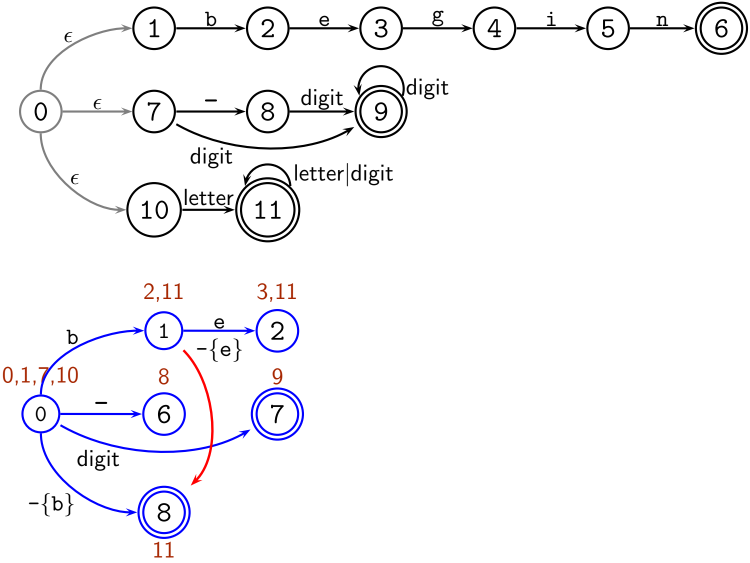

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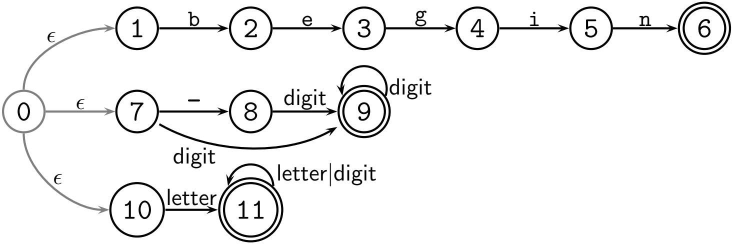

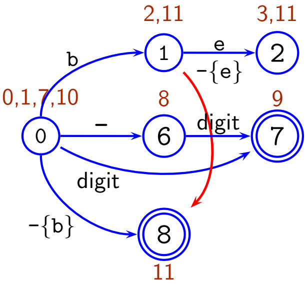

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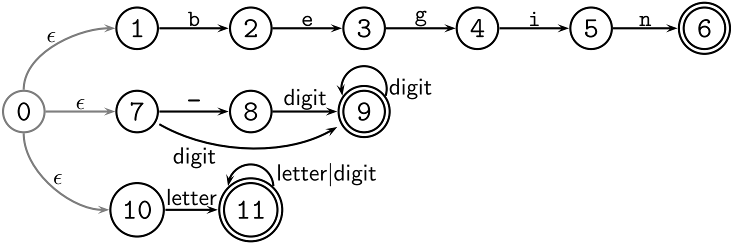

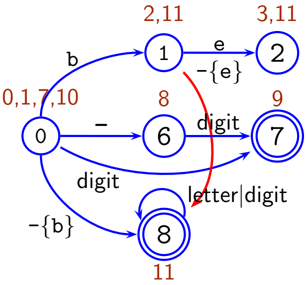

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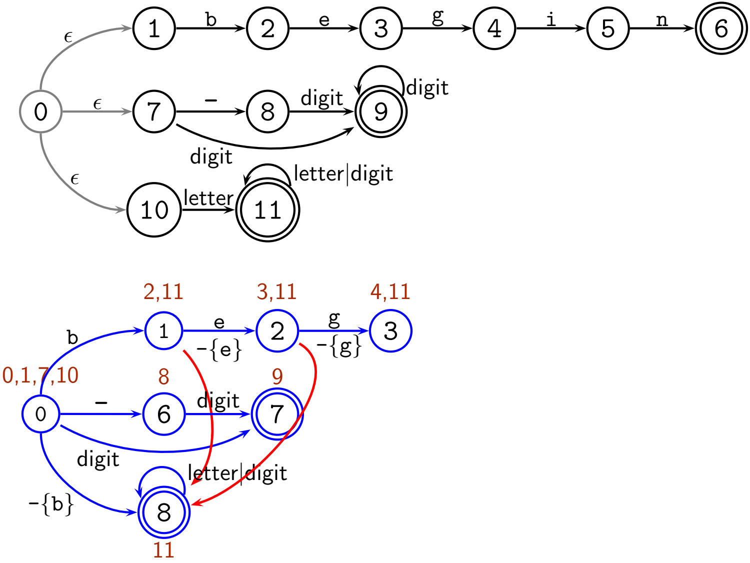

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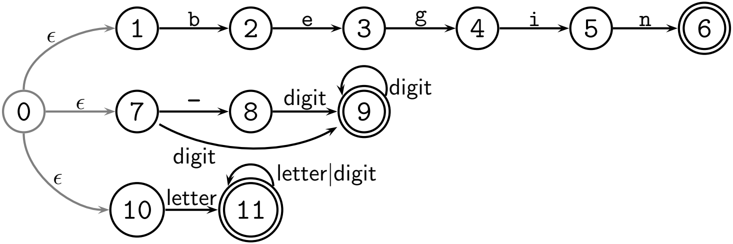

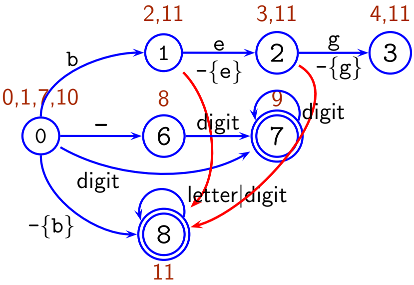

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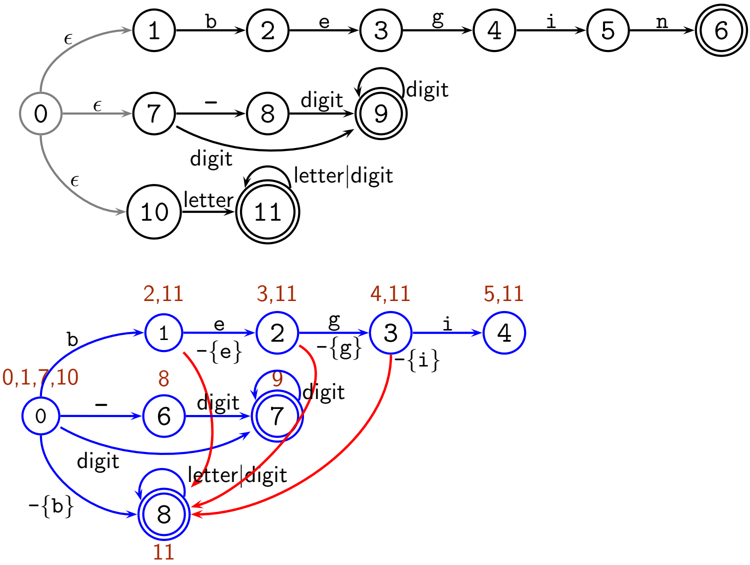

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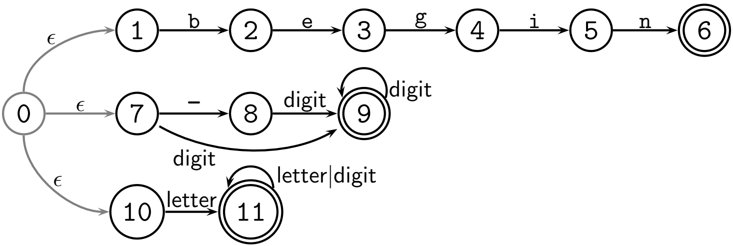

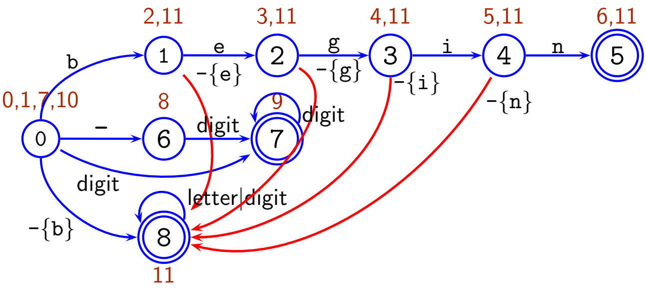

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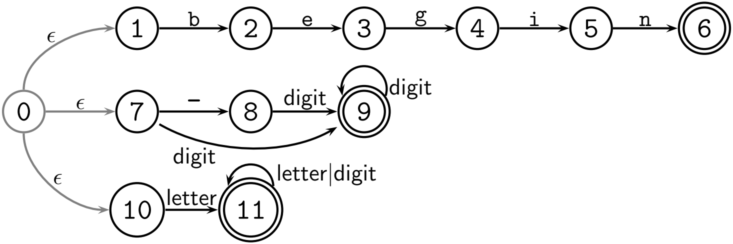
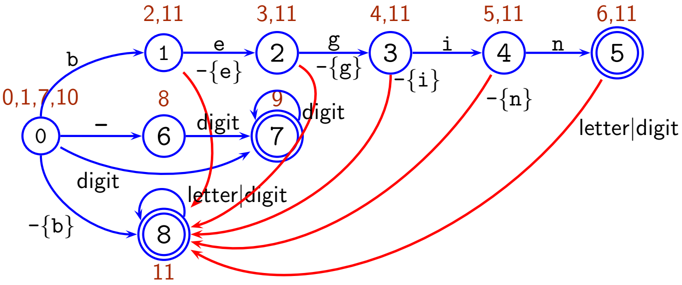




LEXICAL ERRORS

Primarily of two kinds:

1. Lexemes whose length exceed the bound specified by the language.
   - In Fortran, an identifier more than 7 characters long is a lexical error.
   - Most languages have a bound on the precision of numeric constants. A constant whose length exceeds this bound is a lexical error.
2. Illegal characters in the program.
   - The characters `~`, `&` and `@` occuring in a Pascal program (but not within a string or a comment) are lexical errors.
3. Unterminated strings or comments.

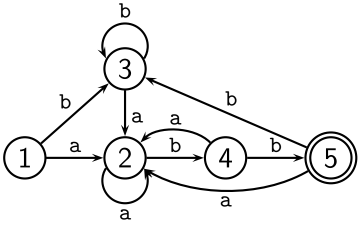
Handling Lexical Errors

issuing an error message, the action taken on detection of an error are:

1. Issue an appropriate error message.
2. - Error of the first type – the entire lexeme is read and then truncated to the specified length.
   - Error of the second type –
     - skip illegal character.
     - pass the character to the parser which has better knowledge of the context in which error has occurred. more possibilities of recovery - *replacement* instead of *deletion*.
   - Error of the third type – wait till end of file an issue error message.

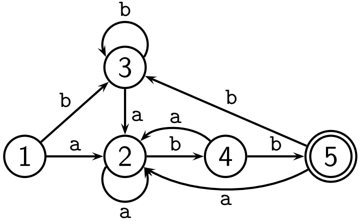
MINIMIZING THE NUMBER OF STATES

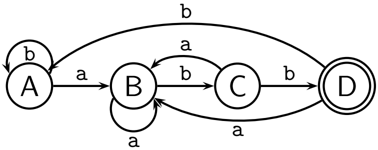
- The DFA constructed for $(b|\epsilon)(a|b)^*abb$.



MINIMIZING THE NUMBER OF STATES

- The DFA constructed for $(b|\epsilon)(a|b)^*abb$.
- There is another DFA for the same regular expression with lesser number of states.





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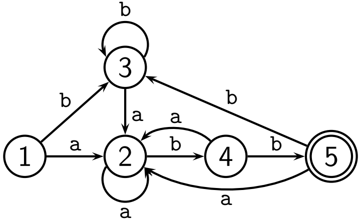

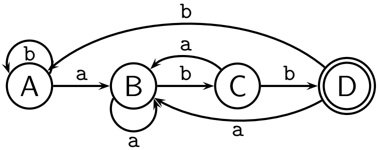

- For a typical language, the number of states of the DFA is in order of hundreds.

MINIMIZING THE NUMBER OF STATES

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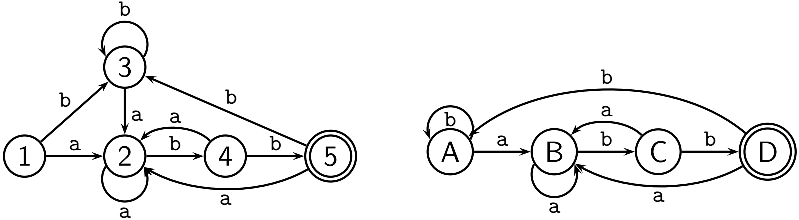
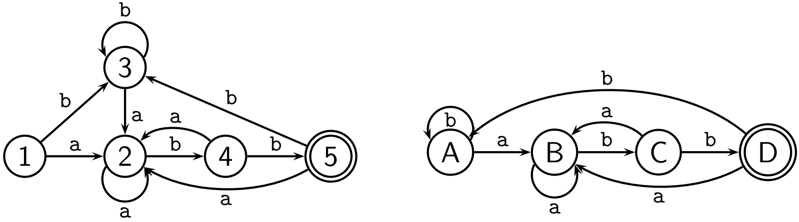
- There is another DFA for the same regular expression with lesser number of states.



- For a typical language, the number of states of the DFA is in order of hundreds.
- Therefore we should try to minimize the number of states.

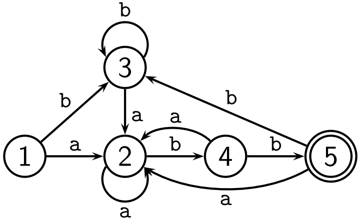
MINIMIZING THE NUMBER OF STATES

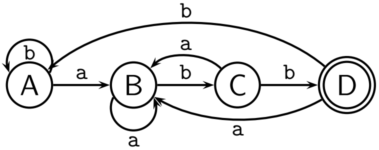
- The second DFA has been obtained by merging states 1 and 3 of the first DFA.



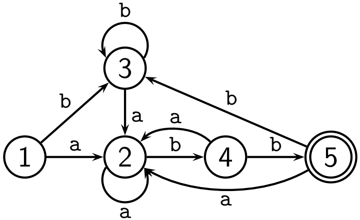
MINIMIZING THE NUMBER OF STATES

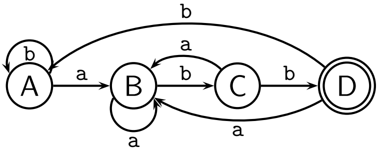
- The second DFA has been obtained by merging states 1 and 3 of the first DFA.
- Under what conditions can this merging take place?





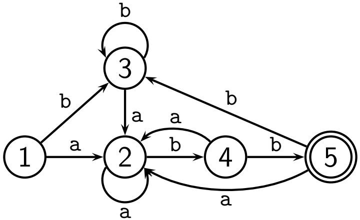
MINIMIZING THE NUMBER OF STATES

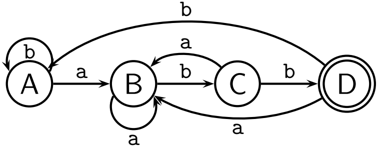




- The string bb takes both states 1 and 3 to a final state.

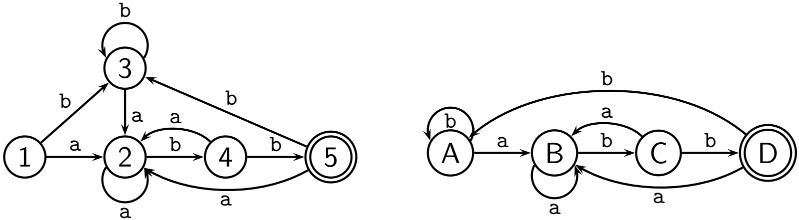
MINIMIZING THE NUMBER OF STATES





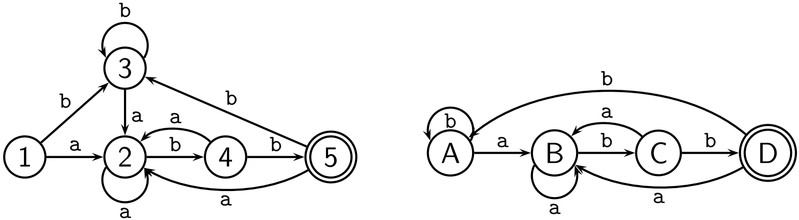
- The string bb takes both states 1 and 3 to a final state.
- The string aba takes both states 1 and 3 to a `non-final state`.

MINIMIZING THE NUMBER OF STATES



- The string bb takes both states 1 and 3 to a final state.
- The string aba takes both states 1 and 3 to a `non-final state`.
- The string ϵ takes both states 1 and 3 to a non-final state.

MINIMIZING THE NUMBER OF STATES



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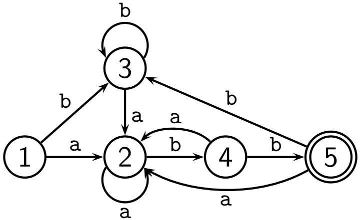
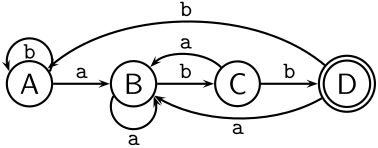
MINIMIZING THE NUMBER OF STATES



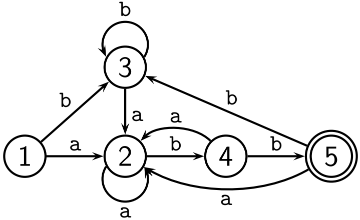


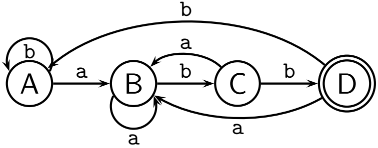
- The string bb takes both states 1 and 3 to a final state.
- The string aba takes both states 1 and 3 to a non-final state.
- The string ϵ takes both states 1 and 3 to a non-final state.
- The string bbabb takes both states 1 and 3 to a final state.

Conclusion:

Any string that takes state 1 to a final state also takes 3 to a final state. Conversely, any string that takes state 1 to a non-final state also takes 3 to a non-final state.

MINIMIZING THE NUMBER OF STATES





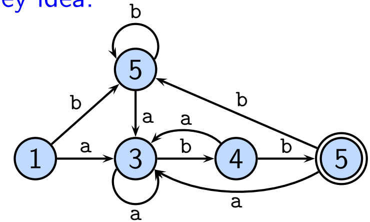
- States 1 and 3 are said to be *indistinguishable*.
- Minimimization strategy:
  - Find indistinguishable states.
  - Merge them.
- Question: How does one find indistingushable states?

MINIMIZING THE NUMBER OF STATES

Key idea:

MINIMIZING THE NUMBER OF STATES

Key idea:



- Initially assume all states to be indistinguishable. Put them in a single set.

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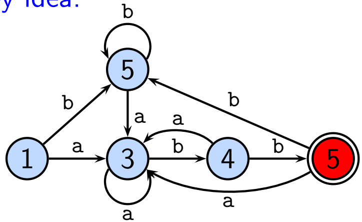

- The string ϵ distinguishes between final states and non-final states. Create two partitions.

MINIMIZING THE NUMBER OF STATES

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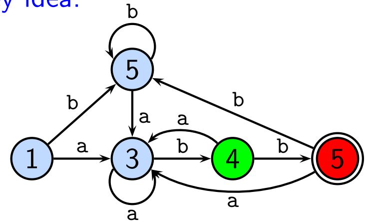
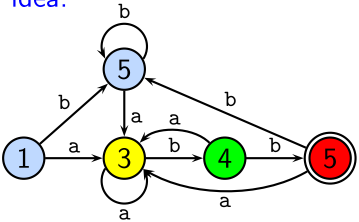


- b takes 4 to a red partition and retains other blue states in blue partition. Put 4 in a separate partition.

MINIMIZING THE NUMBER OF STATES

Key idea:



- The string b distinguishes 3 from other states in the blue partition.

MINIMIZING THE NUMBER OF STATES

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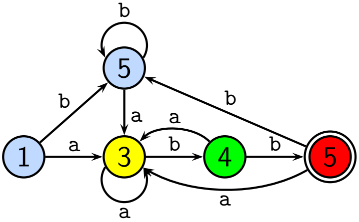
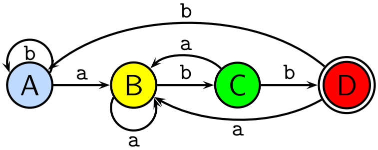




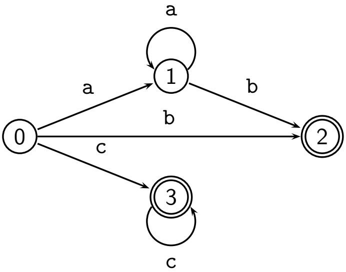
- No other partition possible. Merge all states in the same partition.

Summary of the Method

1. Construct an initial partition $\pi = \{S - F, F_1, \ldots, F_n,\}$, where $F = F_1 \cup F_2 \cup \ldots F_n s$, and each F_i is the set of final states for some token i.
2. for each set G in π do
   partition G into subsets such that two states
   s and t of G are in the same subset if and only if
   for all input symbols a, states s and t have transitions
   onto states in the same set of π;
   replace G in π_{new} by the set of all subsets formed
3. If $\pi_{new} = \pi$, let $\pi_{final} := \pi$ and continue with step 4. Otherwise repeat step 2 with $\pi := \pi_{new}$.
4. Merge states in the same set of the partition.
5. Remove any dead states.

EFFICIENT REPRESENTATION OF DFA

A naive method to represent a DFA uses a two dimensional array.



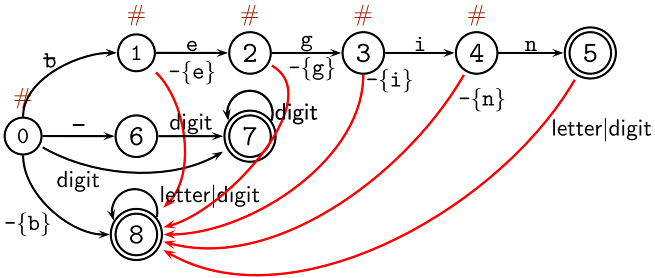
x

	a	b	c
0	1	2	3
2	1	2	–
2	–	–	–
3	–	–	3

- For a typical language:
  - the number of DFA states is in the order of hundreds (sometimes 1000),
  - the number of input symbols is greater than 100.
- It is desirable to find a space-efficient representation of the DFA.

The Four Arrays Scheme

Key Observation For a DFA that we have seen earlier, the states marked with # behave like state 8 on all symbols *except for one symbol*.

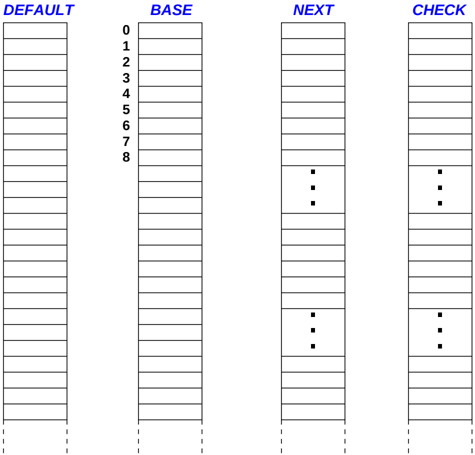


Therefore information about state 8 can also be used for these states.

Four Arrays Representation of DFA

Symbols and their numbering

a–z	0–25
0–9	26–35
-	36



Four Arrays Representation of DFA

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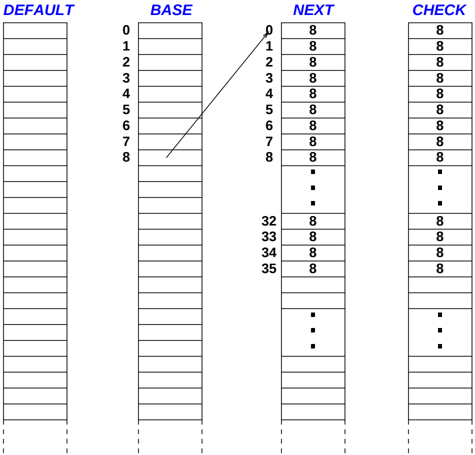

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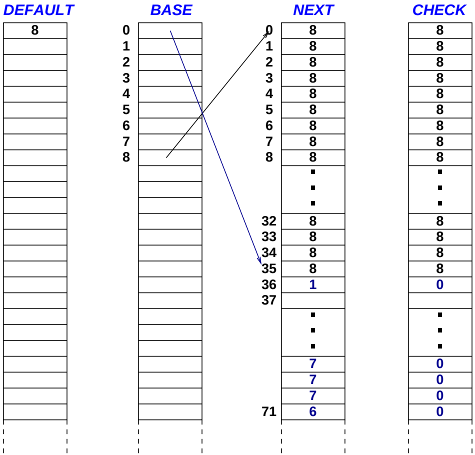

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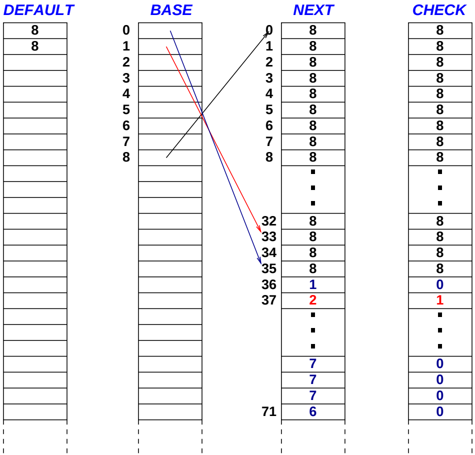

Four Arrays Representation of DFA

If s is a state and a is the numeric representation of a symbol, then

1. $BASE[s]$ gives the base location for the information stored about state s.
2. $NEXT[BASE[s]{+}a]$ gives the next state for s and symbol a, only if $CHECK[BASE[s]{+}a] = s$.
3. If $CHECK[BASE[s]{+}a] \neq s$, then the next state information is associated with $DEFAULT[s]$.

function *nextstate*(s,a);
begin
    if $CHECK[BASE[s] + a] = s$ then $NEXT[BASE[s]{+}a]$
    else return(*nextstate*($DEFAULT[s]$,a))
end

Four Arrays Representation of DFA

- All the entries for state 8 have been stored in the array *NEXT*. The *CHECK* array shows that the entries are valid for state 8.
- State 1 has a transition on e(4), which is different from the corresponding transition on state 8. This differing entry is stored in $NEXT[37]$. Therefore $BASE[1]$ is set to $37 - 4 = 33$.
- By a similar reasoning $BASE[0]$ is set to 36.
- To find $nextstate[1, 0]$, we first refer to $NEXT[33 + 0]$, But since $CHECK[33 + 0]$ is not 1 we have to refer to $DEFAULT[1]$ which is 8. So the correct next state is found from $NEXT[BASE[8] + 0] = 8$.
- To fill up the four arrays, we have to use a heuristic method. One possibility, which works well in practice, is to find for a given state, the lowest *BASE*, so that the special entries can be filled without conflicting with existing entries.