# *General Data Flow Framekworks*

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## *Part 1*

# *About These Slides*

# Copyright

These slides constitute the lecture notes for CS618 Program Analysis course at IIT Bombay and have been made available as teaching material accompanying the book:

- Uday Khedker, Amitabha Sanyal, and Bageshri Karkare. *Data Flow Analysis: Theory and Practice*. CRC Press (Taylor and Francis Group). 2009.

Apart from the above book, some slides are based on the material from the following book

- M. S. Hecht. *Flow Analysis of Computer Programs*. Elsevier North-Holland Inc. 1977.

# Outline

- Modelling General Flows
- Constant Propagation
- Faint Variables Analysis
- Pointer Analyses
- Heap Reference Analysis

## Part 2

# Modelling General Flows

# Modelling Flow Functions for General Flows

- General flow functions can be written as

$$f_n(X) \quad = \quad (X - \mathsf{Kill}_n(X)) \cup \mathsf{Gen}_n(X)$$

where Gen and Kill have constant and dependent parts

$$\begin{aligned} \mathsf{Gen}_n(X) &= ConstGen_n \cup DepGen_n(X) \\ \mathsf{Kill}_n(X) &= ConstKill_n \cup DepKill_n(X) \end{aligned}$$

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- The dependent parts take care of
  - dependence across different entities as well as
  - dependence on the value of the same entity in the argument $X$
- Bit vector frameworks are a special case

$$DepGen_n(X) = DepKill_n(X) = \emptyset$$

# Separability

$f : L \mapsto L$ is $\langle \widehat{h}_1, \widehat{h}_2, \ldots, \widehat{h}_m \rangle$ where $\widehat{h}_i$ computes the value of $\widehat{x}_i$

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| Separable | Non-Separable |
|---|---|
| Example: All bit vector frameworks | Example: Constant Propagation |

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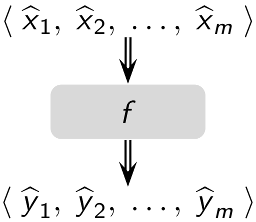
**Separable**



$$\langle\ \widehat{x}_1,\ \widehat{x}_2,\ \ldots,\ \widehat{x}_m\ \rangle$$

$$\Downarrow$$

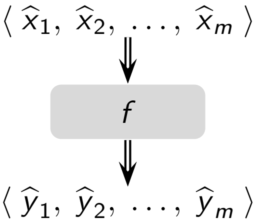
$$f$$

$$\Downarrow$$

$$\langle\ \widehat{y}_1,\ \widehat{y}_2,\ \ldots,\ \widehat{y}_m\ \rangle$$

Example: All bit vector frameworks

**Non-Separable**



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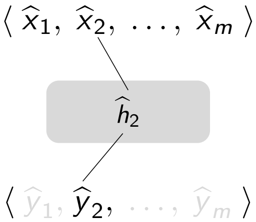

$$\langle\ \widehat{x}_1,\ \widehat{x}_2,\ \ldots,\ \widehat{x}_m\ \rangle$$

$$\widehat{h}_2$$

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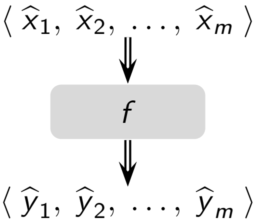

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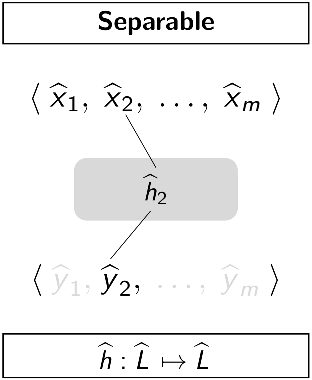

**Separable**

$\langle\ \widehat{x}_1,\ \widehat{x}_2,\ \ldots,\ \widehat{x}_m\ \rangle$

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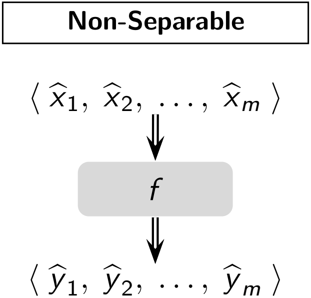
$\widehat{h} : \widehat{L} \mapsto \widehat{L}$



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Example: All bit vector frameworks

Example: Constant Propagation

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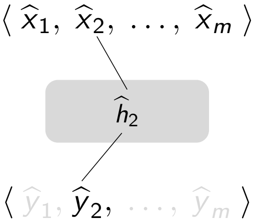

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Example: All bit vector frameworks

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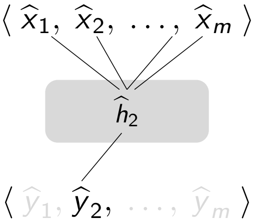

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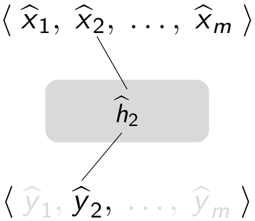

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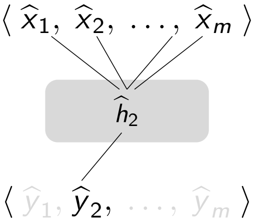

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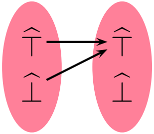
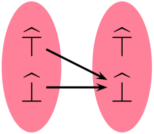
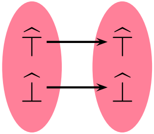
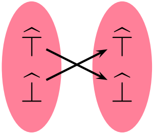
Example: Constant Propagation

# Larger Values of Loop Closure Bounds

- Fast Frameworks ≡ 2-bounded frameworks (eg. bit vector frameworks)
  Both these conditions must be satisfied
  - *Separability*
    Data flow values of different entities are independent
  - *Constant or Identity Flow Functions*
    Flow functions for an entity are either constant or identity
- Non-fast frameworks
  At least one of the above conditions is violated

# Separability of Bit Vector Frameworks

- $\widehat{L}$ is $\{0,1\}$, $L$ is $\{0,1\}^m$
- $\widehat{\sqcap}$ is either boolean AND or boolean OR
- $\widehat{\top}$ and $\widehat{\bot}$ are 0 or 1 depending on $\widehat{\sqcap}$.
- $\widehat{h}$ is a *bit function* and could be one of the following:

| *Raise* | *Lower* | *Propagate* | *Negate* |
|---|---|---|---|
|  |  |  |  |

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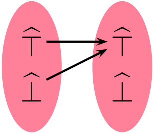
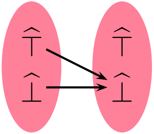
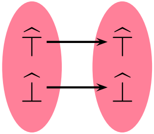
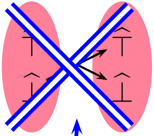
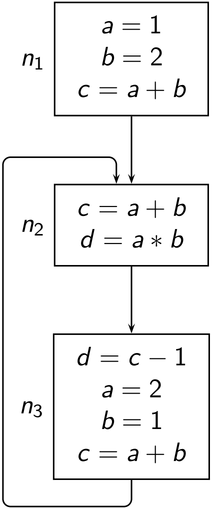
|  |  |  |  |

Non-monotonicity

### *Part 3*

# *Constant Propagation*

# Example of Constant Propagation



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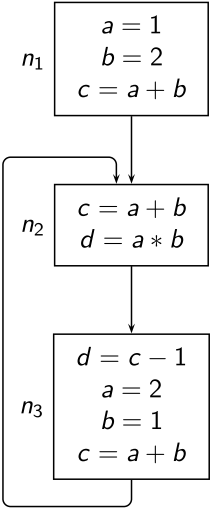


MoP

$\langle \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top} \rangle$

$\langle 1, 2, 3, \widehat{\top} \rangle$

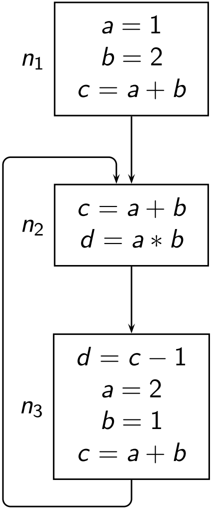
$\langle \widehat{\bot}, \widehat{\bot}, 3, 2 \rangle$

$\langle \widehat{\bot}, \widehat{\bot}, 3, 2 \rangle$

$\langle \widehat{\bot}, \widehat{\bot}, 3, 2 \rangle$

$\langle 2, 1, 3, 2 \rangle$

# Example of Constant Propagation



$n_1$

$a = 1$

$b = 2$

$c = a + b$

$n_2$

$c = a + b$

$d = a * b$

$n_3$

$d = c - 1$

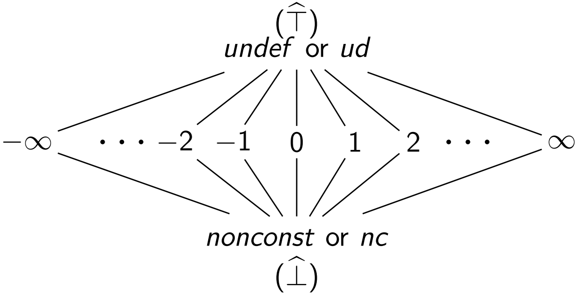
$a = 2$

$b = 1$

$c = a + b$

| MoP | MFP |
|---|---|
| $\langle \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top} \rangle$ | $\langle \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top} \rangle$ |
| $\langle 1, 2, 3, \widehat{\top} \rangle$ | $\langle 1, 2, 3, \widehat{\top} \rangle$ |
| $\langle \widehat{\bot}, \widehat{\bot}, 3, 2 \rangle$ | $\langle \widehat{\bot}, \widehat{\bot}, 3, \widehat{\bot} \rangle$ |
| $\langle \widehat{\bot}, \widehat{\bot}, 3, 2 \rangle$ | $\langle \widehat{\bot}, \widehat{\bot}, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $\langle \widehat{\bot}, \widehat{\bot}, 3, 2 \rangle$ | $\langle \widehat{\bot}, \widehat{\bot}, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $\langle 2, 1, 3, 2 \rangle$ | $\langle 2, 1, 3, \widehat{\bot} \rangle$ |

# Component Lattice for Integer Constant Propagation



$(\widehat{\top})$

*undef* or *ud*

$-\infty \quad \cdots \; -2 \quad -1 \quad 0 \quad 1 \quad 2 \; \cdots \quad \infty$

*nonconst* or *nc*

$(\widehat{\bot})$

- Overall lattice $L$ is the product of $\widehat{L}$ for all variables.
- $\sqcap$ and $\widehat{\sqcap}$ get defined by $\sqsubseteq$ and $\widehat{\sqsubseteq}$.

| $\widehat{\sqcap}$ | $\langle v, ud \rangle$ | $\langle v, nc \rangle$ | $\langle v, c_1 \rangle$ |
|---|---|---|---|
| $\langle v, ud \rangle$ | $\langle v, ud \rangle$ | $\langle v, nc \rangle$ | $\langle v, c_1 \rangle$ |
| $\langle v, nc \rangle$ | $\langle v, nc \rangle$ | $\langle v, nc \rangle$ | $\langle v, nc \rangle$ |
| $\langle v, c_2 \rangle$ | $\langle v, c_2 \rangle$ | $\langle v, nc \rangle$ | If $c_1 = c_2$ then $\langle v, c_1 \rangle$ else $\langle v, nc \rangle$ |

# Flow Functions for Constant Propagation

- Flow function for $r = a_1 * a_2$

| $mult$ | $\langle a_1, ud \rangle$ | $\langle a_1, nc \rangle$ | $\langle a_1, c_1 \rangle$ |
|---|---|---|---|
| $\langle a_2, ud \rangle$ | $\langle r, ud \rangle$ | $\langle r, nc \rangle$ | $\langle r, ud \rangle$ |
| $\langle a_2, nc \rangle$ | $\langle r, nc \rangle$ | $\langle r, nc \rangle$ | $\langle r, nc \rangle$ |
| $\langle a_2, c_2 \rangle$ | $\langle r, ud \rangle$ | $\langle r, nc \rangle$ | $\langle r, (c_1 * c_2) \rangle$ |

# Defining Data Flow Equations for Constant Propagation

| | $ConstGen_n$ | $DepGen_n(X)$ | $ConstKill_n$ | $DepKill_n(X)$ |
|---|---|---|---|---|
| $v = c$,<br>$c \in \mathbb{C}$onst | $\{\langle v, c\rangle\}$ | $\emptyset$ | $\emptyset$ | $\{\langle v, d\rangle \mid \langle v, d\rangle \in X\}$ |
| $v = e$,<br>$e \in \mathbb{E}$xpr | $\emptyset$ | $\{\langle v, eval(e,X)\rangle\}$ | $\emptyset$ | $\{\langle v, d\rangle \mid \langle v, d\rangle \in X\}$ |
| $read(v)$ | $\{\langle v, nc\rangle\}$ | $\emptyset$ | $\emptyset$ | $\{\langle v, d\rangle \mid \langle v, d\rangle \in X\}$ |
| $other$ | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |

# Defining Data Flow Equations for Constant Propagation

| | $ConstGen_n$ | $DepGen_n(X)$ | $ConstKill_n$ | $DepKill_n(X)$ |
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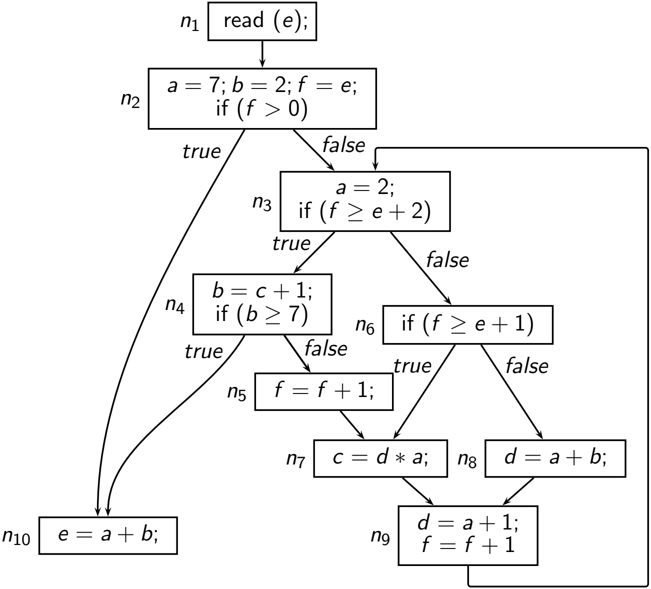
| $v = c$,<br>$c \in \mathbb{C}\text{onst}$ | $\{\langle v, c\rangle\}$ | $\emptyset$ | $\emptyset$ | $\{\langle v, d\rangle \mid \langle v, d\rangle \in X\}$ |
| $v = e$,<br>$e \in \mathbb{E}\text{xpr}$ | $\emptyset$ | $\{\langle v, eval(e,X)\rangle\}$ | $\emptyset$ | $\{\langle v, d\rangle \mid \langle v, d\rangle \in X\}$ |
| $read(v)$ | $\{\langle v, nc\rangle\}$ | $\emptyset$ | $\emptyset$ | $\{\langle v, d\rangle \mid \langle v, d\rangle \in X\}$ |
| $other$ | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |

| $eval(a_1\ op\ a_2, X)$ | | | |
|---|---|---|---|
| | $\langle a_1, ud\rangle \in X$ | $\langle a_1, nc\rangle \in X$ | $\langle a_1, c_1\rangle \in X$ |
| $\langle a_2, ud\rangle \in X$ | $ud$ | $nc$ | $ud$ |
| $\langle a_2, nc\rangle \in X$ | $nc$ | $nc$ | $nc$ |
| $\langle a_2, c_2\rangle \in X$ | $ud$ | $nc$ | $c_1\ op\ c_2$ |

# Example Program for Constant Propagation



# Result of Constant Propagation

| | Iteration #1 | Changes in iteration #2 | Changes in iteration #3 | Changes in iteration #4 |
|---|---|---|---|---|
| $In_{n_1}$ | $\widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top}$ | | | |
| $Out_{n_1}$ | $\widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\top}$ | | | |
| $In_{n_2}$ | $\widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\top}$ | | | |
| $Out_{n_2}$ | $7, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | | | |
| $In_{n_3}$ | $7, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | $\widehat{\bot}, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot}$ | $\widehat{\bot}, 2, 6, 3, \widehat{\bot}, \widehat{\bot}$ | $\widehat{\bot}, \widehat{\bot}, 6, 3, \widehat{\bot}, \widehat{\bot}$ |
| $Out_{n_3}$ | $2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | $2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot}$ | $2, 2, 6, 3, \widehat{\bot}, \widehat{\bot}$ | $2, \widehat{\bot}, 6, 3, \widehat{\bot}, \widehat{\bot}$ |
| $In_{n_4}$ | $2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | $2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot}$ | $2, 2, 6, 3, \widehat{\bot}, \widehat{\bot}$ | $2, \widehat{\bot}, 6, 3, \widehat{\bot}, \widehat{\bot}$ |
| $Out_{n_4}$ | $2, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | $2, \widehat{\top}, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot}$ | $2, 7, 6, 3, \widehat{\bot}, \widehat{\bot}$ | |
| $In_{n_5}$ | $2, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | $2, \widehat{\top}, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot}$ | $2, 7, 6, 3, \widehat{\bot}, \widehat{\bot}$ | |
| $Out_{n_5}$ | $2, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | $2, \widehat{\top}, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot}$ | $2, 7, 6, 3, \widehat{\bot}, \widehat{\bot}$ | |
| $In_{n_6}$ | $2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | $2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot}$ | $2, 2, 6, 3, \widehat{\bot}, \widehat{\bot}$ | $2, \widehat{\bot}, 6, 3, \widehat{\bot}, \widehat{\bot}$ |
| $Out_{n_6}$ | $2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | $2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot}$ | $2, 2, 6, 3, \widehat{\bot}, \widehat{\bot}$ | $2, \widehat{\bot}, 6, 3, \widehat{\bot}, \widehat{\bot}$ |
| $In_{n_7}$ | $2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | $2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot}$ | $2, \widehat{\bot}, 6, 3, \widehat{\bot}, \widehat{\bot}$ | |
| $Out_{n_7}$ | $2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | $2, 2, 6, 3, \widehat{\bot}, \widehat{\bot}$ | $2, \widehat{\bot}, 6, 3, \widehat{\bot}, \widehat{\bot}$ | |
| $In_{n_8}$ | $2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | $2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot}$ | $2, 2, 6, 3, \widehat{\bot}, \widehat{\bot}$ | $2, \widehat{\bot}, 6, 3, \widehat{\bot}, \widehat{\bot}$ |
| $Out_{n_8}$ | $2, 2, \widehat{\top}, 4, \widehat{\bot}, \widehat{\bot}$ | $2, 2, \widehat{\top}, 4, \widehat{\bot}, \widehat{\bot}$ | $2, 2, 6, 4, \widehat{\bot}, \widehat{\bot}$ | $2, \widehat{\bot}, 6, \widehat{\bot}, \widehat{\bot}, \widehat{\bot}$ |
| $In_{n_9}$ | $2, 2, \widehat{\top}, 4, \widehat{\bot}, \widehat{\bot}$ | $2, 2, 6, \widehat{\bot}, \widehat{\bot}, \widehat{\bot}$ | $2, \widehat{\bot}, 6, \widehat{\bot}, \widehat{\bot}, \widehat{\bot}$ | |
| $Out_{n_9}$ | $2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot}$ | $2, 2, 6, 3, \widehat{\bot}, \widehat{\bot}$ | $2, \widehat{\bot}, 6, 3, \widehat{\bot}, \widehat{\bot}$ | |
| $In_{n_{10}}$ | $\widehat{\bot}, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | $\widehat{\bot}, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot}$ | $\widehat{\bot}, \widehat{\bot}, 6, 3, \widehat{\bot}, \widehat{\bot}$ | |
| $Out_{n_{10}}$ | $\widehat{\bot}, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot}$ | $\widehat{\bot}, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot}$ | $\widehat{\bot}, \widehat{\bot}, 6, 3, \widehat{\bot}, \widehat{\bot}$ | |

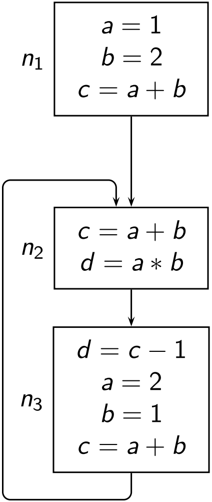
# Monotonicity of Constant Propagation

- Flow function $f_n(X) = (X - \text{Kill}_n(X)) \cup \text{Gen}_n(X)$ where

$$\begin{array}{rcl} \text{Gen}_n(X) & = & ConstGen_n \cup DepGen_n(X) \\ \text{Kill}_n(X) & = & ConstKill_n \cup DepKill_n(X) \end{array}$$

- $ConstGen_n$ and $ConstKill_n$ are trivially monotonic
- To show $X_1 \sqsubseteq X_2 \;\Rightarrow\; DepGen_n(X_1) \sqsubseteq DepGen_n(X_2)$ we need to show that $X_1 \sqsubseteq X_2 \;\Rightarrow\; eval(e, X_1) \sqsubseteq eval(e, X_2)$. This follows from definition of $eval(e, X)$.
- To show $X_1 \sqsubseteq X_2 \;\Rightarrow\; (X_1 - DepKill_n(X_1)) \sqsubseteq (X_1 - DepKill_n(X_2))$ observe that $DepKill_n$ removes the pair corresponding to the variable modified in statement $n$. Data flow values of other variables remain unaffected.

# Non-Distributivity of Constant Propagation



$n_1$

$a = 1$
$b = 2$
$c = a + b$

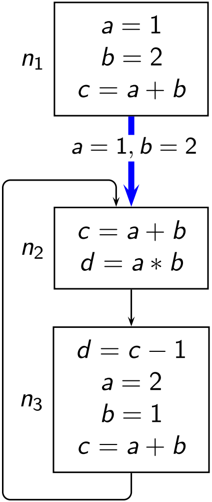
$n_2$

$c = a + b$
$d = a * b$

$n_3$

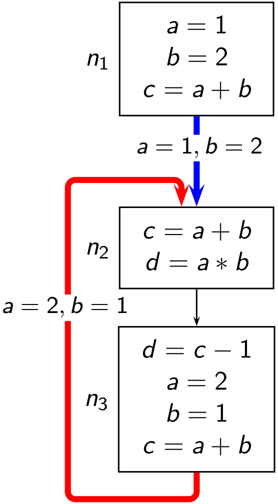
$d = c - 1$
$a = 2$
$b = 1$
$c = a + b$

# Non-Distributivity of Constant Propagation



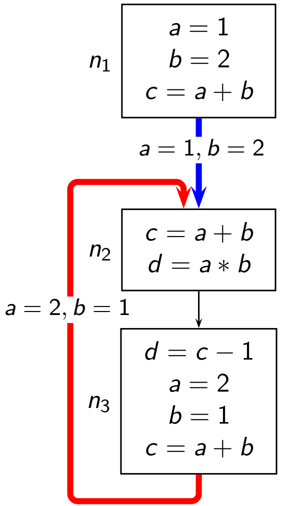
- $x = \langle 1, 2, 3, ? \rangle$ (Along $Out_{n_1} \rightarrow In_{n_2}$)

# Non-Distributivity of Constant Propagation



- $x = \langle 1, 2, 3, ? \rangle$ (Along $Out_{n_1} \rightarrow In_{n_2}$)
- $y = \langle 2, 1, 3, 2 \rangle$ (Along $Out_{n_3} \rightarrow In_{n_2}$)

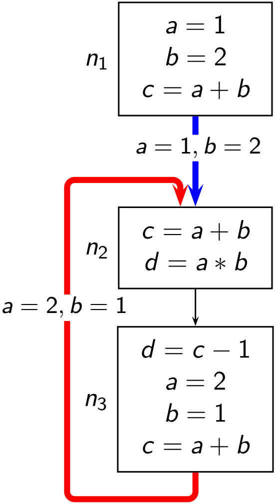
# Non-Distributivity of Constant Propagation



- $x = \langle 1, 2, 3, ? \rangle$ (Along $Out_{n_1} \rightarrow In_{n_2}$)
- $y = \langle 2, 1, 3, 2 \rangle$ (Along $Out_{n_3} \rightarrow In_{n_2}$)
- Function application before merging

$$\begin{aligned} f(x) \sqcap f(y) &= f(\langle 1, 2, 3, ? \rangle) \sqcap f(\langle 2, 1, 3, 2 \rangle) \\ &= \langle 1, 2, 3, 2 \rangle \sqcap \langle 2, 1, 3, 2 \rangle \\ &= \langle \widehat{\bot}, \widehat{\bot}, 3, 2 \rangle \end{aligned}$$

# Non-Distributivity of Constant Propagation



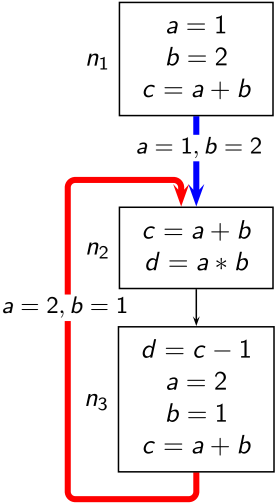
- $x = \langle 1, 2, 3, ? \rangle$ (Along $Out_{n_1} \rightarrow In_{n_2}$)
- $y = \langle 2, 1, 3, 2 \rangle$ (Along $Out_{n_3} \rightarrow In_{n_2}$)
- Function application before merging

$$\begin{aligned} f(x) \sqcap f(y) &= f(\langle 1, 2, 3, ? \rangle) \sqcap f(\langle 2, 1, 3, 2 \rangle) \\ &= \langle 1, 2, 3, 2 \rangle \sqcap \langle 2, 1, 3, 2 \rangle \\ &= \langle \widehat{\perp}, \widehat{\perp}, 3, 2 \rangle \end{aligned}$$

- Function application after merging

$$\begin{aligned} f(x \sqcap y) &= f(\langle 1, 2, 3, ? \rangle \sqcap \langle 2, 1, 3, 2 \rangle) \\ &= f(\langle \widehat{\perp}, \widehat{\perp}, 3, 2 \rangle) \\ &= \langle \widehat{\perp}, \widehat{\perp}, \widehat{\perp}, \widehat{\perp} \rangle \end{aligned}$$

# Non-Distributivity of Constant Propagation



- $x = \langle 1, 2, 3, ? \rangle$ (Along $Out_{n_1} \rightarrow In_{n_2}$)
- $y = \langle 2, 1, 3, 2 \rangle$ (Along $Out_{n_3} \rightarrow In_{n_2}$)
- Function application before merging

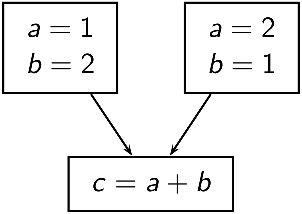
$$\begin{aligned} f(x) \sqcap f(y) &= f(\langle 1, 2, 3, ? \rangle) \sqcap f(\langle 2, 1, 3, 2 \rangle) \\ &= \langle 1, 2, 3, 2 \rangle \sqcap \langle 2, 1, 3, 2 \rangle \\ &= \langle \widehat{\bot}, \widehat{\bot}, 3, 2 \rangle \end{aligned}$$

- Function application after merging

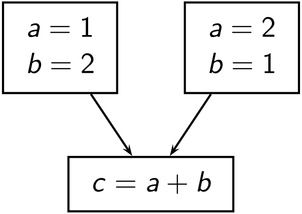
$$\begin{aligned} f(x \sqcap y) &= f(\langle 1, 2, 3, ? \rangle \sqcap \langle 2, 1, 3, 2 \rangle) \\ &= f(\langle \widehat{\bot}, \widehat{\bot}, 3, 2 \rangle) \\ &= \langle \widehat{\bot}, \widehat{\bot}, \widehat{\bot}, \widehat{\bot} \rangle \end{aligned}$$

- $f(x \sqcap y) \sqsubset f(x) \sqcap f(y)$

# Why is Constant Propagation Non-Distributive?



# Why is Constant Propagation Non-Distribitive?



$a = 1$
$b = 2$

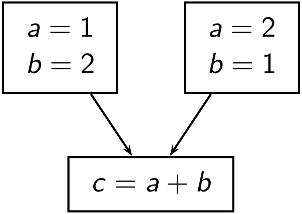
$a = 2$
$b = 1$

$c = a + b$

Possible combinations due to merging

$a = 1$ $a = 2$ $b = 1$ $b = 2$

# Why is Constant Propagation Non-Distribitive?



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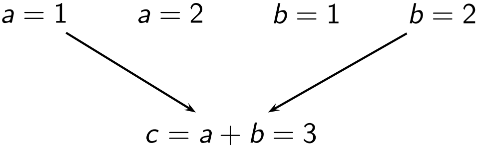
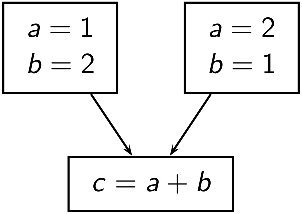


- Correct combination.

# Why is Constant Propagation Non-Distribitive?

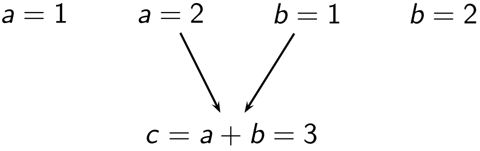


$a = 1$
$b = 2$

$a = 2$
$b = 1$

$c = a + b$

Possible combinations due to merging

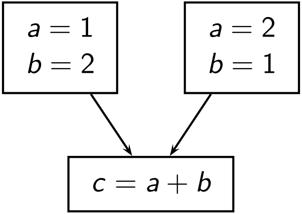


$a = 1$ $a = 2$ $b = 1$ $b = 2$

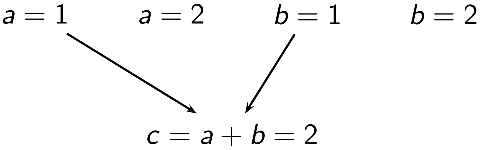
$c = a + b = 3$

- Correct combination.

# Why is Constant Propagation Non-Distribitive?

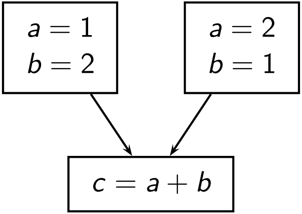


Possible combinations due to merging

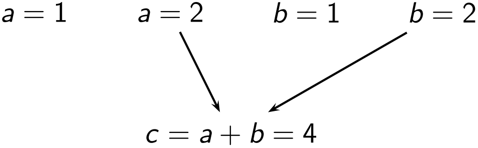


- Wrong combination.
- Mutually exclusive information.
- No execution path along which this information holds.

# Why is Constant Propagation Non-Distribitive?

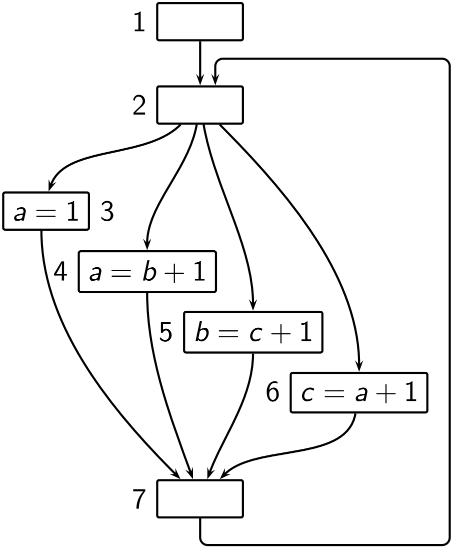


Possible combinations due to merging



- Wrong combination.
- Mutually exclusive information.
- No execution path along which this information holds.

# Boundedness of Constant Propagation



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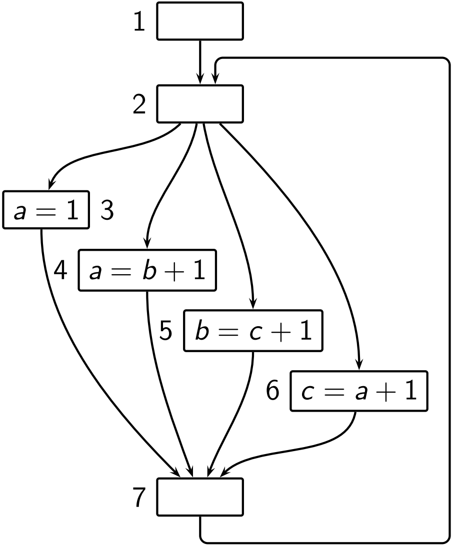
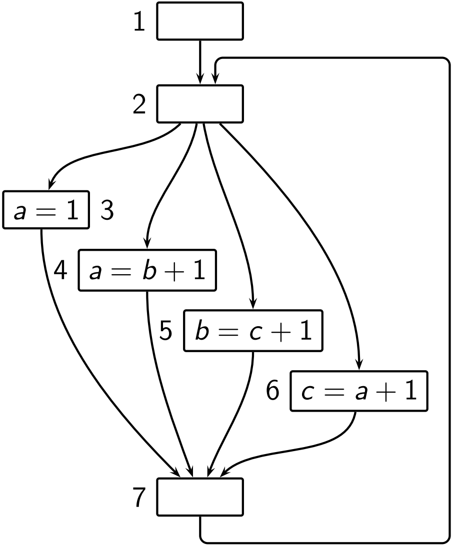


Summary flow function:
(data flow value at node 7)

$$f(\langle v_a, v_b, v_c\rangle) = \langle\ 1 \sqcap (v_b + 1), (v_c + 1), (v_a + 1) \rangle$$

# Boundedness of Constant Propagation



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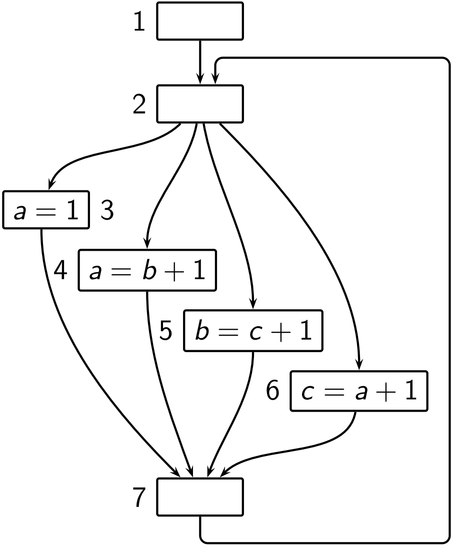
$$f(\langle v_a, v_b, v_c\rangle) = \langle\ 1 \sqcap (v_b + 1), (v_c + 1), (v_a + 1) \rangle$$

$$f^0(\top) = \langle\widehat{\top}, \widehat{\top}, \widehat{\top}\rangle$$
$$f^1(\top) = \langle 1, \widehat{\top}, \widehat{\top}\rangle$$

# Boundedness of Constant Propagation



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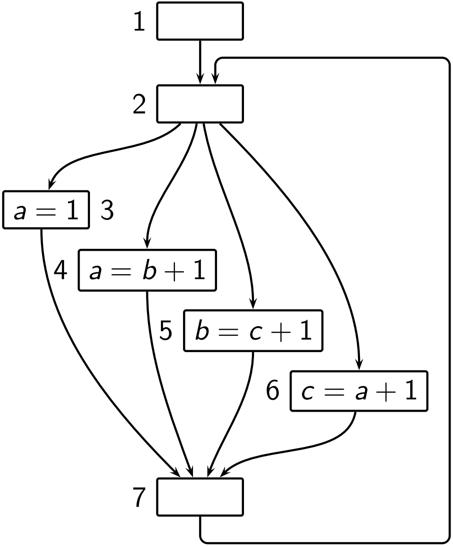
# Boundedness of Constant Propagation



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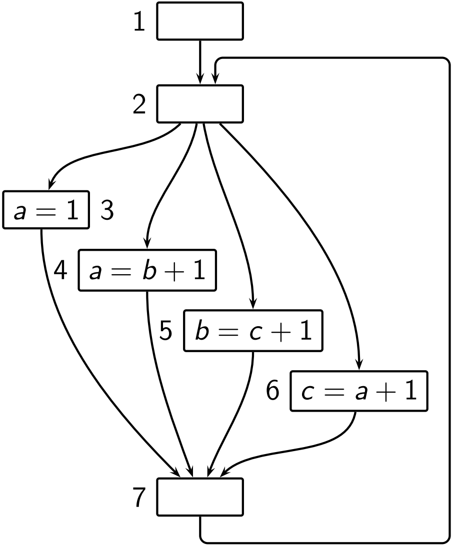

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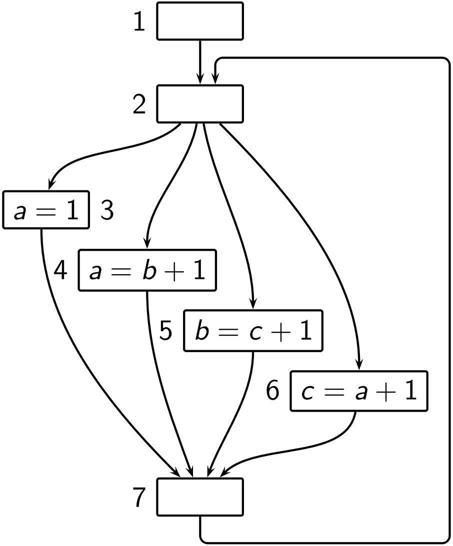
# Boundedness of Constant Propagation



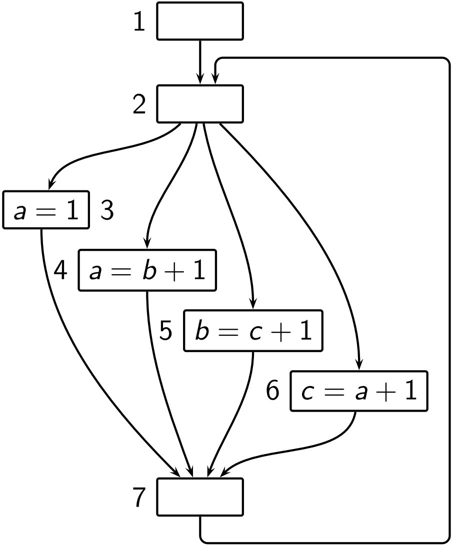
Summary flow function:

(data flow value at node 7)

$$f(\langle v_a, v_b, v_c \rangle) \;=\; \langle\; 1 \sqcap (v_b + 1),\; (v_c + 1),\; (v_a + 1) \;\rangle$$

$$\begin{aligned}
f^0(\top) &= \langle \widehat{\top}, \widehat{\top}, \widehat{\top} \rangle \\
f^1(\top) &= \langle 1, \widehat{\top}, \widehat{\top} \rangle \\
f^2(\top) &= \langle 1, \widehat{\top}, 2 \rangle \\
f^3(\top) &= \langle 1, 3, 2 \rangle \\
f^4(\top) &= \langle \widehat{\bot}, 3, 2 \rangle \\
f^5(\top) &= \langle \widehat{\bot}, 3, \widehat{\bot} \rangle
\end{aligned}$$

# Boundedness of Constant Propagation



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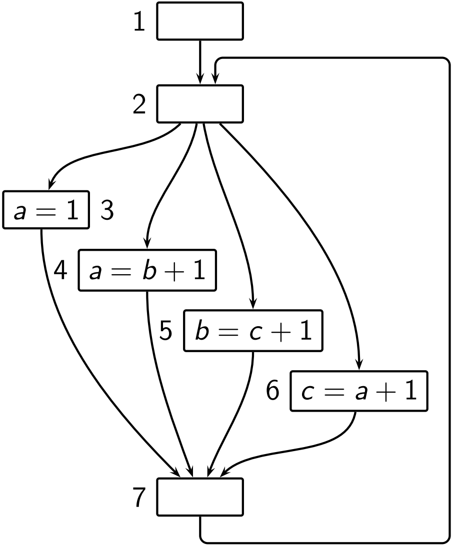
f^6(\top) &= \langle \widehat{\bot}, \widehat{\bot}, \widehat{\bot} \rangle
\end{aligned}$$

# Boundedness of Constant Propagation



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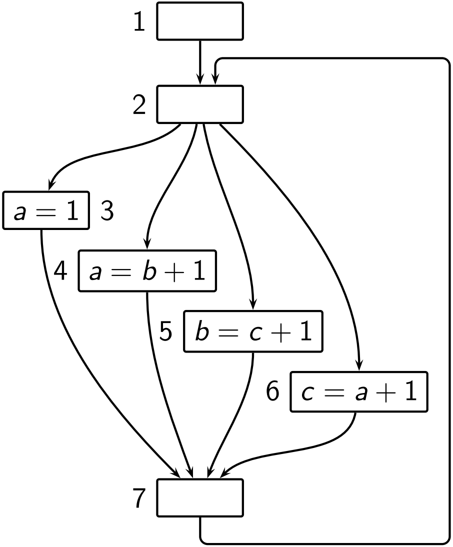
f^5(\top) &= \langle \widehat{\bot}, 3, \widehat{\bot} \rangle \\
f^6(\top) &= \langle \widehat{\bot}, \widehat{\bot}, \widehat{\bot} \rangle \\
f^7(\top) &= \langle \widehat{\bot}, \widehat{\bot}, \widehat{\bot} \rangle
\end{aligned}$$

# Boundedness of Constant Propagation



$$f^*(\top) = \prod_{i=0}^{6} f^i(\top)$$

# Boundedness of Constant Propagation

The moral of the story:

- The data flow value of every variable could change twice

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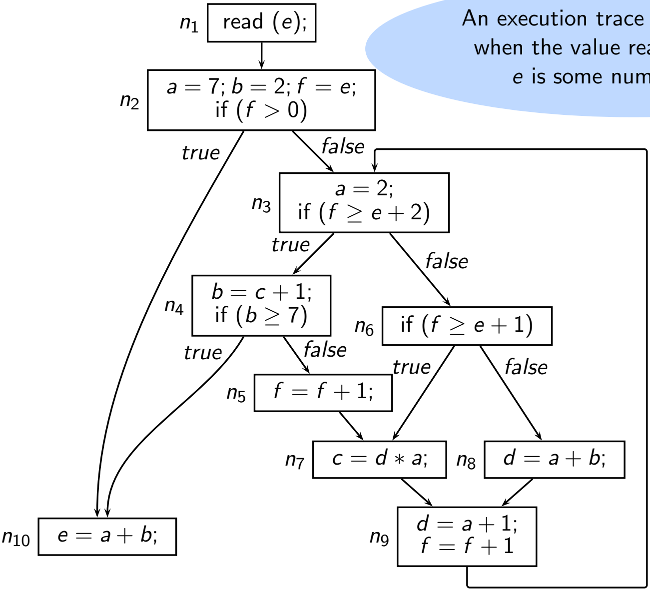
The moral of the story:

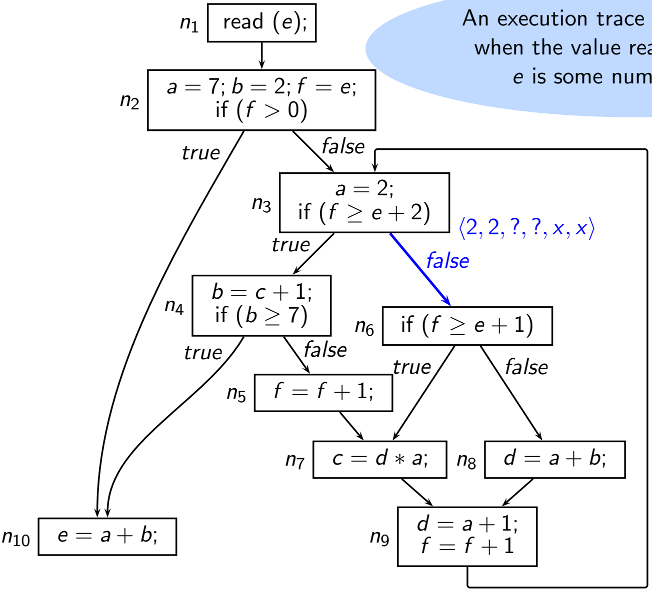
- The data flow value of every variable could change twice
- In the worst case, only one change may happen in every step of a function application
- Maximum number of steps: $2 \times |\mathbb{V}\text{ar}|$
- Boundedness parameter $k$ is $(2 \times |\mathbb{V}\text{ar}|) + 1$

# Conditional Constant Propagation



An execution trace of the program when the value read for variable $e$ is some number $x \leq 0$

# Conditional Constant Propagation



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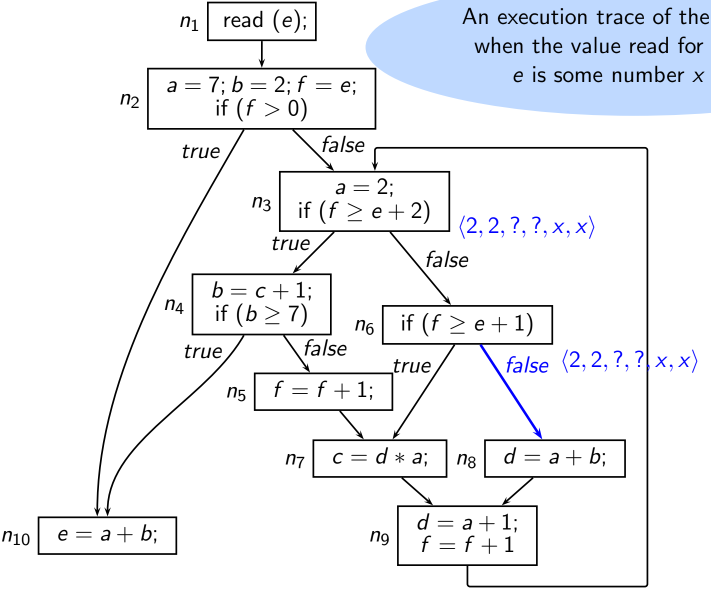
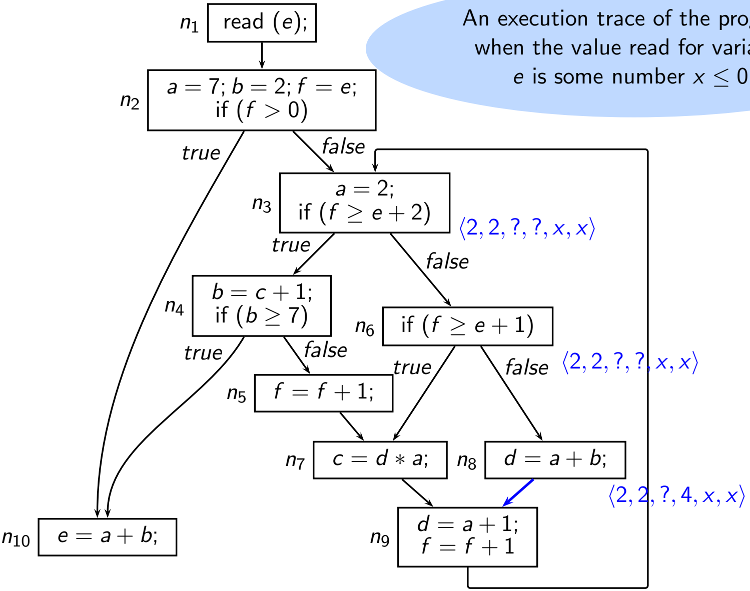
# Conditional Constant Propagation



An execution trace of the program when the value read for variable $e$ is some number $x \leq 0$

# Conditional Constant Propagation

An execution trace of the program when the value read for variable $e$ is some number $x \leq 0$



$n_1$ read $(e)$;

$n_2$ $a = 7; b = 2; f = e;$ if $(f > 0)$

$n_3$ $a = 2;$ if $(f \geq e + 2)$

$n_4$ $b = c + 1;$ if $(b \geq 7)$

$n_5$ $f = f + 1;$

$n_6$ if $(f \geq e + 1)$

$n_7$ $c = d * a;$

$n_8$ $d = a + b;$

$n_9$ $d = a + 1;$ $f = f + 1$

$n_{10}$ $e = a + b;$

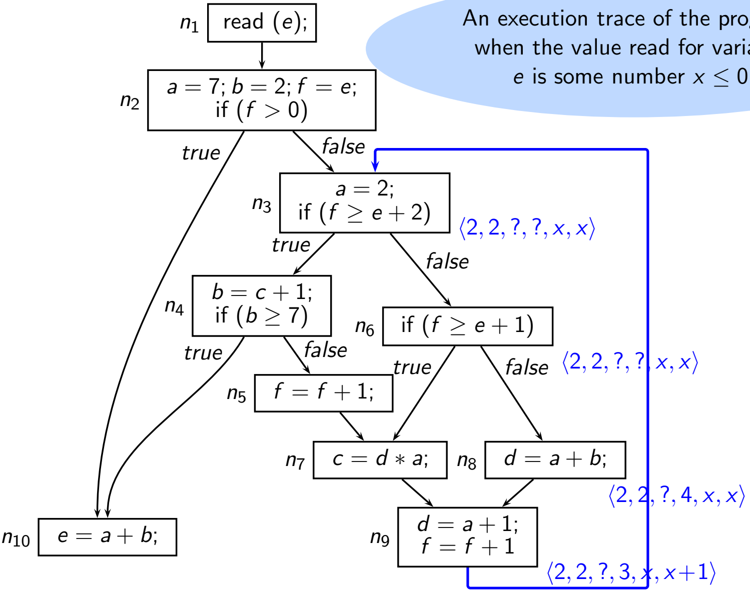
$\langle 2, 2, ?, ?, x, x \rangle$

$\langle 2, 2, ?, ?, x, x \rangle$

$\langle 2, 2, ?, 4, x, x \rangle$

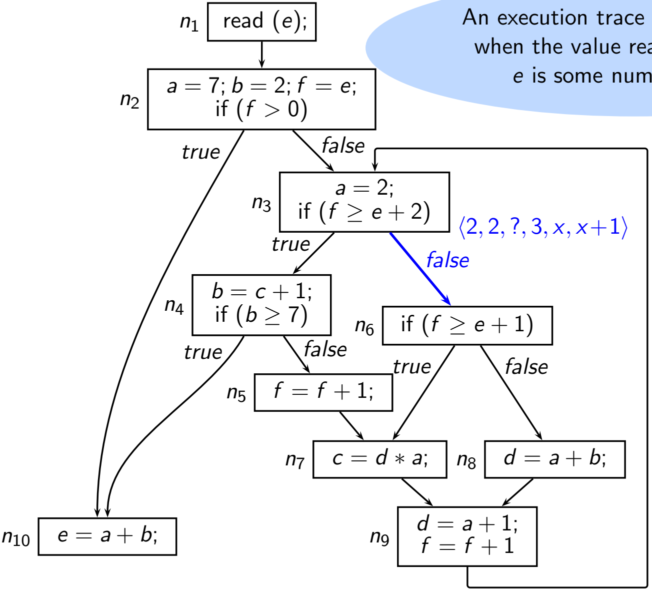
# Conditional Constant Propagation

An execution trace of the program when the value read for variable $e$ is some number $x \leq 0$



# Conditional Constant Propagation

An execution trace of the program when the value read for variable $e$ is some number $x \leq 0$



$n_1$ read $(e)$;

$n_2$ $a = 7; b = 2; f = e;$ if $(f > 0)$

true

false

$n_3$ $a = 2;$ if $(f \geq e + 2)$

$\langle 2, 2, ?, 3, x, x+1 \rangle$

true

false

$n_4$ $b = c + 1;$ if $(b \geq 7)$

true

false

$n_5$ $f = f + 1;$

$n_6$ if $(f \geq e + 1)$

true

false

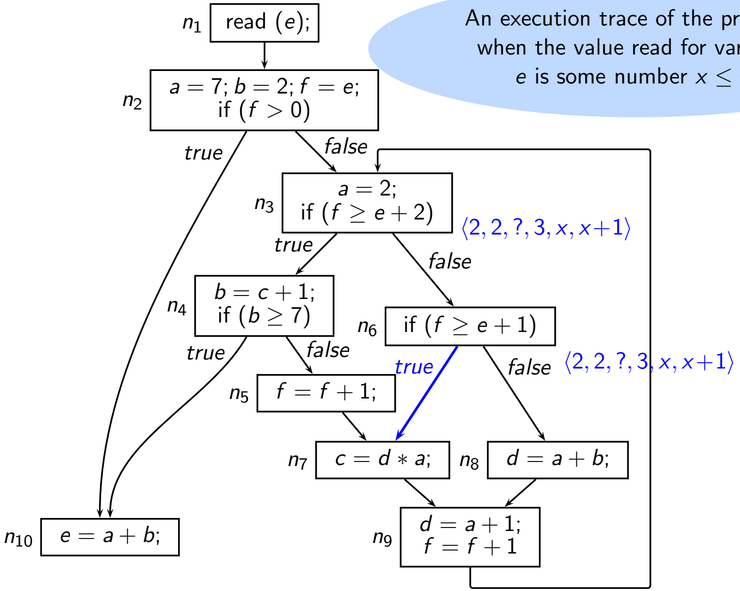
$n_7$ $c = d * a;$

$n_8$ $d = a + b;$

$n_9$ $d = a + 1;$ $f = f + 1$

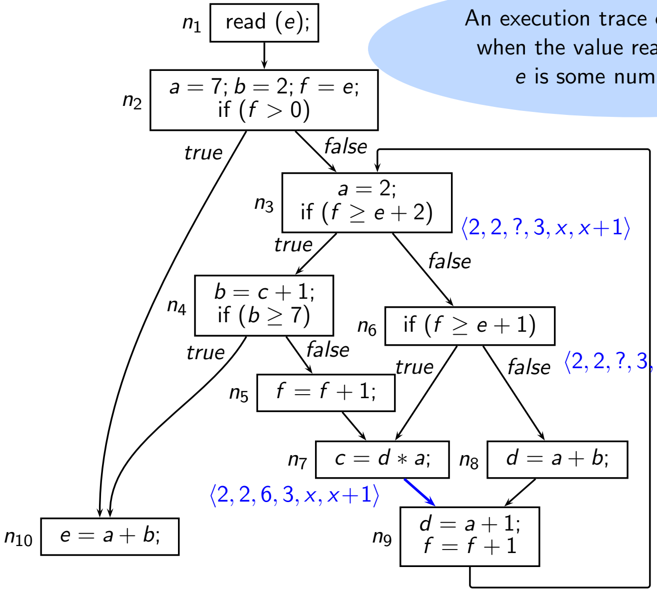
$n_{10}$ $e = a + b;$

# Conditional Constant Propagation



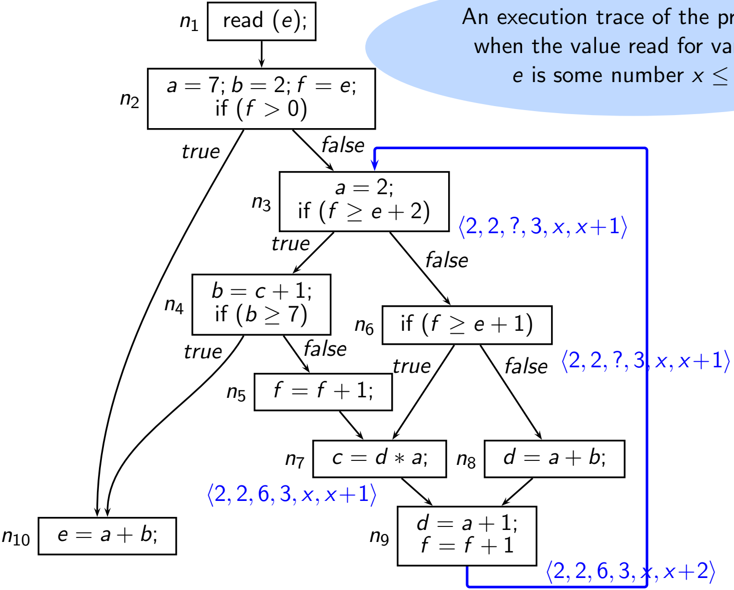
An execution trace of the program when the value read for variable $e$ is some number $x \leq 0$

# Conditional Constant Propagation



An execution trace of the program when the value read for variable $e$ is some number $x \leq 0$

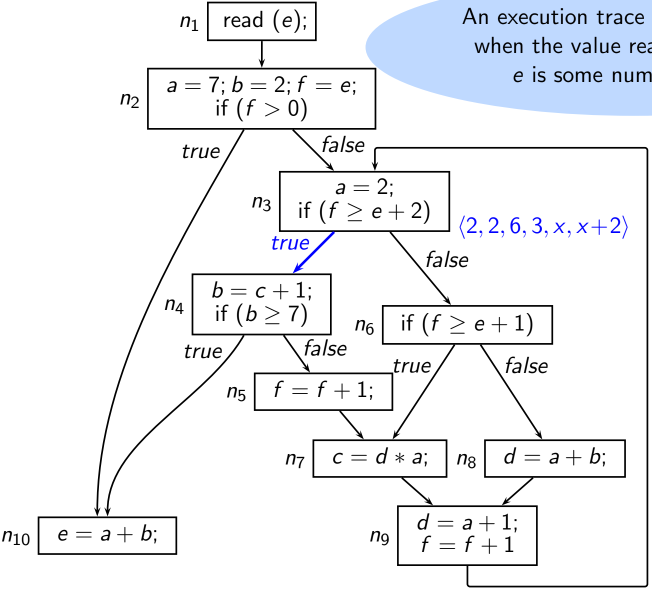
# Conditional Constant Propagation



An execution trace of the program when the value read for variable $e$ is some number $x \leq 0$

# Conditional Constant Propagation

An execution trace of the program
when the value read for variable
$e$ is some number $x \leq 0$



$n_1$ read $(e)$;

$n_2$ $a = 7; b = 2; f = e;$ if $(f > 0)$

true

false

$n_3$ $a = 2;$ if $(f \geq e + 2)$

$\langle 2, 2, 6, 3, x, x{+}2 \rangle$

true

false

$n_4$ $b = c + 1;$ if $(b \geq 7)$

true

false

$n_5$ $f = f + 1;$

$n_6$ if $(f \geq e + 1)$

true

false

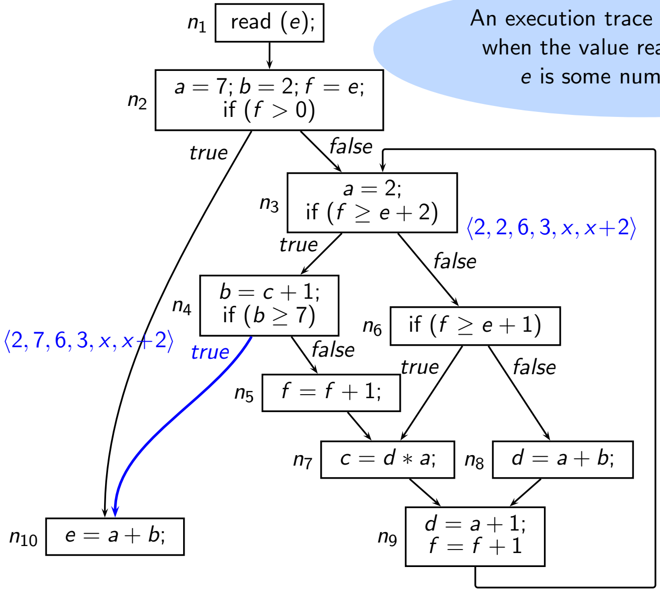
$n_7$ $c = d * a;$

$n_8$ $d = a + b;$

$n_9$ $d = a + 1;$ $f = f + 1$

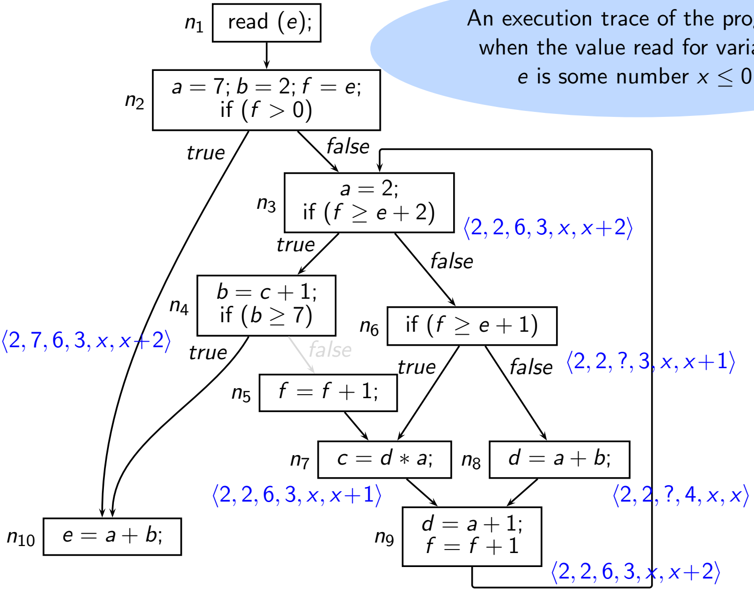
$n_{10}$ $e = a + b;$

# Conditional Constant Propagation



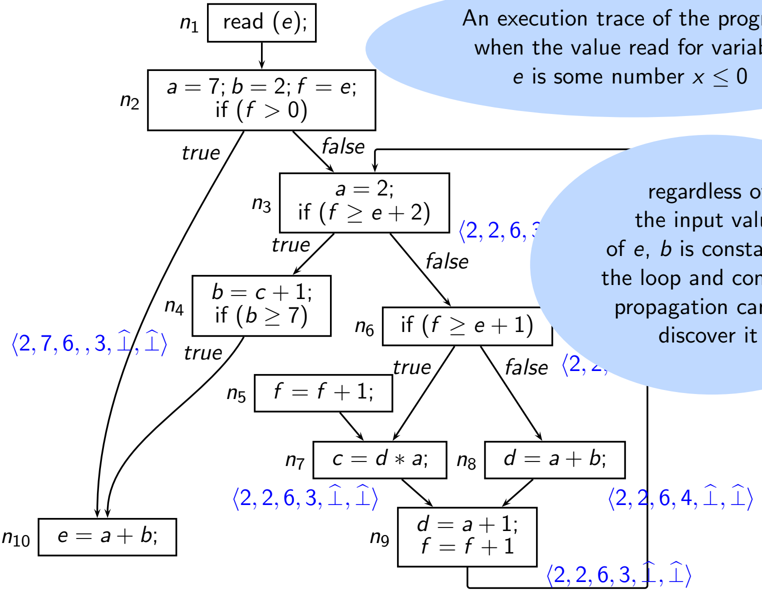
An execution trace of the program when the value read for variable $e$ is some number $x \leq 0$

# Conditional Constant Propagation



An execution trace of the program when the value read for variable $e$ is some number $x \leq 0$

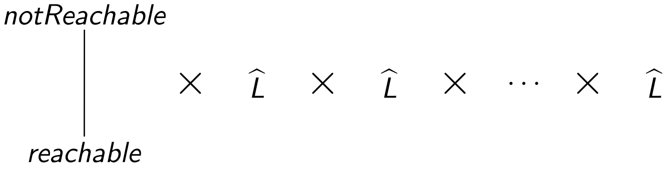
# Conditional Constant Propagation



An execution trace of the program when the value read for variable $e$ is some number $x \leq 0$

regardless of the input value of $e$, $b$ is constant in the loop and constant propagation cannot discover it

# Lattice for Conditional Constant Propagation



*notReachable*

*reachable*

$$\times \quad \widehat{L} \quad \times \quad \widehat{L} \quad \times \quad \cdots \quad \times \quad \widehat{L}$$

- Let $\langle s, X\rangle$ denote an augmented data flow value where $s \in \{reachable, notReachable\}$ and $X \in L$.
- If we can maintain the invariant $s = notReachable \Rightarrow X = \top$, then the meet can be defined as

$$\langle s_1, X_1\rangle \ \sqcap_c \ \langle s_2, X_2\rangle = \langle s_1 \sqcap_c s_2, X_1 \sqcap X_2\rangle$$

# Data Flow Equations for Conditional Constant Propagation

$$In_n = \begin{cases} \langle reachable, BI \rangle & n \text{ is } Start \\ \displaystyle\prod_{p \in pred(n)}{}_{c}\; g_{p \rightarrow n}(Out_p) & \text{otherwise} \end{cases}$$

$$Out_n = \begin{cases} \langle reachable, f_n(X) \rangle & In_n = \langle reachable, X \rangle \\ \langle notReachable, \top \rangle & \text{otherwise} \end{cases}$$

$$g_{m \rightarrow n}(s, X) = \begin{cases} \langle notReachable, \top \rangle & evalCond(m, X) \neq undefined \text{ and} \\ & evalCond(m, X) \neq label(m \rightarrow n) \\ \langle s, X \rangle & \text{otherwise} \end{cases}$$

# Conditional Constant Propagation

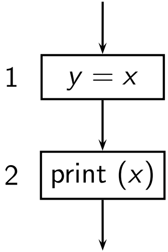
| | Iteration #1 | Changes in iteration #2 | Changes in iteration #3 |
|---|---|---|---|
| $In_{n_1}$ | $R, \langle \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top} \rangle$ | | |
| $Out_{n_1}$ | $R, \langle \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\top} \rangle$ | | |
| $In_{n_2}$ | $R, \langle \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | | |
| $Out_{n_2}$ | $R, \langle 7, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | | |
| $In_{n_3}$ | $R, \langle 7, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle \widehat{\bot}, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle \widehat{\bot}, 2, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $Out_{n_3}$ | $R, \langle 2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $In_{n_4}$ | $R, \langle 2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $Out_{n_4}$ | $R, \langle 2, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, \widehat{\top}, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 7, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $In_{n_5}$ | $R, \langle 2, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, \widehat{\top}, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $N, \top = \langle \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top} \rangle$ |
| $Out_{n_5}$ | $R, \langle 2, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, \widehat{\top}, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $N, \top = \langle \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top}, \widehat{\top} \rangle$ |
| $In_{n_6}$ | $R, \langle 2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $Out_{n_6}$ | $R, \langle 2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $In_{n_7}$ | $R, \langle 2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $Out_{n_7}$ | $R, \langle 2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $In_{n_8}$ | $R, \langle 2, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $Out_{n_8}$ | $R, \langle 2, 2, \widehat{\top}, 4, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, \widehat{\top}, 4, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, 6, 4, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $In_{n_9}$ | $R, \langle 2, 2, \widehat{\top}, 4, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, 6, \widehat{\bot}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, 6, \widehat{\bot}, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $Out_{n_9}$ | $R, \langle 2, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle 2, 2, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $In_{n_{10}}$ | $R, \langle \widehat{\bot}, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle \widehat{\bot}, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle \widehat{\bot}, \widehat{\bot}, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ |
| $Out_{n_{10}}$ | $R, \langle \widehat{\bot}, 2, \widehat{\top}, \widehat{\top}, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle \widehat{\bot}, 2, \widehat{\top}, 3, \widehat{\bot}, \widehat{\bot} \rangle$ | $R, \langle \widehat{\bot}, \widehat{\bot}, 6, 3, \widehat{\bot}, \widehat{\bot} \rangle$ |

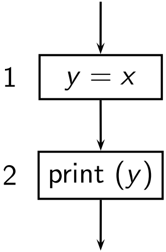
### Part 4

# Faint Variables Analysis

# Faint Variables Analysis

A variable is faint if it is dead or is used in computing faint variables.





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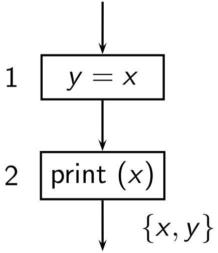
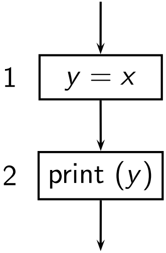
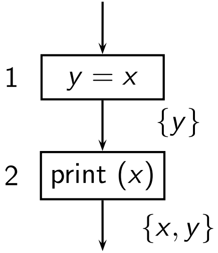


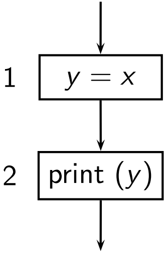


$\text{Gen}_2 = \emptyset$
$\text{Kill}_2 = \{x\}$

# Faint Variables Analysis

A variable is faint if it is dead or is used in computing faint variables.



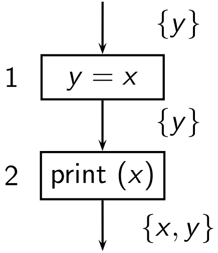


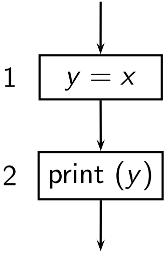
$\text{Gen}_2 = \emptyset$
$\text{Kill}_2 = \{x\}$

$\text{Gen}_1 = \{y\}$
$\text{Kill}_1 = \emptyset$

# Faint Variables Analysis

A variable is faint if it is dead or is used in computing faint variables.



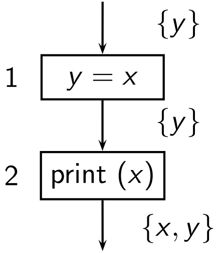


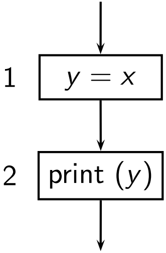
$\text{Gen}_2 = \emptyset$ $\text{Gen}_1 = \{y\}$

$\text{Kill}_2 = \{x\}$ $\text{Kill}_1 = \emptyset$

# Faint Variables Analysis

A variable is faint if it is dead or is used in computing faint variables.





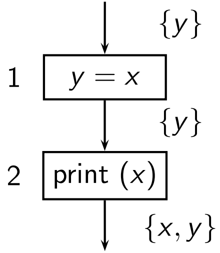
$\text{Gen}_2 = \emptyset$ $\text{Gen}_1 = \{y\}$

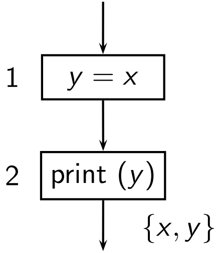
$\text{Kill}_2 = \{x\}$ $\text{Kill}_1 = \emptyset$

Faintness of $x$ is killed
by the print statement
(i.e. $x$ becomes live)

# Faint Variables Analysis

A variable is faint if it is dead or is used in computing faint variables.





$\text{Gen}_2 = \emptyset$
$\text{Kill}_2 = \{x\}$

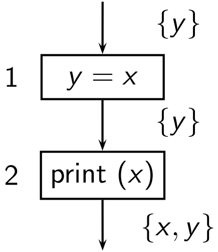
$\text{Gen}_1 = \{y\}$
$\text{Kill}_1 = \emptyset$

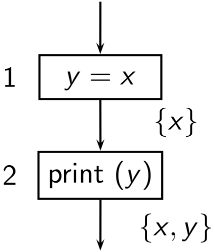
$\text{Gen}_2 = \emptyset$
$\text{Kill}_2 = \{y\}$

Faintness of $x$ is killed
by the print statement
(i.e. $x$ becomes live)

# Faint Variables Analysis

A variable is faint if it is dead or is used in computing faint variables.





$\text{Gen}_2 = \emptyset$ $\quad$ $\text{Gen}_1 = \{y\}$

$\text{Kill}_2 = \{x\}$ $\quad$ $\text{Kill}_1 = \emptyset$

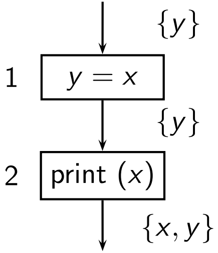
$\text{Gen}_2 = \emptyset$ $\quad$ $\text{Gen}_1 = \{y\}$

$\text{Kill}_2 = \{y\}$ $\quad$ $\text{Kill}_1 = \{x\}$

Faintness of $x$ is killed
by the print statement
(i.e. $x$ becomes live)

# Faint Variables Analysis

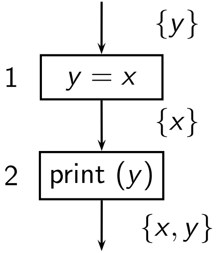
A variable is faint if it is dead or is used in computing faint variables.



$\text{Gen}_2 = \emptyset$ $\text{Gen}_1 = \{y\}$

$\text{Kill}_2 = \{x\}$ $\text{Kill}_1 = \emptyset$

Faintness of $x$ is killed by the print statement (i.e. $x$ becomes live)

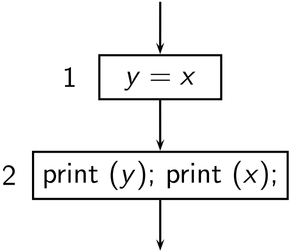


$\text{Gen}_2 = \emptyset$ $\text{Gen}_1 = \{y\}$

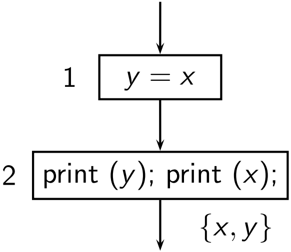
$\text{Kill}_2 = \{y\}$ $\text{Kill}_1 = \{x\}$

Faintness of $x$ is killed by the assignment to $y$ (i.e. $x$ becomes live)

# Faint Variables Analysis

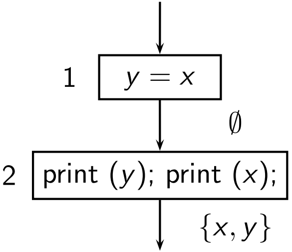


# Faint Variables Analysis



$\text{Gen}_2 = \emptyset$
$\text{Kill}_2 = \{x, y\}$

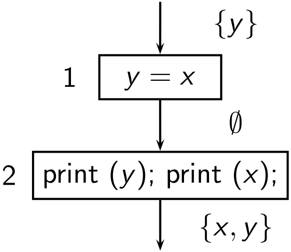
# Faint Variables Analysis



$\text{Gen}_2 = \emptyset$ $\text{Gen}_1 = \{y\}$

$\text{Kill}_2 = \{x, y\}$ $\text{Kill}_1 = \{x\}$

# Faint Variables Analysis



$\text{Gen}_2 = \emptyset$

$\text{Kill}_2 = \{x, y\}$

$\text{Gen}_1 = \{y\}$

$\text{Kill}_1 = \{x\}$

Faintness of $x$ is killed both by the print statement and by the assignment to $y$ (i.e. $x$ becomes live)

# Data Flow Equations for Faint Variables Analysis

$$
\begin{aligned}
In_n &= f_n(Out_n) \\
Out_n &= \begin{cases} BI & n \text{ is } End \\ \displaystyle\bigcap_{s \in succ(n)} In_s & \text{otherwise} \end{cases}
\end{aligned}
$$

where,

$$
\begin{aligned}
f_n(X) &= (X - (ConstKill_n \cup DepKill_n(X))) \\
&\quad \cup (ConstGen_n \cup DepGen_n(X))
\end{aligned}
$$

# Flow Function Components for Faint Variables Analysis

| | Statement | | |
|---|---|---|---|
| | $x = e,\ e \in \mathbb{E}\text{xpr}$ | $read(x)$ (assigning value from input) | $use(x)$ (not in assignment) |
| $ConstGen_n$ | $x \notin Opd(e) \Rightarrow \{x\}$<br>$x \in Opd(e) \Rightarrow \emptyset$ | $\{x\}$ | $\emptyset$ |
| $ConstKill_n$ | $\emptyset$ | $\emptyset$ | $\{x\}$ |
| $DepGen_n(X)$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| $DepKill_n(X)$ | $x \notin X \Rightarrow Opd(e) \cap \mathbb{V}\text{ar}$<br>$x \in X \Rightarrow \emptyset$ | $\emptyset$ | $\emptyset$ |

# Why is $DepGen_n(X) = \emptyset$ in Faint Variables Analysis?

Faintness can only be generated by an assignment statement, a read statement, or BI.

Consider an assignment statement $x = e$ where $e \in \mathbb{E}$xpr

# Why is $DepGen_n(X) = \emptyset$ in Faint Variables Analysis?

Faintness can only be generated by an assignment statement, a read statement, or BI.

Consider an assignment statement $x = e$ where $e \in \mathbb{E}\text{xpr}$

- If $x \notin Opd(e)$ then $x$ becomes faint unconditionally.
  Case covered by $ConstGen_n$.

# Why is $DepGen_n(X) = \emptyset$ in Faint Variables Analysis?

Faintness can only be generated by an assignment statement, a read statement, or BI.

Consider an assignment statement $x = e$ where $e \in \mathbb{E}\text{xpr}$

- If $x \notin Opd(e)$ then $x$ becomes faint unconditionally.
  Case covered by $ConstGen_n$.
- For the operands of $e$ (including $x$ if it is in $Opd(\text{e})$)

# Why is $DepGen_n(X) = \emptyset$ in Faint Variables Analysis?

Faintness can only be generated by an assignment statement, a read statement, or BI.

Consider an assignment statement $x = e$ where $e \in \mathbb{E}\text{xpr}$

- If $x \notin Opd(e)$ then $x$ becomes faint unconditionally.
  Case covered by $ConstGen_n$.
- For the operands of $e$ (including $x$ if it is in $Opd(\text{e})$)
  - If $x \notin X$ (i.e. $x$ is not faint after the assignment), operands cannot be faint before the assignment.

# Why is $DepGen_n(X) = \emptyset$ in Faint Variables Analysis?

Faintness can only be generated by an assignment statement, a read statement, or BI.

Consider an assignment statement $x = e$ where $e \in \mathbb{E}\mathrm{xpr}$

- If $x \notin Opd(e)$ then $x$ becomes faint unconditionally.
  Case covered by $ConstGen_n$.
- For the operands of $e$ (including $x$ if it is in $Opd(\mathrm{e})$)
  - If $x \notin X$ (i.e. $x$ is not faint after the assignment), operands cannot be faint before the assignment.
  - If $x \in X$ (i.e. is faint after the assignment), faintness of operands depends on their uses after the assignment.

# Why is $DepGen_n(X) = \emptyset$ in Faint Variables Analysis?

Faintness can only be generated by an assignment statement, a read statement, or BI.

Consider an assignment statement $x = e$ where $e \in \mathbb{E}\text{xpr}$

- If $x \notin Opd(e)$ then $x$ becomes faint unconditionally.
  Case covered by $ConstGen_n$.
- For the operands of $e$ (including $x$ if it is in $Opd(\text{e})$)
    - If $x \notin X$ (i.e. $x$ is not faint after the assignment), operands cannot be faint before the assignment.
    - If $x \in X$ (i.e. is faint after the assignment), faintness of operands depends on their uses after the assignment.
        - If they are faint after the assignment, they continue to remain faint.

# Why is $DepGen_n(X) = \emptyset$ in Faint Variables Analysis?

Faintness can only be generated by an assignment statement, a read statement, or BI.

Consider an assignment statement $x = e$ where $e \in \mathbb{E}\text{xpr}$

- If $x \notin Opd(e)$ then $x$ becomes faint unconditionally.
  Case covered by $ConstGen_n$.
- For the operands of $e$ (including $x$ if it is in $Opd$(e))
  - If $x \notin X$ (i.e. $x$ is not faint after the assignment), operands cannot be faint before the assignment.
  - If $x \in X$ (i.e. is faint after the assignment), faintness of operands depends on their uses after the assignment.
    - If they are faint after the assignment, they continue to remain faint.
    - If the are not faint after the assignment, they connot become faint before the assignment.

# Faint Variable Analysis

- What is $\widehat{L}$ for faint variables analysis?
- Is faint variables analysis a bit vector framework?
- Is faint variables analysis distributive? Monotonic?

# Distributivity of Faint Variables Analysis

Prove that

$$\forall X_1, X_2 \in L,\ f_n(X_1 \cap X_2) = f_n(X_1) \cap f_n(X_2)$$

# Distributivity of Faint Variables Analysis

Prove that

$$\forall X_1, X_2 \in L,\ f_n(X_1 \cap X_2) = f_n(X_1) \cap f_n(X_2)$$

- *ConstGen$_n$*, *DepGen$_n$*, and *ConstKill$_n$* are trivially distributive.
  Assume that *DepKill$_n$* is $\emptyset$

  $$f_n(X) = (X - ConstKill_n) \cup ConstGen_n \cup DepGen_n(X)$$

  Since $DepGen_n(X) = \emptyset$, the flow function has only constant parts!

# Distributivity of Faint Variables Analysis

To show that

$$
\begin{aligned}
&(X_1 \cap X_2) - \mathit{DepKill}_n(X_1 \cap X_2) \\
=\ \ &(X_1 - \mathit{DepKill}_n(X_1)) \ \cap \ (X_2 - \mathit{DepKill}_n(X_2))
\end{aligned}
$$

# Distributivity of Faint Variables Analysis

To show that

$$\begin{aligned}&(X_1 \cap X_2) - \mathit{DepKill}_n(X_1 \cap X_2)\\ =\ &(X_1 - \mathit{DepKill}_n(X_1)) \ \cap\ (X_2 - \mathit{DepKill}_n(X_2))\end{aligned}$$

- If $n$ is an assignment statement $x = e$, and $x \notin X_1 \cap X_2$. Assume that $x$ is neither in $X_1$ nor in $X_2$.

$$\begin{aligned}&(X_1 \cap X_2) - \mathit{DepKill}_n(X_1 \cap X_2)\\ &= (X_1 \cap X_2) - (\mathit{Opd}(e) \cap \mathbb{V}\text{ar})\\ &= (X_1 - (\mathit{Opd}(e) \cap \mathbb{V}\text{ar})) \ \cap\ (X_2 - (\mathit{Opd}(e) \cap \mathbb{V}\text{ar}))\\ &= (X_1 - \mathit{DepKill}_n(X_1)) \ \cap\ (X_2 - \mathit{DepKill}_n(X_2))\end{aligned}$$

What if $x$ is in $X_1$ but not in $X_2$?

# Distributivity of Faint Variables Analysis

To show that

$$\begin{aligned}&(X_1 \cap X_2) - \mathit{DepKill}_n(X_1 \cap X_2)\\ = \;\; &(X_1 - \mathit{DepKill}_n(X_1)) \;\cap\; (X_2 - \mathit{DepKill}_n(X_2))\end{aligned}$$

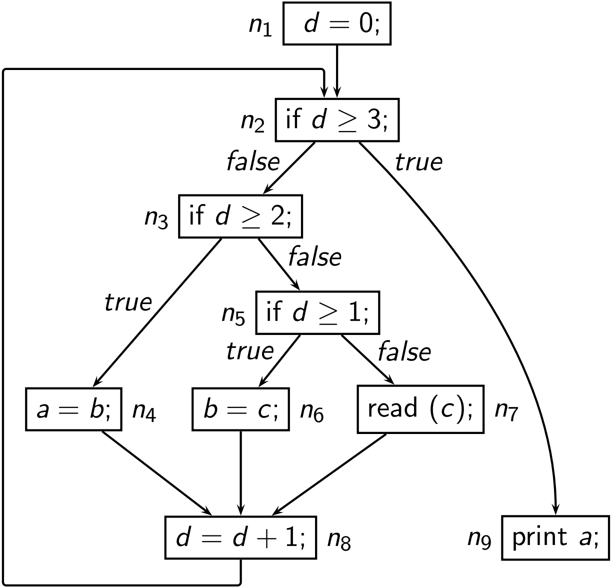
- If $n$ is an assignment statement $x = e$, and $x \notin X_1 \cap X_2$. Assume that $x$ is neither in $X_1$ nor in $X_2$.

$$\begin{aligned}&(X_1 \cap X_2) - \mathit{DepKill}_n(X_1 \cap X_2)\\ &= (X_1 \cap X_2) - (\mathit{Opd}(e) \cap \mathbb{V}\text{ar})\\ &= (X_1 - (\mathit{Opd}(e) \cap \mathbb{V}\text{ar})) \;\cap\; (X_2 - (\mathit{Opd}(e) \cap \mathbb{V}\text{ar}))\\ &= (X_1 - \mathit{DepKill}_n(X_1)) \;\cap\; (X_2 - \mathit{DepKill}_n(X_2))\end{aligned}$$

What if $x$ is in $X_1$ but not in $X_2$?

- In all other cases, $\mathit{DepKill}_n(X) = \emptyset$.

# Example Program for Faint Variables Analysis



# Result of Faint Variables Analysis

| Node | Iteration #1 | | Changes in Iteration #2 | | Changes in Iteration #3 | | Changes in Iteration #4 | |
|---|---|---|---|---|---|---|---|---|
| | $Out_n$ | $In_n$ | $Out_n$ | $In_n$ | $Out_n$ | $In_n$ | $Out_n$ | $In_n$ |
| $n_9$ | $\{a, b, c, d\}$ | $\{b, c, d\}$ | | | | | | |
| $n_8$ | $\{a, b, c, d\}$ | $\{a, b, c, d\}$ | $\{b, c\}$ | $\{b, c\}$ | $\{c\}$ | $\{c\}$ | $\emptyset$ | $\emptyset$ |
| $n_7$ | $\{a, b, c, d\}$ | $\{a, b, c, d\}$ | $\{b, c\}$ | $\{b, c\}$ | $\{c\}$ | $\{c\}$ | $\emptyset$ | |
| $n_6$ | $\{a, b, c, d\}$ | $\{a, b, c, d\}$ | $\{b, c\}$ | $\{b, c\}$ | $\{c\}$ | $\emptyset$ | $\emptyset$ | |
| $n_5$ | $\{a, b, c, d\}$ | $\{a, b, c\}$ | $\{b, c\}$ | $\{b, c\}$ | $\emptyset$ | $\emptyset$ | | |
| $n_4$ | $\{a, b, c, d\}$ | $\{a, b, c, d\}$ | $\{b, c\}$ | $\{a, c\}$ | $\{c\}$ | | $\emptyset$ | $\emptyset$ |
| $n_3$ | $\{a, b, c\}$ | $\{a, b, c\}$ | $\{c\}$ | $\{c\}$ | $\emptyset$ | $\emptyset$ | | |
| $n_2$ | $\{b, c\}$ | $\{b, c\}$ | $\{c\}$ | $\{c\}$ | $\emptyset$ | $\emptyset$ | $\emptyset$ | |
| $n_1$ | $\{b, c\}$ | $\{b, c, d\}$ | $\{c\}$ | $\{c, d\}$ | $\emptyset$ | $\{d\}$ | | |

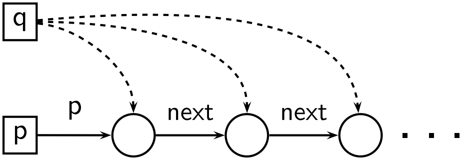
## Part 5

# Pointer Analyses

# Code Optimization In Presence of Pointers

```
1.  q = p;
2.  while (. . . ) {
3.        q = q↣next;
4.  }
5.  p↣data = r1;
6.  print (q↣data);
7.  p↣data = r2;
8.  r4 = p↣data + r3;
```

*Program*



*Memory graph at statement 5*

- Is p↣data live at the exit of line 5? Can we delete line 5?

# Code Optimization In Presence of Pointers

```
1.  q = p;
2.  do {
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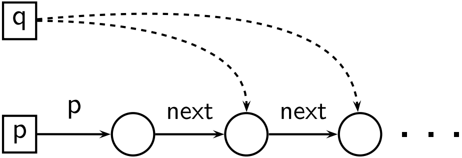
```

*Program*



*Memory graph at statement 5*

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# Code Optimization In Presence of Pointers

```
1.  q = p;
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3.        q = q→next;
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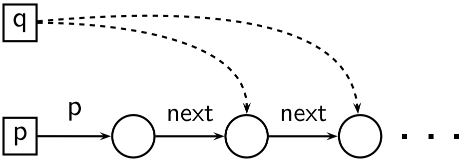
6.  print (q→data);
7.  p→data = r2;
8.  r4 = p→data + r3;
```

*Program*



*Memory graph at statement 5*

- Is p→data live at the exit of line 5? Can we delete line 5?
- No, if p and q can be possibly aliased.

# Code Optimization In Presence of Pointers

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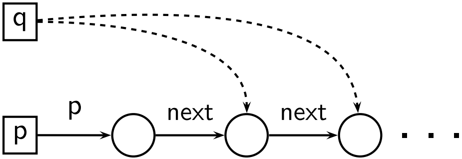
```

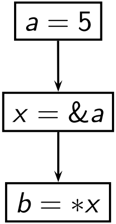
*Program*



*Memory graph at statement 5*

- Is p↣data live at the exit of line 5? Can we delete line 5?
- No, if p and q can be possibly aliased.
- Yes, if p and q are definitely not aliased.

# Code Optimization In Presence of Pointers



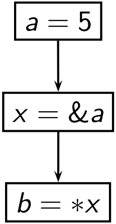
$a = 5$

$x = \&a$

$b = *x$

Original Program

# Code Optimization In Presence of Pointers



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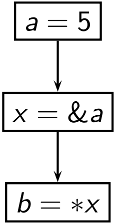


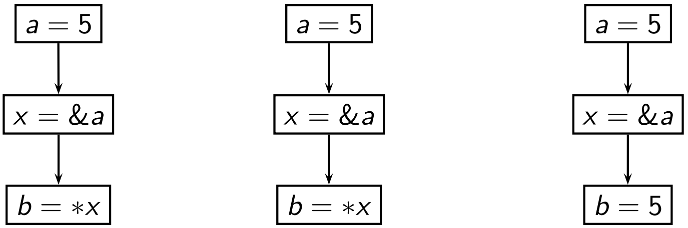
$a = 5$

$x = \&a$

$b = *x$

Constant Propagation
without aliasing

# Code Optimization In Presence of Pointers

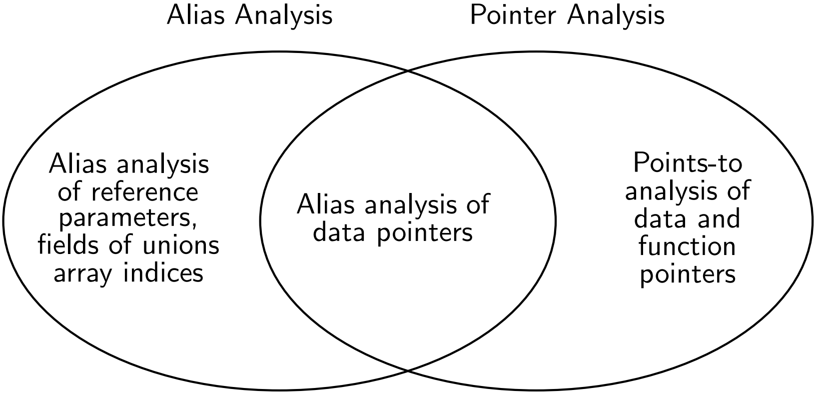


Original Program

Constant Propagation without aliasing

Constant Propagation with aliasing

# The World of Pointer Analysis



Alias Analysis

Pointer Analysis

Alias analysis
of reference
parameters,
fields of unions
array indices

Alias analysis of
data pointers

Points-to
analysis of
data and
function
pointers

# The Mathematics of Pointer Analysis

In the most general situation

- Alias analysis is undecidable.
  Landi-Ryder [POPL 1991], Landi [LOPLAS 1992],
  Ramalingam [TOPLAS 1994]
- Flow insensitive alias analysis is NP-hard
  Horwitz [TOPLAS 1997]
- Points-to analysis is undecidable
  Chakravarty [POPL 2003]

# Motivation for a Good Science of Pointer Analysis

- To quote Hind [PASTE 2001]

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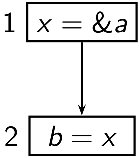
- Needs to scale to large programs.

# Motivation for a Good Science of Pointer Analysis

- To quote Hind [PASTE 2001]
    - Fortunately many approximations exist
    - Unfortunately too many approximations exist!
- Pointer analysis enables not only precise data analysis but also precise control flow analysis.
- Needs to scale to large programs.
- Engineering of pointer analysis is much more dominant than the science of pointer analysis.

⇒ Results in many questionable perceptions.

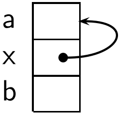
# Alias Information Vs. Points-To Information

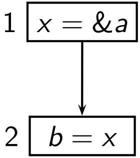


1 $x = \&a$

2 $b = x$

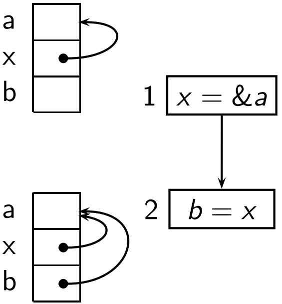
# Alias Information Vs. Points-To Information





"*x Points-To a*"
denoted $x \rightarrowtail a$

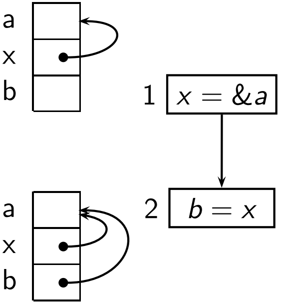
# Alias Information Vs. Points-To Information



"*x Points-To a*"
denoted $x \rightarrowtail a$

"*x* and *b* are *Aliases*"
denoted $x \overset{\circ}{=} b$

# Alias Information Vs. Points-To Information

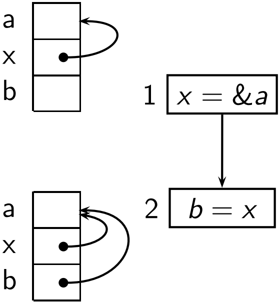


"*x Points-To a*"
denoted $x \rightarrowtail a$

"$x$ and $b$ are *Aliases*"
denoted $x \circeq b$

Symmetric
and
Reflexive

# Alias Information Vs. Points-To Information



"$x$ *Points-To* $a$"
denoted $x \rightarrowtail a$

Neither
Symmetric
Nor Reflexive

"$x$ and $b$ are *Aliases*"
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Symmetric
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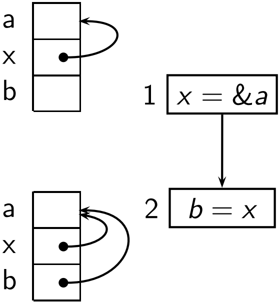
# Alias Information Vs. Points-To Information



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"$x$ and $b$ are *Aliases*"
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Symmetric
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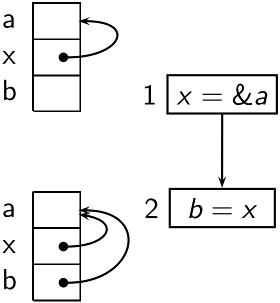
Reflexive

- What about transitivity?

# Alias Information Vs. Points-To Information



"*x Points-To a*"
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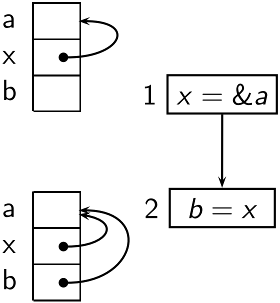
Neither
Symmetric
Nor Reflexive

"*x* and *b* are *Aliases*"
denoted $x \circeq b$

Symmetric
and
Reflexive

- What about transitivity?
  - Points-To: No.

# Alias Information Vs. Points-To Information



"*x Points-To a*"
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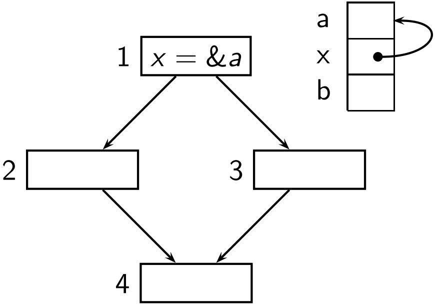
Neither
Symmetric
Nor Reflexive

"*x* and *b* are *Aliases*"
denoted $x \circeq b$

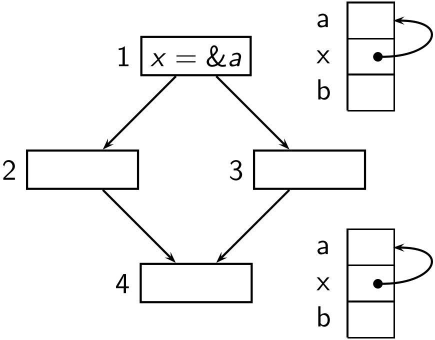
Symmetric
and
Reflexive

- What about transitivity?
  - Points-To: No.
  - Alias: Depends.

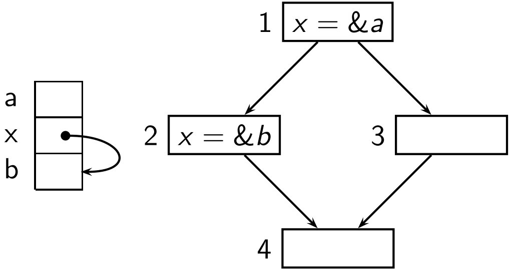
# Must Points-To Information



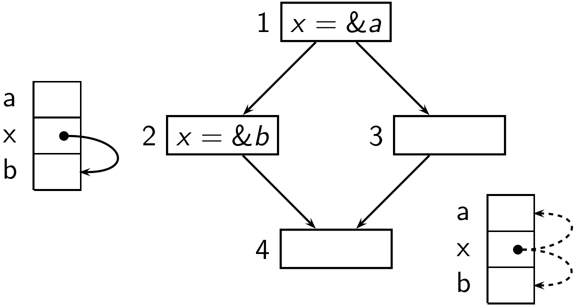
# Must Points-To Information



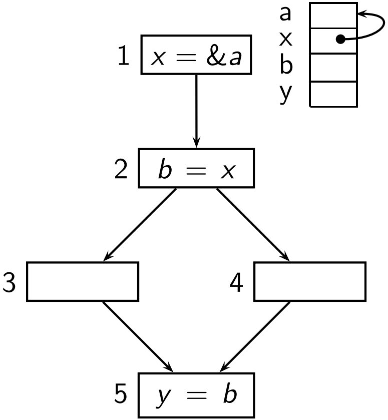
# May Points-To Information



# May Points-To Information



# Must Alias Information



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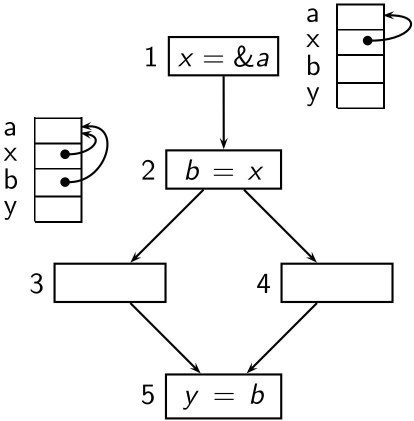

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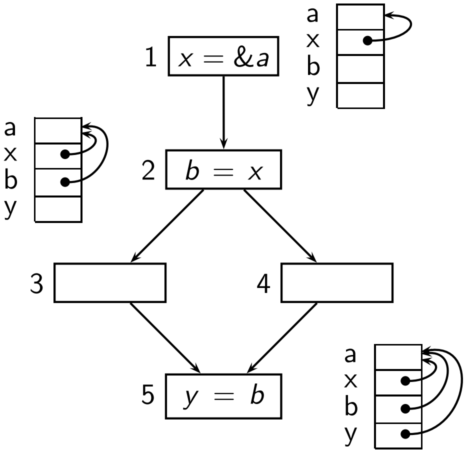

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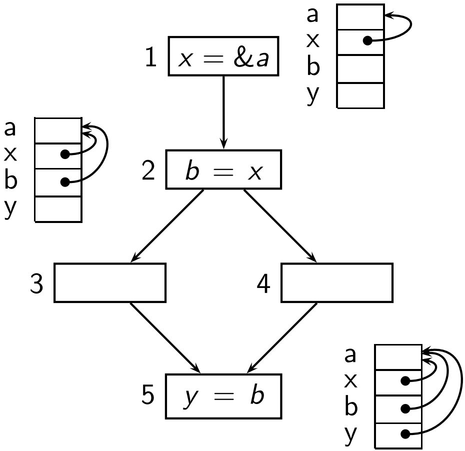
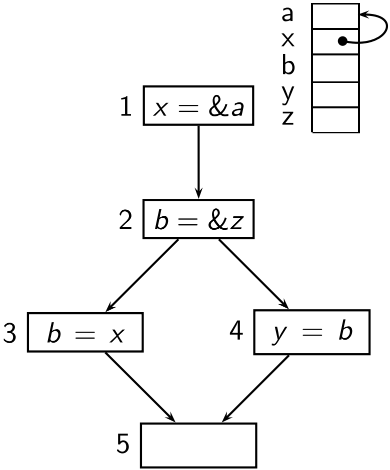


$$x \overset{\circ}{=} b \text{ and } b \overset{\circ}{=} y \Rightarrow x \overset{\circ}{=} y$$

# May Alias Information



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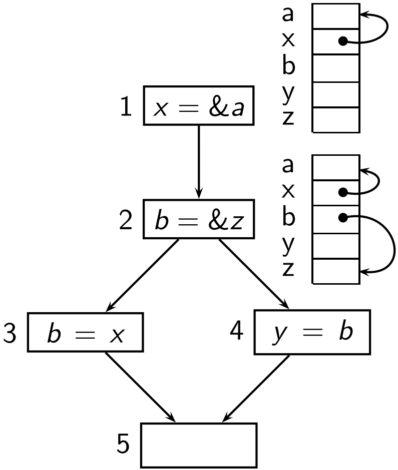

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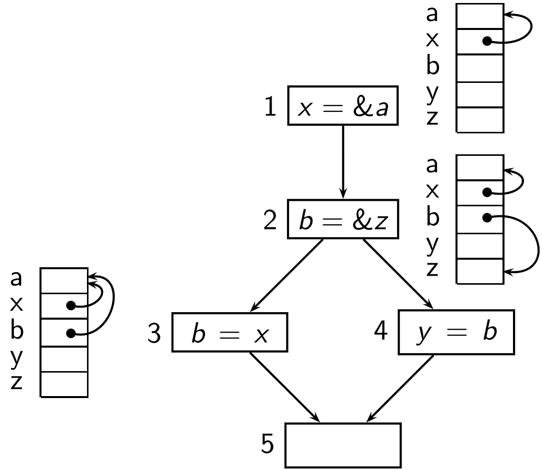

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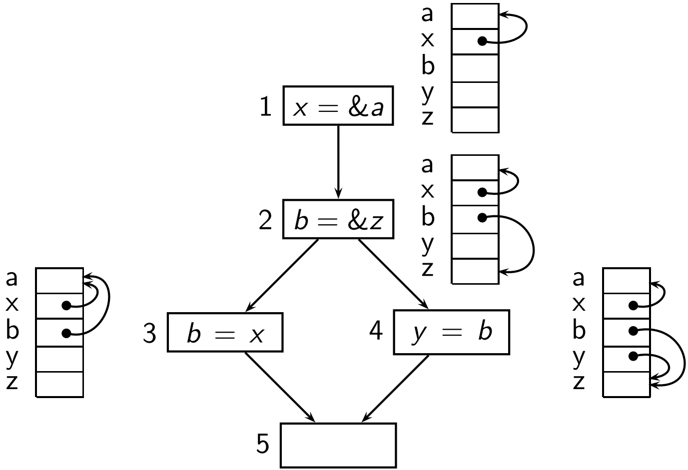

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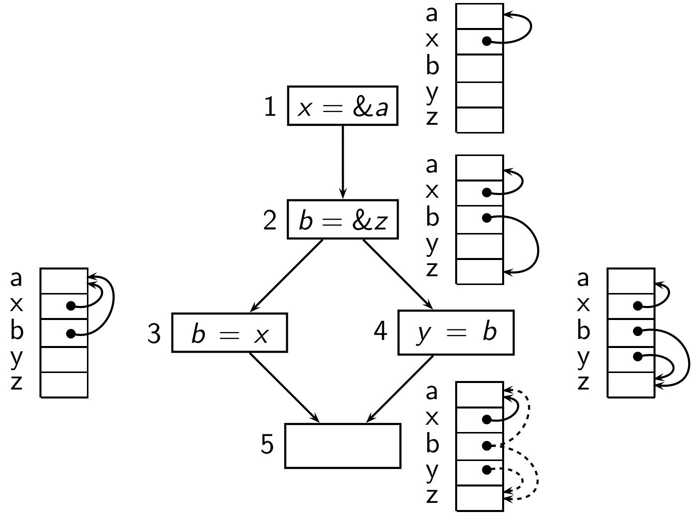

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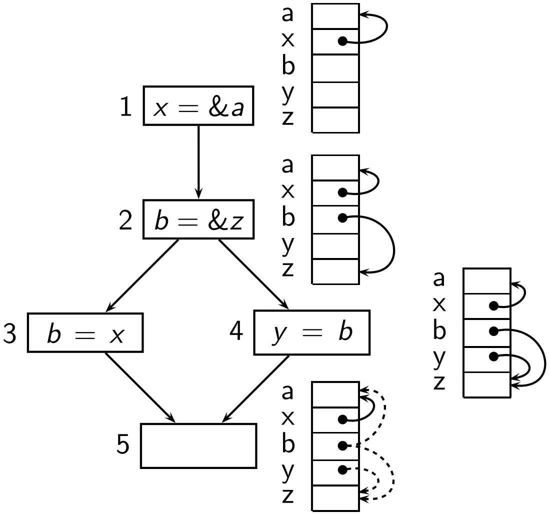
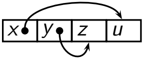
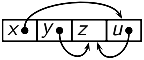
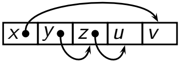
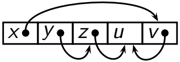
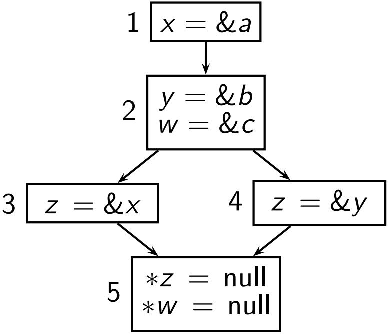


$$x \stackrel{\circ}{=} b \text{ and } b \stackrel{\circ}{=} y \not\Rightarrow x \stackrel{\circ}{=} y$$

# A Comparison of Points-To and Alias Relations

| Asgn. | Memory | Points-to | | Aliases | |
|---|---|---|---|---|---|
| $*x = y$ | Before  <br>After  | Existing | $x \rightarrowtail u$<br>$y \rightarrowtail z$ | Existing | $*x \circeq u$<br>$*y \circeq z$ |
| | | New | $u \rightarrowtail z$ | New Direct | $*x \circeq y$<br>$y \circeq u$ |
| | | | | New Indirect | $*u \circeq z$<br>$**x \circeq z$ |
| $*x = *y$ | Before  <br>After  | Existing | $x \rightarrowtail v$<br>$y \rightarrowtail z$<br>$z \rightarrowtail u$ | Existing | $*x \circeq v$<br>$*y \circeq z$<br>$*z \circeq u$<br>$**y \circeq u$ |
| | | New | $v \rightarrowtail u$ | New Direct | $*x \circeq *y$<br>$*x \circeq z$<br>$v \circeq z$<br>$v \circeq *y$ |
| | | | | New Indirect | $**x \circeq u$<br>$*v \circeq u$ |

# Strong and Weak Updates



1 $x = \&a$

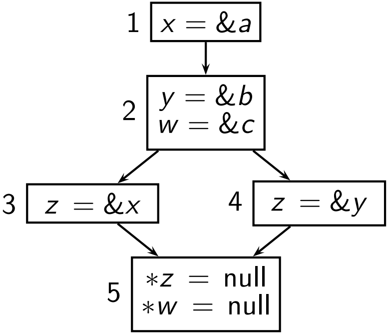
2 $y = \&b$ $w = \&c$

3 $z = \&x$

4 $z = \&y$

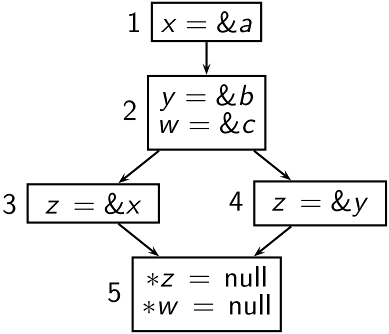
5 $*z = \text{null}$ $*w = \text{null}$

# Strong and Weak Updates



Weak update: Modification of $x$ or $y$ due to $*z$ in block 5

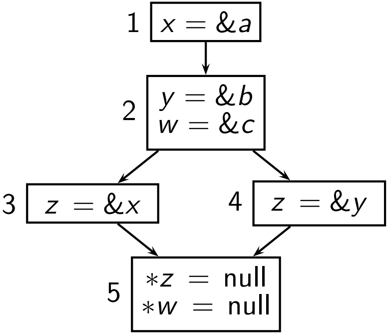
# Strong and Weak Updates



Weak update: Modification of $x$ or $y$ due to $*z$ in block 5

Strong update: Modification of $c$ due to $*w$ in block 5

# Strong and Weak Updates



Weak update: Modification of $x$ or $y$ due to $*z$ in block 5

Strong update: Modification of $c$ due to $*w$ in block 5

How is this concept related to May/Must nature of information?

# What About Heap Data?

- Compile time entities, abstract entities, or summarized entities
- Three options:
  - Represent all heap locations by a single abstract heap location
  - Represent all heap locations of a particular type by a single abstract heap location
  - Represent all heap locations allocated at a givem memory allocation site by a single abstract heap location
- Summarization: Usually based on the length of pointer expression
- No clean and elegant solution exists

# Left and Right Locations in Pointer Assignments

For an assignment statement $lhs_n = rhs_n$

- Left Locations

| Left Locations | | |
|---|---|---|
| $lhs_n$ | $ConstLeftL_n$ | $DepLeftL_n(X)$ |
| $x$ | $\{x\}$ | $\emptyset$ |
| $*x$ | $\emptyset$ | $\{y \mid (x \rightarrowtail y) \in X\}$ |

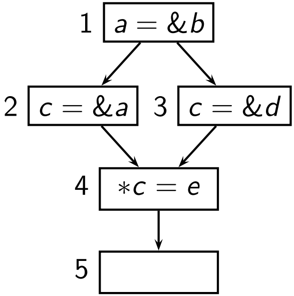
- Right Locations

| Right Locations | | |
|---|---|---|
| $rhs_n$ | $ConstRightL_n$ | $DepRightL_n(X)$ |
| $x$ | $\emptyset$ | $\{y \mid (x \rightarrowtail y) \in X\}$ |
| $*x$ | $\emptyset$ | $\{z \mid \{x \rightarrowtail y, y \rightarrowtail z\} \subseteq X\}$ |
| $\&x$ | $\{x\}$ | $\emptyset$ |

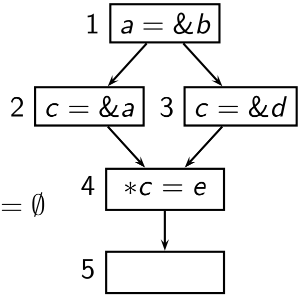
# Gen and Kill Components

$$\begin{aligned}
ConstGen_n &= \{x \rightarrowtail y \mid x \in ConstLeftL_n, y \in ConstRightL_n\} \\
DepGen_n(X) &= \{x \rightarrowtail y \mid (x \in ConstLeftL_n, y \in DepRightL_n(X)), \text{ or} \\
&\qquad (x \in DepLeftL_n(X), y \in ConstRightL_n), \text{ or} \\
&\qquad (x \in DepLeftL_n(X), y \in DepRightL_n(X))\} \\
ConstKill_n &= \{x \rightarrowtail y \mid x \in ConstLeftL_n\} \\
DepKill_n(X) &= \{x \rightarrowtail y \mid x \in DepLeftL_n(X)\}
\end{aligned}$$

# *DepKill*(*X*) in May and Must Points-To Analysis



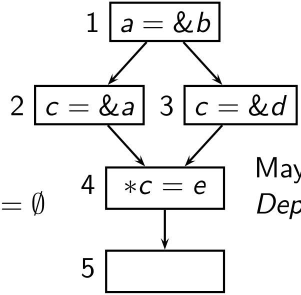
# $DepKill(X)$ in May and Must Points-To Analysis



$\text{MustIn}_4 = \{a \rightarrowtail b\}$

$DepLeftL_4(MustIn_4) = \emptyset$

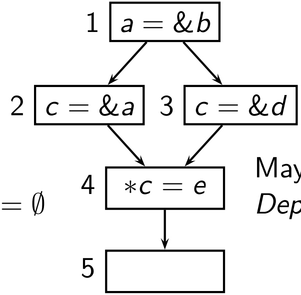
# $DepKill(X)$ in May and Must Points-To Analysis



$\text{MustIn}_4 = \{a \rightarrowtail b\}$
$DepLeftL_4(MustIn_4) = \emptyset$

$\text{MayIn}_4 = \{a \rightarrowtail b, c \rightarrowtail a, c \rightarrowtail d\}$
$DepLeftL_4(MayIn_4) = \{a, d\}$

# $DepKill(X)$ in May and Must Points-To Analysis



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- $a \rightarrowtail b$ at block 5 along path $1, 3, 4, 5$ but not along path $1, 2, 4, 5$.

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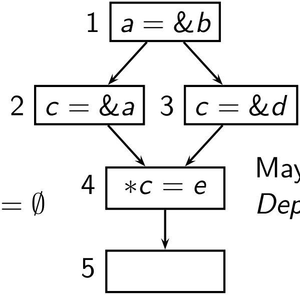

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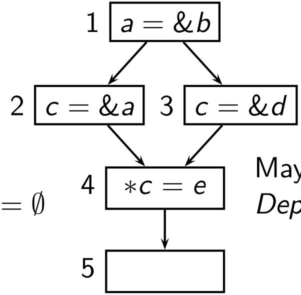
- $a \rightarrowtail b$ at block 5 along path $1, 3, 4, 5$ but not along path $1, 2, 4, 5$.
- $a \rightarrowtail b \in MayIn_5$ but $a \rightarrowtail b \notin MustIn_5$

# $DepKill(X)$ in May and Must Points-To Analysis



$\text{MustIn}_4 = \{a \rightarrowtail b\}$
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- If $DepKill_n$ for $MayOut_4$ is defined in terms of $MayIn_4$ then $a \rightarrowtail b \notin MayOut_4$ because $a$ is in $DepLeftL_4(MayIn_4)$

# $DepKill(X)$ in May and Must Points-To Analysis

$\text{MustIn}_4 = \{a \rightarrowtail b\}$
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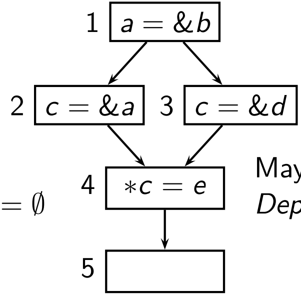

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- If $DepKill_4$ for $MustOut_4$ is defined in terms of $MustIn_4$ then $a \rightarrowtail b \in MustOut_4$ because $a$ is not in $DepLeftL_4(MustIn_4)$

# *DepKill*($X$) in May and Must Points-To Analysis

- May Points-To analysis
    - A points-to pair should be removed only if it must be removed along all paths
    - *DepKill*($X$) should remove only strong updates
    - $X$ should be Must Points-To information
- Must Points-To analysis
    - A points-to pair should be removed if it can be removed along some path
    - *DepKill*($X$) should remove all weak updates
    - $X$ should be May Points-To information
- Must Points-To $\subseteq$ May Points-To

# Data Flow Equations for Points-To Analysis

$$\begin{aligned}
\mathit{MayIn}_n &= \begin{cases} \mathit{BI} & n \text{ is } \mathit{Start} \\ \displaystyle\bigcup_{p\in \mathit{pred}(n)} \mathit{MayOut}_n & \text{otherwise} \end{cases} \\
\mathit{MayOut}_n &= f_n(\mathit{MayIn}_n, \mathit{MustIn}_n) \\
\mathit{MustIn}_n &= \begin{cases} \mathit{BI} & n \text{ is } \mathit{Start} \\ \displaystyle\bigcap_{p\in \mathit{pred}(n)} \mathit{MustOut}_n & \text{otherwise} \end{cases} \\
\mathit{MustOut}_n &= f_n(\mathit{MustIn}_n, \mathit{MayIn}_n)
\end{aligned}$$

$$f_n(X_1, X_2) = (X_1 - \mathrm{Kill}_n(X_2)) \cup \mathrm{Gen}_n(X_1)$$

# Approximating May and Must Alias and Points-To Information

- May Alias: Every pointer variable is aliased to every pointer variable.
- Must Alias: Every pointer variable is alised only to itself.

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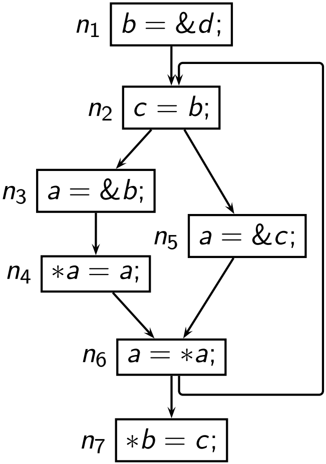
- Both May and Must analyses need not be performed.

*In every case, the approximation uses the* $\perp$ *element of the lattice.*

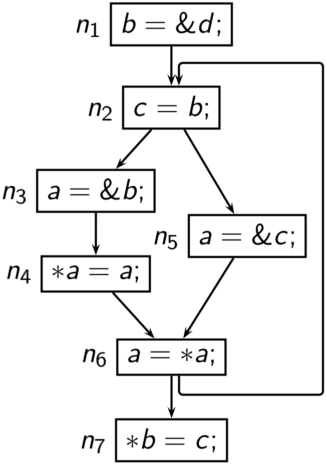
# Example Program for Points-To Analysis



- Variables and points-to sets:

$$
\begin{array}{rcl}
\mathbb{V}\text{ar} & = & \{a, b, c, d\} \\
\mathbb{U} & = & \{\ a \rightarrowtail a,\ a \rightarrowtail b,\ a \rightarrowtail c,\ a \rightarrowtail d, \\
 & & \quad b \rightarrowtail a,\ b \rightarrowtail b,\ b \rightarrowtail d,\ b \rightarrowtail d, \\
 & & \quad c \rightarrowtail a,\ c \rightarrowtail b,\ c \rightarrowtail c,\ c \rightarrowtail d, \\
 & & \quad d \rightarrowtail a,\ d \rightarrowtail b,\ d \rightarrowtail c,\ d \rightarrowtail d\ \}
\end{array}
$$

# Example Program for Points-To Analysis

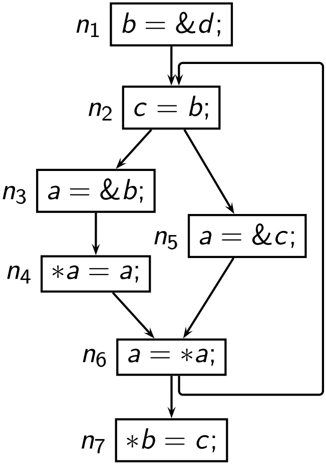


- Variables and points-to sets:

$$\begin{aligned} \mathbb{V}\text{ar} &= \{a, b, c, d\} \\ \mathbb{U} &= \{\ a \rightarrowtail a,\ a \rightarrowtail b,\ a \rightarrowtail c,\ a \rightarrowtail d, \\ &\qquad b \rightarrowtail a,\ b \rightarrowtail b,\ b \rightarrowtail d,\ b \rightarrowtail d, \\ &\qquad c \rightarrowtail a,\ c \rightarrowtail b,\ c \rightarrowtail c,\ c \rightarrowtail d, \\ &\qquad d \rightarrowtail a,\ d \rightarrowtail b,\ d \rightarrowtail c,\ d \rightarrowtail d\ \} \end{aligned}$$

- $L_{may} = \langle 2^{\mathbb{U}}, \supseteq \rangle,\ \top_{may} = \emptyset, \bot_{may} = \mathbb{U}$

# Example Program for Points-To Analysis

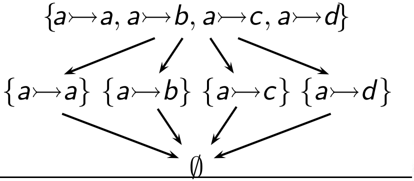


- Variables and points-to sets:

$$\begin{aligned} \mathbb{V}\text{ar} &= \{a, b, c, d\} \\ \mathbb{U} &= \{\ a \rightarrowtail a,\ a \rightarrowtail b,\ a \rightarrowtail c,\ a \rightarrowtail d, \\ &\quad\ \ b \rightarrowtail a,\ b \rightarrowtail b,\ b \rightarrowtail d,\ b \rightarrowtail d, \\ &\quad\ \ c \rightarrowtail a,\ c \rightarrowtail b,\ c \rightarrowtail c,\ c \rightarrowtail d, \\ &\quad\ \ d \rightarrowtail a,\ d \rightarrowtail b,\ d \rightarrowtail c,\ d \rightarrowtail d\ \} \end{aligned}$$

- $L_{may} = \langle 2^{\mathbb{U}}, \supseteq \rangle,\ \top_{may} = \emptyset, \bot_{may} = \mathbb{U}$

- $L_{must} = \widehat{L}_a \times \widehat{L}_b \times \widehat{L}_c \times \widehat{L}_d$
  The component lattice $\widehat{L}_a$ is:



$\{a \rightarrowtail a, a \rightarrowtail b, a \rightarrowtail c, a \rightarrowtail d\}$

$\{a \rightarrowtail a\}$ $\{a \rightarrowtail b\}$ $\{a \rightarrowtail c\}$ $\{a \rightarrowtail d\}$

$\emptyset$

# Result of Pointer Analysis

| | Iteration #1 | Changes in Iteration #2 | Changes in Iteration #3 |
|---|---|---|---|
| $\mathit{MayIn}_{n_1}$ | $\emptyset$ | | |
| $\mathit{MustIn}_{n_1}$ | $\emptyset$ | | |
| $\mathit{MayOut}_{n_1}$ | $\{b \rightarrowtail d\}$ | | |
| $\mathit{MustOut}_{n_1}$ | $\{b \rightarrowtail d\}$ | | |
| $\mathit{MayIn}_{n_2}$ | $\{b \rightarrowtail d\}$ | $\{a \rightarrowtail b, a \rightarrowtail d, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail d\}$ | $\{a \rightarrowtail b, a \rightarrowtail d, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail b, c \rightarrowtail d\}$ |
| $\mathit{MustIn}_{n_2}$ | $\{b \rightarrowtail d\}$ | $\emptyset$ | |
| $\mathit{MayOut}_{n_2}$ | $\{b \rightarrowtail d, c \rightarrowtail d\}$ | $\{a \rightarrowtail b, a \rightarrowtail d, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail b, c \rightarrowtail d\}$ | |
| $\mathit{MustOut}_{n_2}$ | $\{b \rightarrowtail d, c \rightarrowtail d\}$ | $\emptyset$ | |
| $\mathit{MayIn}_{n_3}$ | $\{b \rightarrowtail d, c \rightarrowtail d\}$ | $\{a \rightarrowtail b, a \rightarrowtail d, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail b, c \rightarrowtail d\}$ | |
| $\mathit{MustIn}_{n_3}$ | $\{b \rightarrowtail d, c \rightarrowtail d\}$ | $\emptyset$ | |
| $\mathit{MayOut}_{n_3}$ | $\{a \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail d\}$ | $\{a \rightarrowtail b, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail b, c \rightarrowtail d\}$ | |
| $\mathit{MustOut}_{n_3}$ | $\{a \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail d\}$ | $\{a \rightarrowtail b\}$ | |

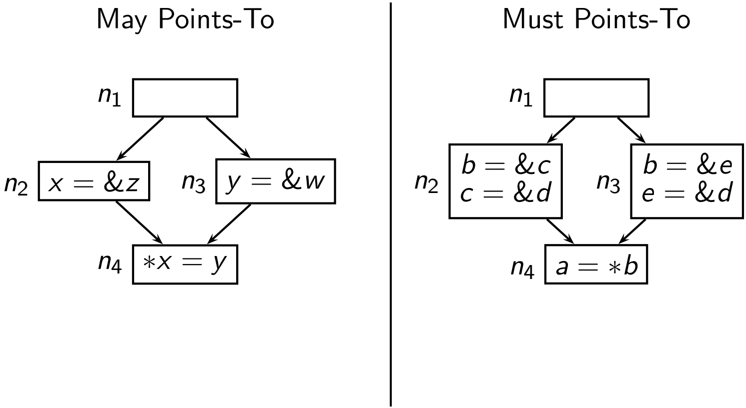
# Result of Pointer Analysis

| | Iteration #1 | Changes in Iteration #2 | Changes in Iteration #3 |
|---|---|---|---|
| $\mathit{MayIn}_{n_4}$ | $\{a \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail d\}$ | $\{a \rightarrowtail b, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail b, c \rightarrowtail d\}$ | |
| $\mathit{MustIn}_{n_4}$ | $\{a \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail d\}$ | $\{a \rightarrowtail b\}$ | |
| $\mathit{MayOut}_{n_4}$ | $\{a \rightarrowtail b, b \rightarrowtail b, c \rightarrowtail d\}$ | $\{a \rightarrowtail b, b \rightarrowtail b, c \rightarrowtail b, c \rightarrowtail d\}$ | |
| $\mathit{MustOut}_{n_4}$ | $\{a \rightarrowtail b, b \rightarrowtail b, c \rightarrowtail d\}$ | $\{a \rightarrowtail b, b \rightarrowtail b\}$ | |
| $\mathit{MayIn}_{n_5}$ | $\{b \rightarrowtail d, c \rightarrowtail d\}$ | $\{a \rightarrowtail b, a \rightarrowtail d, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail b, c \rightarrowtail d\}$ | |
| $\mathit{MustIn}_{n_5}$ | $\{b \rightarrowtail d, c \rightarrowtail d\}$ | $\emptyset$ | |
| $\mathit{MayOut}_{n_5}$ | $\{a \rightarrowtail c, b \rightarrowtail d, c \rightarrowtail d\}$ | $\{a \rightarrowtail c, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail b, c \rightarrowtail d\}$ | |
| $\mathit{MustOut}_{n_5}$ | $\{a \rightarrowtail c, b \rightarrowtail d, c \rightarrowtail d\}$ | $\{a \rightarrowtail c\}$ | |

# Result of Pointer Analysis

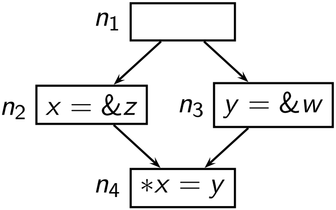
| | Iteration #1 | Changes in Iteration #2 | Changes in Iteration #3 |
|---|---|---|---|
| $MayIn_{n_6}$ | $\{a \rightarrowtail b, a \rightarrowtail c, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail d\}$ | $\{a \rightarrowtail b, a \rightarrowtail c, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail b, c \rightarrowtail d\}$ | |
| $MustIn_{n_6}$ | $\{c \rightarrowtail d\}$ | $\emptyset$ | |
| $MayOut_{n_6}$ | $\{a \rightarrowtail b, a \rightarrowtail d, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail d\}$ | $\{a \rightarrowtail b, a \rightarrowtail d, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail b, c \rightarrowtail d\}$ | |
| $MustOut_{n_6}$ | $\{c \rightarrowtail d\}$ | $\emptyset$ | |
| $MayIn_{n_7}$ | $\{a \rightarrowtail b, a \rightarrowtail d, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail d\}$ | $\{a \rightarrowtail b, a \rightarrowtail d, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail b, c \rightarrowtail d\}$ | |
| $MustIn_{n_7}$ | $\{c \rightarrowtail d\}$ | $\emptyset$ | |
| $MayOut_{n_7}$ | $\{a \rightarrowtail b, a \rightarrowtail d, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail d, d \rightarrowtail d\}$ | $\{a \rightarrowtail b, a \rightarrowtail d, b \rightarrowtail b, b \rightarrowtail d, c \rightarrowtail b, c \rightarrowtail d, d \rightarrowtail b, d \rightarrowtail d\}$ | |
| $MustOut_{n_7}$ | $\{c \rightarrowtail d\}$ | $\emptyset$ | |

# Non-Distributivity of Points-To Analysis



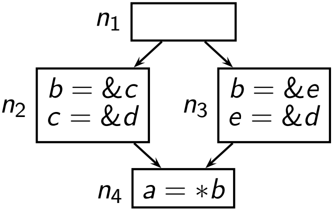
# Non-Distributivity of Points-To Analysis

May Points-To



$z \rightarrowtail w$ is spurious

Must Points-To



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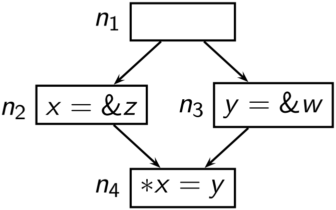

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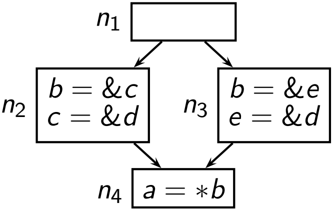
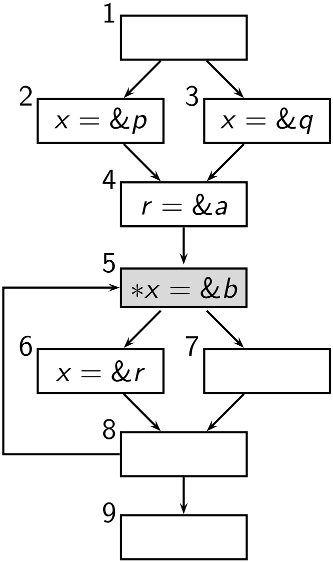
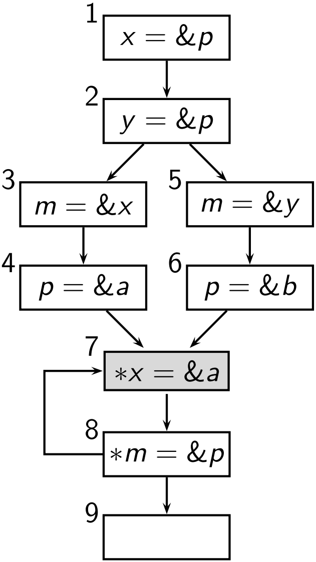


$a \rightarrowtail d$ is missing

# Tutorial Problems for May and Must Points-To Analysis





### *Part 6*

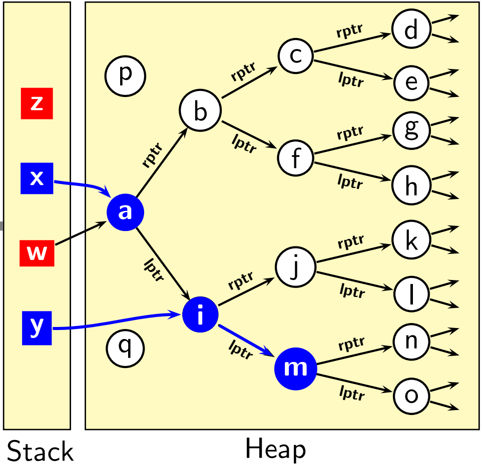
# *Heap Reference Analysis*

# Motivating Example for Heap Liveness Analysis

If the while loop is not executed even once.

```
1   w = x          // x points to $m_a$
2   while  (x.data < max)
3          x = x.rptr
4   y = x.lptr

5   z = New  class_of_z
6   y = y.lptr
7   z.sum = x.data + y.data
```



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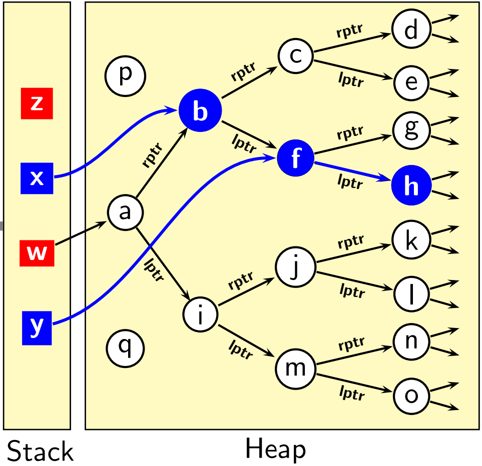
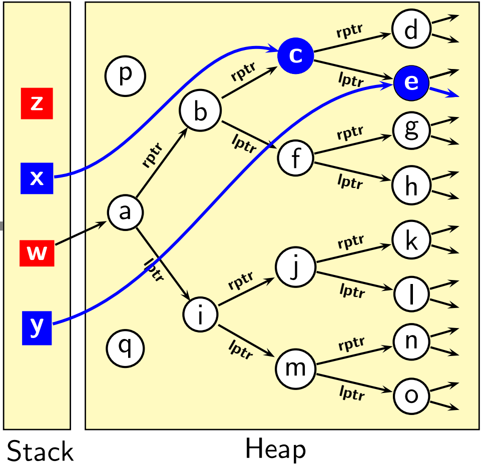


# Motivating Example for Heap Liveness Analysis

If the while loop is executed twice.

```
w = x          // x points to $m_a$
while  (x.data < max)
        x = x.rptr
y = x.lptr

z = New  class_of_z
y = y.lptr
z.sum = x.data + y.data
```



# The Moral of the Story

- Mappings between access expressions and l-values keep changing
- This is a *rule* for heap data
  For stack and static data, it is an *exception*!
- Static analysis of programs has made significant progress for stack and static data.

  What about heap data?

  - Given two access expressions at a program point, do they have the same l-value?
  - Given the same access expression at two program points, does it have the same l-value?

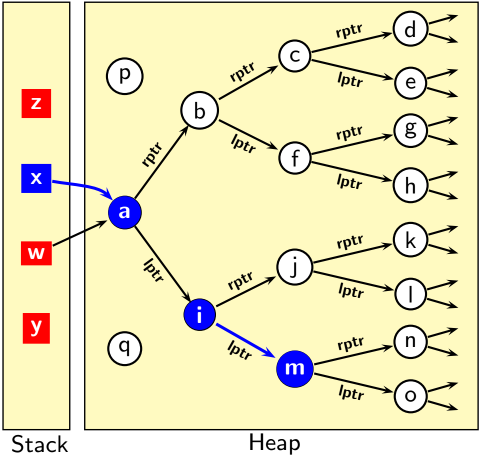
# Our Solution

```
                                  y = z = null
1  w = x
                                  w = null
2  while (x.data < max)
   {                              x.lptr = null
3          x = x.rptr       }
                                  x.rptr = x.lptr.rptr = null
                                  x.lptr.lptr.lptr = null
                                  x.lptr.lptr.rptr = null
4  y = x.lptr
                                  x.lptr = y.rptr = null
                                  y.lptr.lptr = y.lptr.rptr = null
5  z = New  class_of_z
                                  z.lptr = z.rptr = null
6  y = y.lptr
                                  y.lptr = y.rptr = null
7  z.sum = x.data + y.data
                                  x = y = z = null
```

# Our Solution

```
     y = z = null
1    w = x
     w = null
2    while (x.data < max)
     {       x.lptr = null
3            x = x.rptr        }
     x.rptr = x.lptr.rptr = null
     x.lptr.lptr.lptr = null
     x.lptr.lptr.rptr = null
4    y = x.lptr
     x.lptr = y.rptr = null
     y.lptr.lptr = y.lptr.rptr = null
5    z = New  class_of_z
     z.lptr = z.rptr = null
6    y = y.lptr
     y.lptr = y.rptr = null
7    z.sum = x.data + y.data
     x = y = z = null
```

While loop is not executed even once



# Our Solution

```
    y = z = null
1   w = x
    w = null
2   while (x.data < max)
    {        x.lptr = null
3            x = x.rptr        }
    x.rptr = x.lptr.rptr = null
    x.lptr.lptr.lptr = null
    x.lptr.lptr.rptr = null
4   y = x.lptr
    x.lptr = y.rptr = null
    y.lptr.lptr = y.lptr.rptr = null
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    z.lptr = z.rptr = null
6   y = y.lptr
    y.lptr = y.rptr = null
7   z.sum = x.data + y.data
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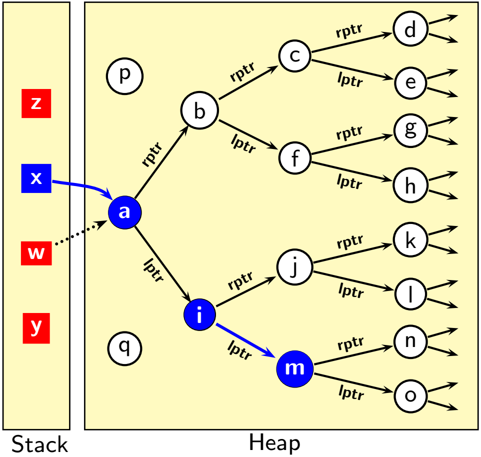

# Our Solution

```
  y = z = null
1 w = x
  w = null
2 while (x.data < max)
  {        x.lptr = null
3          x = x.rptr        }
  x.rptr = x.lptr.rptr = null
  x.lptr.lptr.lptr = null
  x.lptr.lptr.rptr = null
4 y = x.lptr
  x.lptr = y.rptr = null
  y.lptr.lptr = y.lptr.rptr = null
5 z = New  class_of_z
  z.lptr = z.rptr = null
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7 z.sum = x.data + y.data
  x = y = z = null
```

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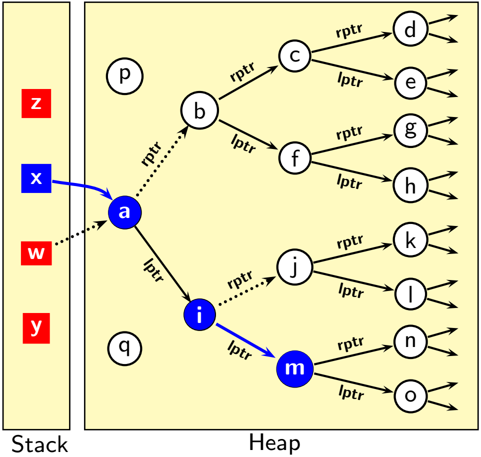

# Our Solution

```
y = z = null
w = x
w = null
while (x.data < max)
{       x.lptr = null
        x = x.rptr       }
x.rptr = x.lptr.rptr = null
x.lptr.lptr.lptr = null
x.lptr.lptr.rptr = null
y = x.lptr
x.lptr = y.rptr = null
y.lptr.lptr = y.lptr.rptr = null
z = New  class_of_z
z.lptr = z.rptr = null
y = y.lptr
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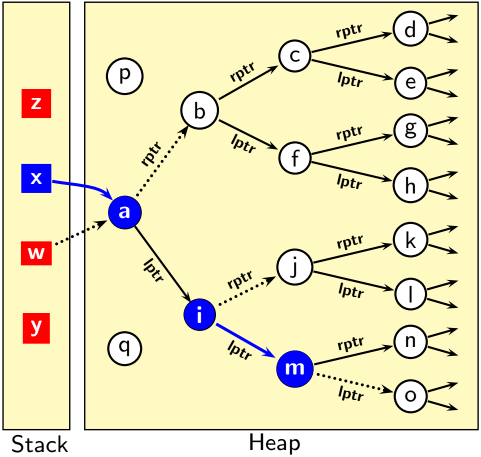
z.sum = x.data + y.data
x = y = z = null
```

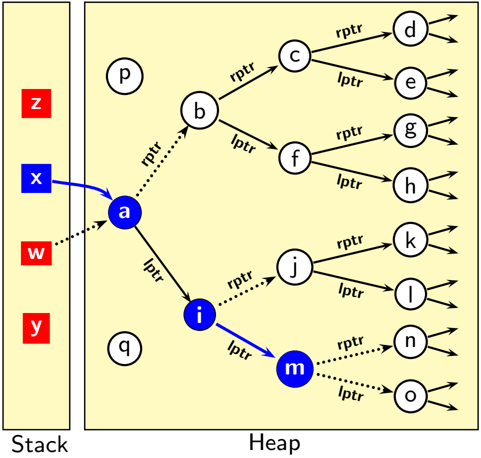
While loop is not executed even once



# Our Solution

```
  y = z = null
1 w = x
  w = null
2 while (x.data < max)
  {        x.lptr = null
3          x = x.rptr        }
  x.rptr = x.lptr.rptr = null
  x.lptr.lptr.lptr = null
  x.lptr.lptr.rptr = null
4 y = x.lptr
  x.lptr = y.rptr = null
  y.lptr.lptr = y.lptr.rptr = null
5 z = New  class_of_z
  z.lptr = z.rptr = null
6 y = y.lptr
  y.lptr = y.rptr = null
7 z.sum = x.data + y.data
  x = y = z = null
```

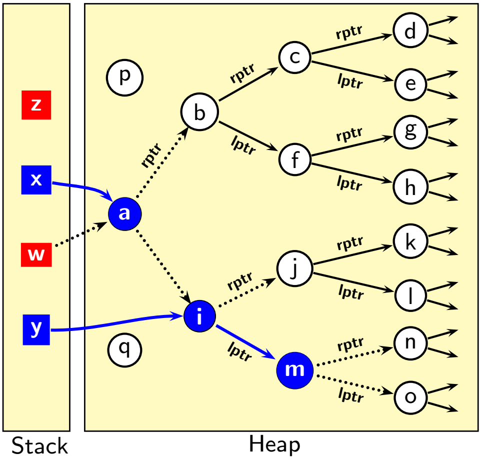
While loop is not executed even once



# Our Solution

```
  y = z = null
1 w = x
  w = null
2 while (x.data < max)
  {       x.lptr = null
3         x = x.rptr         }
  x.rptr = x.lptr.rptr = null
  x.lptr.lptr.lptr = null
  x.lptr.lptr.rptr = null
4 y = x.lptr
  x.lptr = y.rptr = null
  y.lptr.lptr = y.lptr.rptr = null
5 z = New  class_of_z
  z.lptr = z.rptr = null
6 y = y.lptr
  y.lptr = y.rptr = null
7 z.sum = x.data + y.data
  x = y = z = null
```

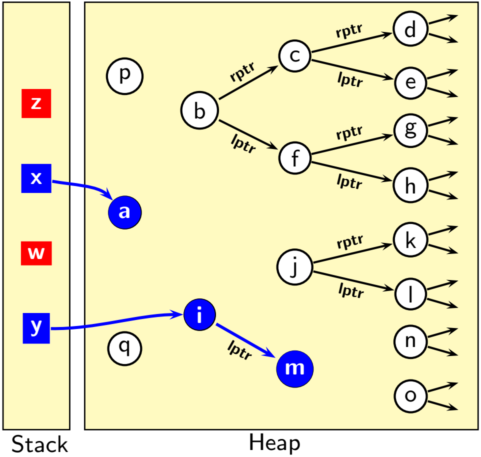
While loop is not executed even once



# Our Solution

```
    y = z = null
1   w = x
    w = null
2   while (x.data < max)
    {       x.lptr = null
3           x = x.rptr       }
    x.rptr = x.lptr.rptr = null
    x.lptr.lptr.lptr = null
    x.lptr.lptr.rptr = null
4   y = x.lptr
    x.lptr = y.rptr = null
    y.lptr.lptr = y.lptr.rptr = null
5   z = New  class_of_z
    z.lptr = z.rptr = null
6   y = y.lptr
    y.lptr = y.rptr = null
7   z.sum = x.data + y.data
    x = y = z = null
```

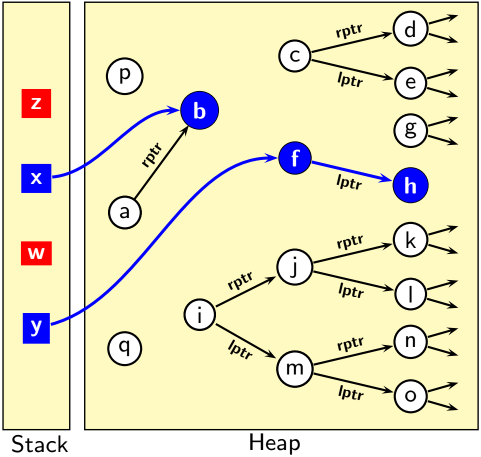
While loop is not executed even once



# Our Solution

```
y = z = null
w = x
w = null
while (x.data < max)
{       x.lptr = null
        x = x.rptr       }
x.rptr = x.lptr.rptr = null
x.lptr.lptr.lptr = null
x.lptr.lptr.rptr = null
y = x.lptr
x.lptr = y.rptr = null
y.lptr.lptr = y.lptr.rptr = null
z = New  class_of_z
z.lptr = z.rptr = null
y = y.lptr
y.lptr = y.rptr = null
z.sum = x.data + y.data
x = y = z = null
```

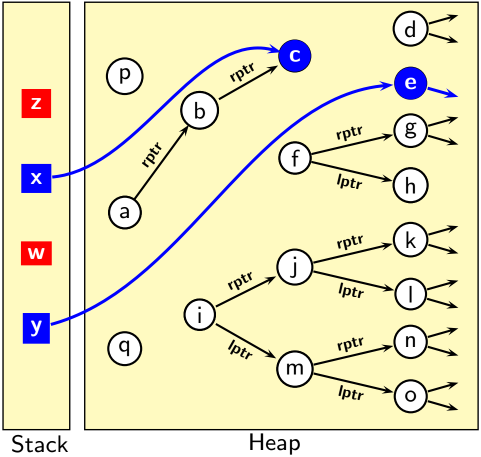
While loop is executed once



# Our Solution

```
    y = z = null
1   w = x
    w = null
2   while (x.data < max)
    {       x.lptr = null
3           x = x.rptr        }
    x.rptr = x.lptr.rptr = null
    x.lptr.lptr.lptr = null
    x.lptr.lptr.rptr = null
4   y = x.lptr
    x.lptr = y.rptr = null
    y.lptr.lptr = y.lptr.rptr = null
5   z = New  class_of_z
    z.lptr = z.rptr = null
6   y = y.lptr
    y.lptr = y.rptr = null
7   z.sum = x.data + y.data
    x = y = z = null
```

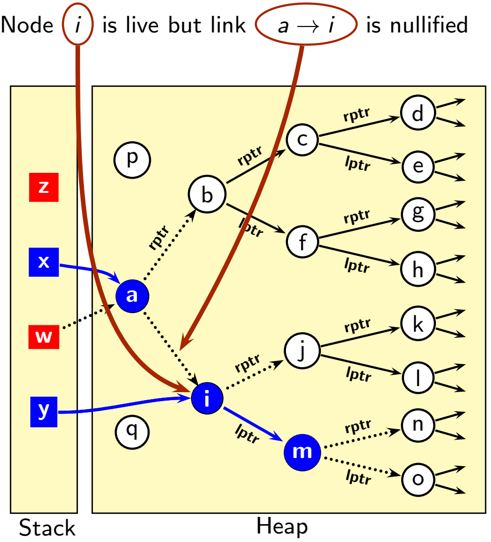
While loop is executed twice



# Some Observations

```
     y = z = null
1    w = x
     w = null
2    while (x.data < max)
     {       x.lptr = null
3            x = x.rptr        }
     x.rptr = x.lptr.rptr = null
     x.lptr.lptr.lptr = null
     x.lptr.lptr.rptr = null
4    y = x.lptr
     x.lptr = y.rptr = null
     y.lptr.lptr = y.lptr.rptr = null
5    z = New  class_of_z
     z.lptr = z.rptr = null
6    y = y.lptr
     y.lptr = y.rptr = null
7    z.sum = x.data + y.data
     x = y = z = null
```

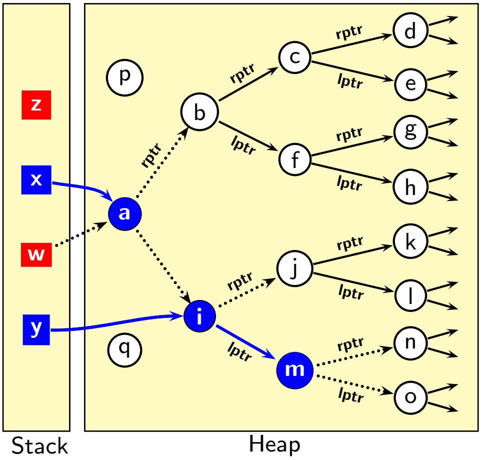
Node $i$ is live but link $a \rightarrow i$ is nullified



# Some Observations

```
y = z = null
w = x
w = null
while (x.data < max)
{       x.lptr = null
        x = x.rptr       }
x.rptr = x.lptr.rptr = null
x.lptr.lptr.lptr = null
x.lptr.lptr.rptr = null
y = x.lptr
x.lptr = y.rptr = null
y.lptr.lptr = y.lptr.rptr = null
z = New  class_of_z
z.lptr = z.rptr = null
y = y.lptr
y.lptr = y.rptr = null
z.sum = x.data + y.data
x = y = z = null
```

New access expressions are created.
Can they cause exceptions?



# An Overview of Heap Reference Analysis

- A reference (called a *link*) can be represented by an *access path*.

  Eg. "$x \rightarrow$ lptr $\rightarrow$ rptr"

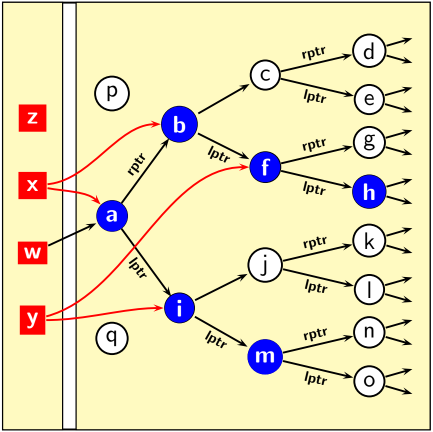
- A link may be accessed in multiple ways
- Setting links to null
  - *Alias Analysis*. Identify all possible ways of accessing a link
  - *Liveness Analysis*. For each program point, identify "dead" links (i.e. links which are not accessed after that program point)
  - *Availability and Anticipability Analyses*. Dead links should be reachable for making null assignment.
  - *Code Transformation*. Set "dead" links to null

# Assumptions

For simplicity of exposition

- Java model of heap access
    - Root variables are on stack and represent references to memory in heap.
    - Root variables cannot be pointed to by any reference.
- Simple extensions for C++
    - Root variables can be pointed to by other pointers.
    - Pointer arithmetic is not handled.

# Key Idea #1 : Access Paths Denote Links



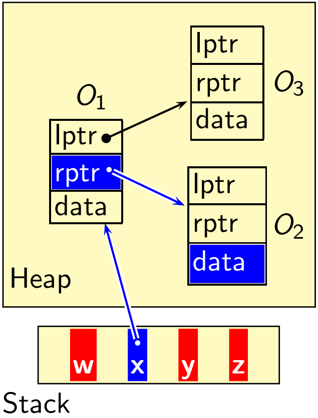
- Root variables : $x, y, z$
- Field names : rptr, lptr
- Access path : $x$→rptr→lptr
  Semantically, sequence of "links"
- Frontier : name of the last link
- Live access path : If the link corresponding to its frontier is used in future

# What Makes a Link Live?

Assuming that a statement is the last statement in the program, if nullifying a link read in the statement can change the semantics of the program, then the link is live.

*Reading a link for accessing the contents of the corresponding target object:*

| Example | Objects read | Live access paths |
|---|---|---|
| sum $=$ $x$.rptr.data | $x, O_1, O_2$ | $x, x \rightarrow$rptr |
| if ($x$.rptr.data $<$ sum) | $x, O_1, O_2$ | $x, x \rightarrow$rptr |

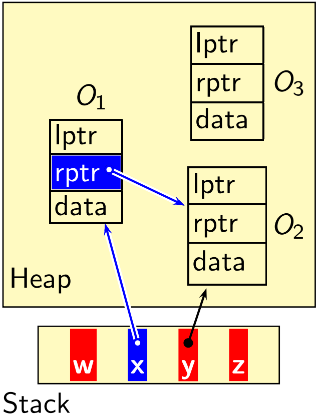


# What Makes a Link Live?

Assuming that a statement is the last statement in the program, if nullifying a link read in the statement can change the semantics of the program, then the link is live.

*Reading a link for copying the contents of the corresponding target object:*

| Example | Objects read | Live access paths |
|---|---|---|
| $y = x.\text{rptr}$ | $x, O_1$ | $x$ |
| | | |

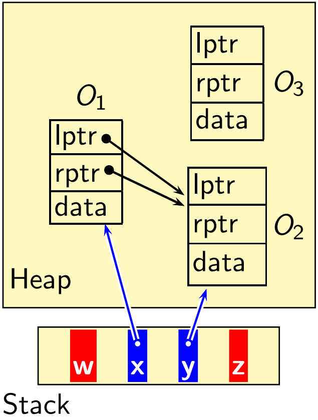


# What Makes a Link Live?

Assuming that a statement is the last statement in the program, if nullifying a link read in the statement can change the semantics of the program, then the link is live.

*Reading a link for copying the contents of the corresponding target object:*

| Example | Objects read | Live access paths |
|---|---|---|
| $y = x.\text{rptr}$ | $x, O_1$ | $x$ |
| $x.\text{lptr} = y$ | $x, O_1, y$ | $x$ |

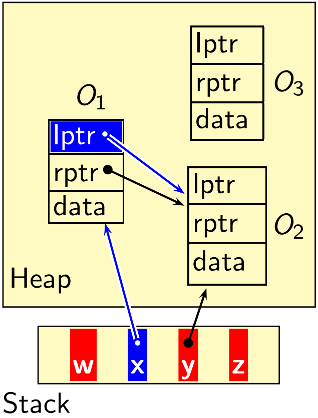


# What Makes a Link Live?

Assuming that a statement is the last statement in the program, if nullifying a link read in the statement can change the semantics of the program, then the link is live.

*Reading a link for comparing the address of the corresponding target object:*

| Example | Objects read | Live access paths |
|---|---|---|
| if ($x$.lptr == null) | $x, O_1$ | $x, x$➔lptr |
| | | |



# What Makes a Link Live?

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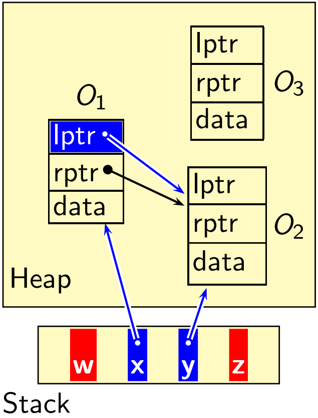
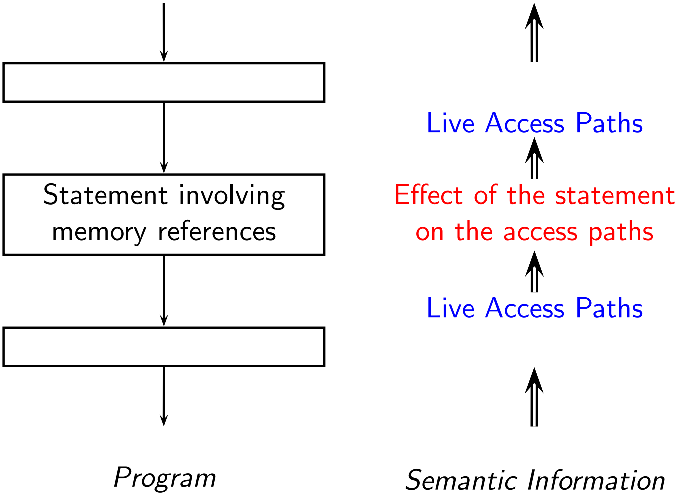
*Reading a link for comparing the address of the corresponding target object:*

| Example | Objects read | Live access paths |
|---|---|---|
| if ($x$.lptr == null) | $x, O_1$ | $x, x\rightarrow$lptr |
| if ($y == x$.lptr) | $x, O_1, y$ | $x, x\rightarrow$lptr$, y$ |



# Liveness Analysis



Statement involving memory references

Live Access Paths

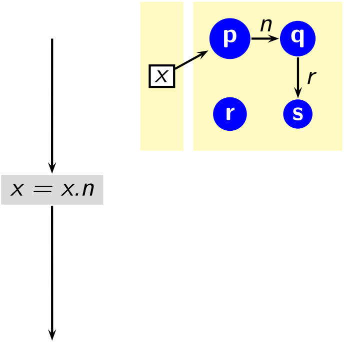
Effect of the statement on the access paths

Live Access Paths

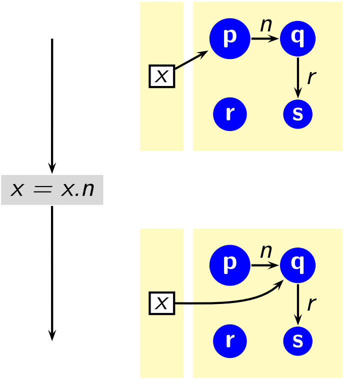
*Program*

*Semantic Information*

# Key Idea #2 : Transfer of Access Paths



# Key Idea #2 : Transfer of Access Paths



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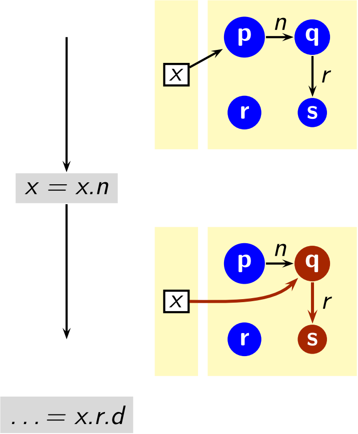

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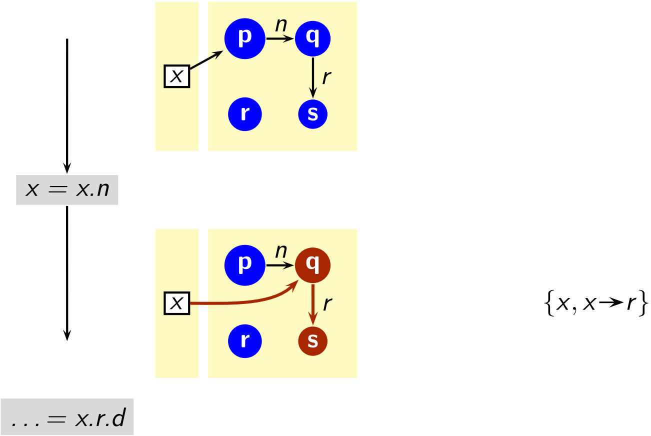

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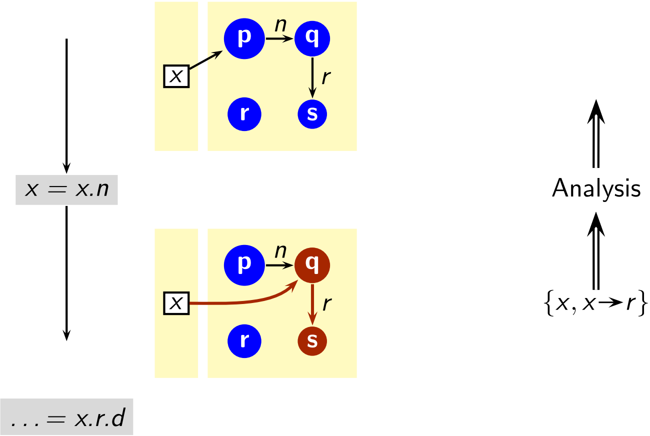


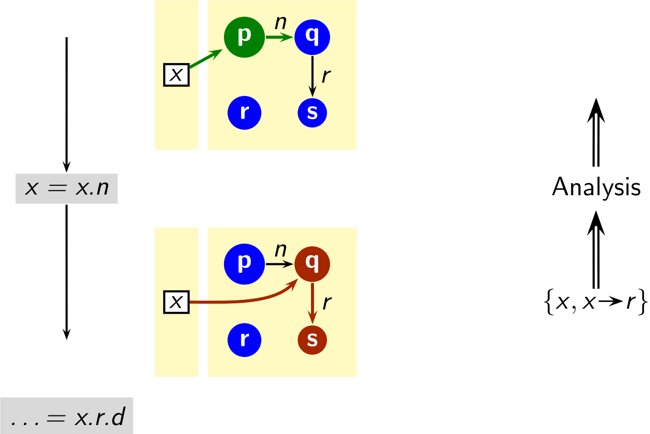
$x = x.n$

$\ldots = x.r.d$

Analysis

$\{x, x \rightarrow r\}$

# Key Idea #2 : Transfer of Access Paths



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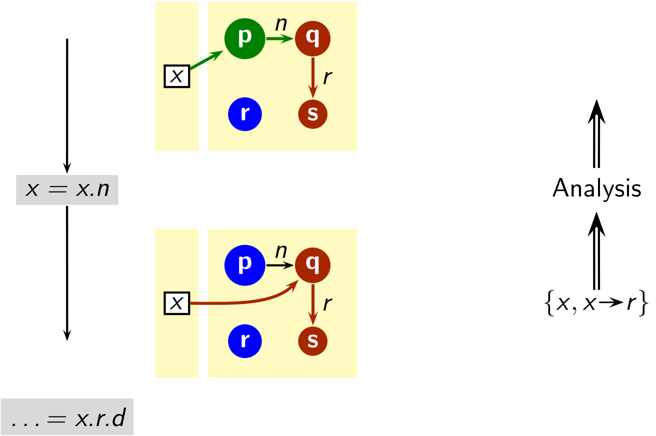

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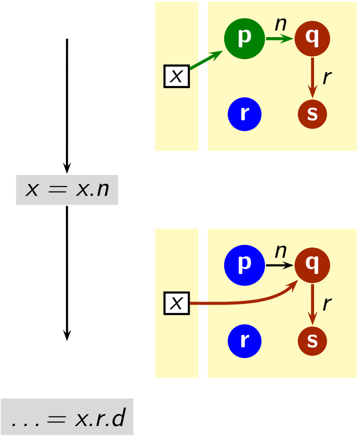
# Key Idea #2 : Transfer of Access Paths



Generated

Constant $\{x\}$

Dependent $\{x \rightarrow n, x \rightarrow n \rightarrow r\}$

Killed

Constant $\{x, x \rightarrow r\}$

Dependent $\emptyset$

$\Uparrow$

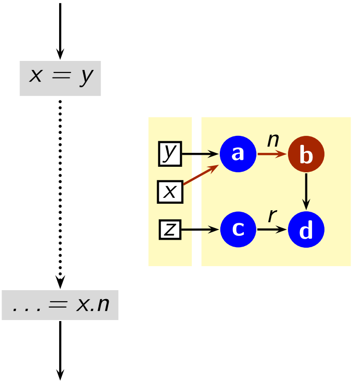
Analysis

$\Uparrow$

$\{x, x \rightarrow r\}$

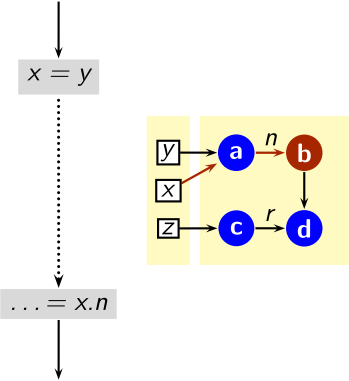
$x$ after the assinment is same as the $x \rightarrow n$ before the assignment

# Key Idea #3 : Liveness Closure Under Link Aliasing



$x$ and $y$ are node aliases

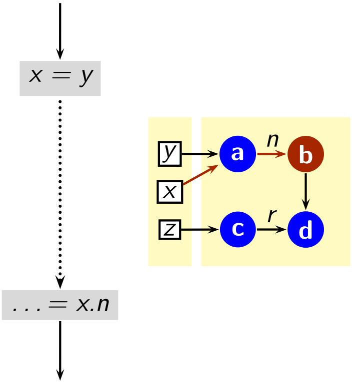
# Key Idea #3 : Liveness Closure Under Link Aliasing



$x$ and $y$ are node aliases

$x.n$ and $y.n$ are link aliases

# Key Idea #3 : Liveness Closure Under Link Aliasing

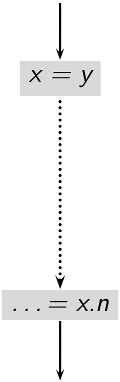


$x$ and $y$ are node aliases

$x.n$ and $y.n$ are link aliases

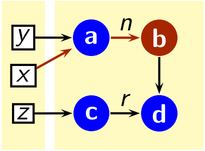
$x \rightarrow n$ is live $\Rightarrow$ $y \rightarrow n$ is live

# Key Idea #3 : Liveness Closure Under Link Aliasing



$x = y$

$\ldots = x.n$



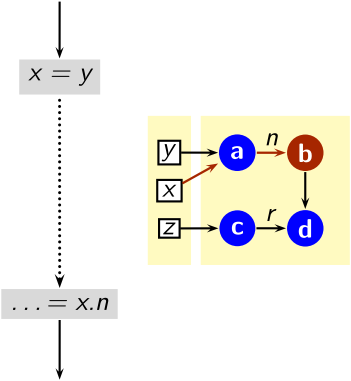
$x$ and $y$ are node aliases

$x.n$ and $y.n$ are link aliases

$x \rightarrow n$ is live $\Rightarrow$ $y \rightarrow n$ is live

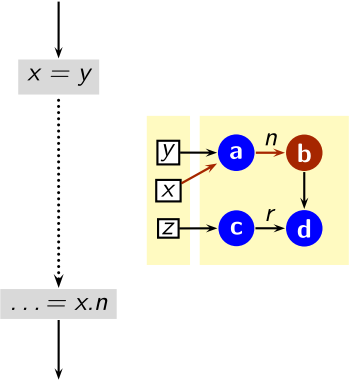
Nullifying $y \rightarrow n$ will have the side effect of nullifying $x \rightarrow n$

# Explicit and Implicit Liveness



$x \rightarrow n$ is live $\Rightarrow$ $y \rightarrow n$ is live

# Explicit and Implicit Liveness



$x \rightarrow n$ is live $\Rightarrow$ $y \rightarrow n$ is live

$y \rightarrow n$ is implicitly live
$x \rightarrow n$ is explicitly live

# Key Idea #4: Explicit Liveness Covers Entire Heap Usage

- Explicit Liveness at $p$
  Liveness purely due to the program beyond $p$.
  The effect of execution before $p$ is not incorporated.

# Key Idea #4: Explicit Liveness Covers Entire Heap Usage

- Explicit Liveness at *p*
  Liveness purely due to the program beyond *p*.
  The effect of execution before *p* is not incorporated.
- Implicit Liveness at *p*
  Access paths that become live under link alias closure.

# Key Idea #4: Explicit Liveness Covers Entire Heap Usage

- Explicit Liveness at $p$
  Liveness purely due to the program beyond $p$.
  The effect of execution before $p$ is not incorporated.
- Implicit Liveness at $p$
  Access paths that become live under link alias closure.
  - The set of implicitly live access paths may not be prefix closed.

# Key Idea #4: Explicit Liveness Covers Entire Heap Usage

- Explicit Liveness at $p$
  Liveness purely due to the program beyond $p$.
  The effect of execution before $p$ is not incorporated.
- Implicit Liveness at $p$
  Access paths that become live under link alias closure.
  - The set of implicitly live access paths may not be prefix closed.
  - These *paths* are not accessed, their frontiers are accessed through some other access path

# Key Idea #4: Explicit Liveness Covers Entire Heap Usage

- Explicit Liveness at $p$
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- Implicit Liveness at $p$
  Access paths that become live under link alias closure.
    - The set of implicitly live access paths may not be prefix closed.
    - These *paths* are not accessed, their frontiers are accessed through some other access path

*Every live link in the heap is the Frontier of some explicitly live access path.*

# Notation for Defining Flow Functions for Explicit Liveness

$$\begin{aligned} \mathit{base}(\rho_x) &= \text{longest proper prefix of } \rho_x \\ \mathit{prefixes}(\rho_x) &= \{\rho'_x \mid \rho'_x \text{ is a prefixe of } \rho_x\} \\ \mathit{summary}(S) &= \{\rho_x \rightarrow * \mid \rho_x \in S\} \end{aligned}$$

| $\rho_x$ | frontier($\rho_x$) | $\mathit{base}(\rho_x)$ | $\mathit{prefixes}(\rho_x)$ | $\mathit{summary}(\{\rho_x\})$ |
|---|---|---|---|---|
| $x \rightarrow n \rightarrow r$ | $r$ | $x \rightarrow n$ | $\{x, x \rightarrow n, x \rightarrow n \rightarrow r\}$ | $\{x \rightarrow n \rightarrow r \rightarrow *\}$ |
| $x \rightarrow r \rightarrow n$ | $n$ | $x \rightarrow r$ | $\{x, x \rightarrow r, x \rightarrow r \rightarrow n\}$ | $\{x \rightarrow r \rightarrow n \rightarrow *\}$ |
| $x \rightarrow n$ | $n$ | $x$ | $\{x, x \rightarrow n\}$ | $\{x \rightarrow n \rightarrow *\}$ |
| $x \rightarrow r$ | $r$ | $x$ | $\{x, x \rightarrow r\}$ | $\{x \rightarrow r \rightarrow *\}$ |
| $x$ | $x$ | $\mathcal{E}$ | $\{x\}$ | $\{x \rightarrow *\}$ |

empty access path

0 or more occurrences of any field name

# Notation for Defining Flow Functions for Explicit Liveness

$$\begin{array}{rcl} \mathit{base}(\rho_x) & = & \text{longest proper prefix of } \rho_x \\ \mathit{prefixes}(\rho_x) & = & \{\rho'_x \mid \rho'_x \text{ is a prefixe of } \rho_x\} \\ \mathit{summary}(S) & = & \{\rho_x \rightarrow * \mid \rho_x \in S\} \end{array}$$

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|---|---|---|---|---|
| $x \rightarrow n \rightarrow r$ | $r$ | $x \rightarrow n$ | $\{x, x \rightarrow n, x \rightarrow n \rightarrow r\}$ | $\{x \rightarrow n \rightarrow r \rightarrow *\}$ |
| $x \rightarrow r \rightarrow n$ | $n$ | $x \rightarrow r$ | $\{x, x \rightarrow r, x \rightarrow r \rightarrow n\}$ | $\{x \rightarrow r \rightarrow n \rightarrow *\}$ |
| $x \rightarrow n$ | $n$ | $x$ | $\{x, x \rightarrow n\}$ | $\{x \rightarrow n \rightarrow *\}$ |
| $x \rightarrow r$ | $r$ | $x$ | $\{x, x \rightarrow r\}$ | $\{x \rightarrow r \rightarrow *\}$ |
| $x$ | $x$ | $\mathcal{E}$ | $\{x\}$ | $\{x \rightarrow *\}$ |

# Flow Functions for Explicit Liveness Analysis

access expression

corresponding access path

| Statement | $ConstKill$ | $DepKill(X)$ | $ConstGen$ | $DepGen(X)$ |
|---|---|---|---|---|
| $Use\ \alpha_y$ | $\emptyset$ | $\emptyset$ | $prefixes(base(\rho_y))$ | $\emptyset$ |
| $Use\ \alpha_y.d$ | $\emptyset$ | $\emptyset$ | $prefixes(\rho_y)$ | $\emptyset$ |
| $\alpha_x = new$ | $\{\rho_x \rightarrow *\}$ | $\emptyset$ | $prefixes(base(\rho_x))$ | $\emptyset$ |
| $\alpha_x = Null$ | $\{\rho_x \rightarrow *\}$ | $\emptyset$ | $prefixes(base(\rho_x))$ | $\emptyset$ |
| $\alpha_x = \alpha_y$ | $\{\rho_x \rightarrow *\}$ | $\emptyset$ | $prefixes(base(\rho_x)) \cup$<br>$prefixes(base(\rho_y))$ | $\{\rho_y \rightarrow \sigma \mid \rho_x \rightarrow \sigma \in X\}$ |
| $End$ | $\emptyset$ | $\emptyset$ | $summary(Globals)$ | $\emptyset$ |
| other | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |

# Flow Functions for Explicit Liveness Analysis

| Statement | $ConstKill$ | $DepKill(X)$ | $ConstGen$ | $DepGen(X)$ |
|---|---|---|---|---|
| $Use\ \alpha_y$ | $\emptyset$ | $\emptyset$ | $prefixes(base(\rho_y))$ | $\emptyset$ |
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| $\alpha_x = new$ | $\{\rho_x \rightarrow *\}$ | $\emptyset$ | $prefixes(base(\rho_x))$ | $\emptyset$ |
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| $End$ | $\emptyset$ | $\emptyset$ | $summary(Globals)$ | $\emptyset$ |
| other | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |

Transfer

# Flow Functions for Explicit Liveness Analysis

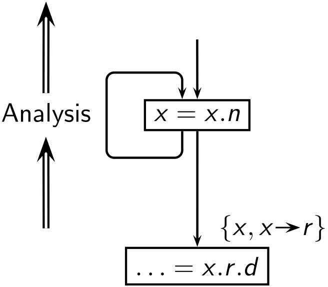
| Statement | *ConstKill* | *DepKill*$(X)$ | *ConstGen* | *DepGen*$(X)$ |
|---|---|---|---|---|
| *Use* $\alpha_y$ | $\emptyset$ | $\emptyset$ | $prefixes(base(\rho_y))$ | $\emptyset$ |
| *Use* $\alpha_y.d$ | $\emptyset$ | $\emptyset$ | $prefixes(\rho_y)$ | $\emptyset$ |
| $\alpha_x = new$ | $\{\rho_x \rightarrow *\}$ | $\emptyset$ | $prefixes(base(\rho_x))$ | $\emptyset$ |
| $\alpha_x = Null$ | $\{\rho_x \rightarrow *\}$ | $\emptyset$ | $prefixes(base(\rho_x))$ | $\emptyset$ |
| $\alpha_x = \alpha_y$ | $\{\rho_x \rightarrow *\}$ | $\emptyset$ | $prefixes(base(\rho_x)) \cup$<br>$prefixes(base(\rho_y))$ | $\{\rho_y \rightarrow \sigma \mid \rho_x \rightarrow \sigma \in X\}$ |
| *End* | $\emptyset$ | $\emptyset$ | $summary(Globals)$ | $\emptyset$ |
| other | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |

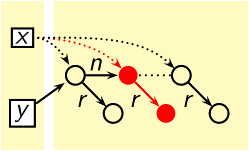
End of procedure

# Flow Functions for Handling Procedure Calls in Computing Explicit Liveness

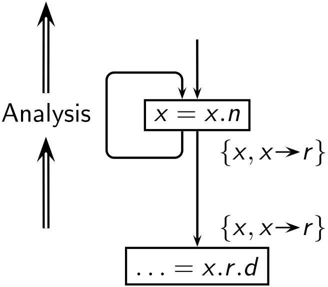
| Statement | $ConstKill$ | $DepKill(X)$ | $ConstGen$ | $DepGen(X)$ |
|---|---|---|---|---|
| $\alpha_x = f(\alpha_y)$ | $\{\rho_x \rightarrow *\}$ | $\emptyset$ | $prefixes(base(\rho_x)) \cup$<br>$prefixes(base(\rho_y)) \cup$<br>$summary(\{\rho_y\} \cup Globals)$ | $\emptyset$ |
| $return\ \alpha_y$ | $\emptyset$ | $\emptyset$ | $prefixes(base(\rho_y)) \cup$<br>$summary(\{\rho_y\})$ | $\emptyset$ |

# Computing Explicit Liveness Using Sets of Access Paths





# Computing Explicit Liveness Using Sets of Access Paths



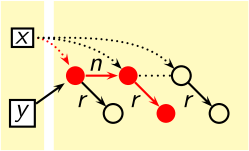
Analysis

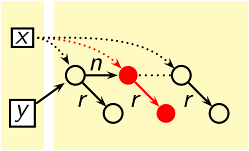
$x = x.n$

$\{x, x \rightarrow r\}$

$\{x, x \rightarrow r\}$

$\ldots = x.r.d$





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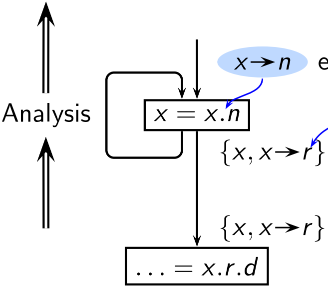


$x \rightarrow n$ extended with $r$

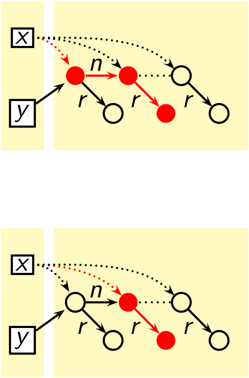
Analysis

$x = x.n$

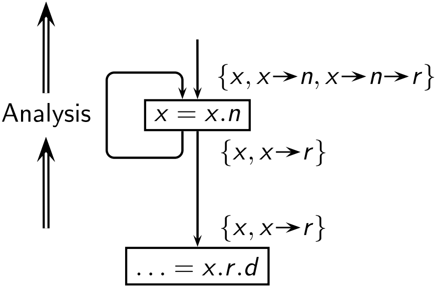
$\{x, x \rightarrow r\}$

$\{x, x \rightarrow r\}$

$\ldots = x.r.d$



# Computing Explicit Liveness Using Sets of Access Paths



Analysis

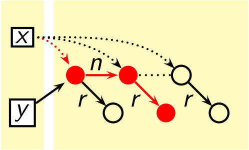
$\{x, x\rightarrow n, x\rightarrow n\rightarrow r\}$

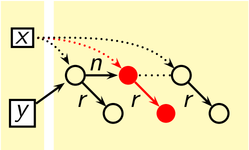
$x = x.n$

$\{x, x\rightarrow r\}$

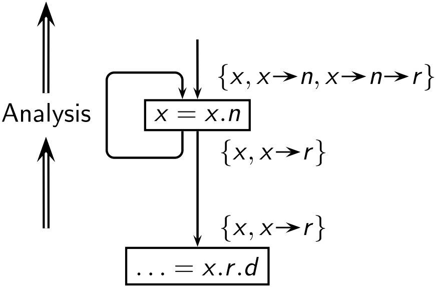
$\{x, x\rightarrow r\}$

$\ldots = x.r.d$





# Computing Explicit Liveness Using Sets of Access Paths



Analysis

$\{x, x\rightarrow n, x\rightarrow n\rightarrow r\}$

$x = x.n$

$\{x, x\rightarrow r\}$

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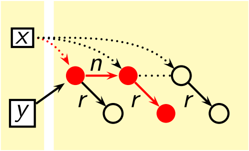
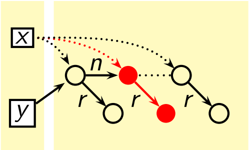
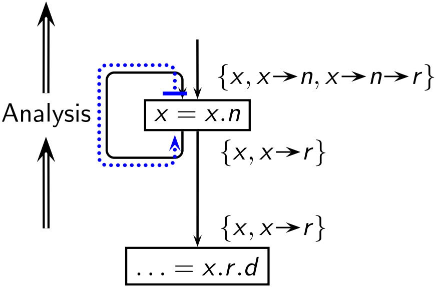




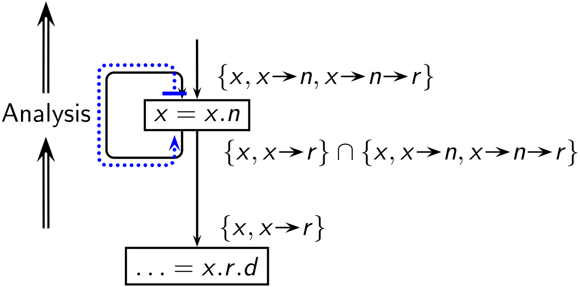
# Computing Explicit Liveness Using Sets of Access Paths

Anticipability of Heap References: An *All Paths* problem



# Computing Explicit Liveness Using Sets of Access Paths

Anticipability of Heap References: An *All Paths* problem



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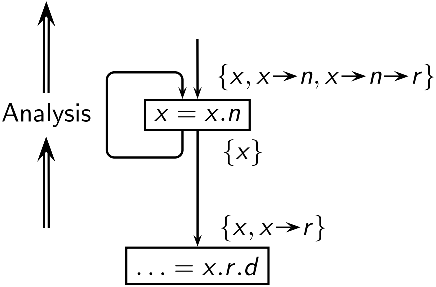
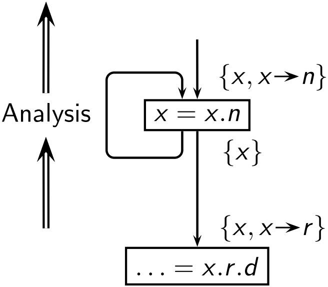
Anticipability of Heap References: An *All Paths* problem



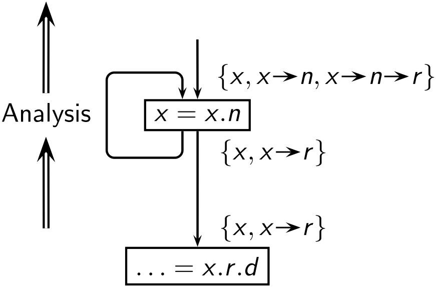
# Computing Explicit Liveness Using Sets of Access Paths

Anticipability of Heap References: An *All Paths* problem



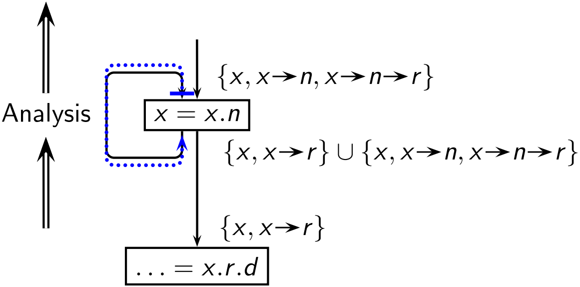
# Computing Explicit Liveness Using Sets of Access Paths

Liveness of Heap References: An *Any Path* problem



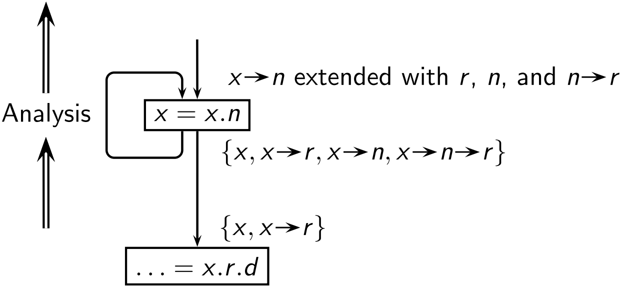
# Computing Explicit Liveness Using Sets of Access Paths

Liveness of Heap References: An *Any Path* problem



# Computing Explicit Liveness Using Sets of Access Paths

Liveness of Heap References: An *Any Path* problem



Analysis

$x \rightarrow n$ extended with $r$, $n$, and $n \rightarrow r$

$x = x.n$

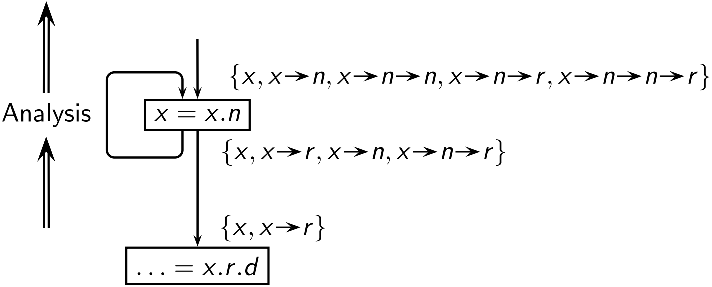
$\{x, x \rightarrow r, x \rightarrow n, x \rightarrow n \rightarrow r\}$

$\{x, x \rightarrow r\}$

$\ldots = x.r.d$

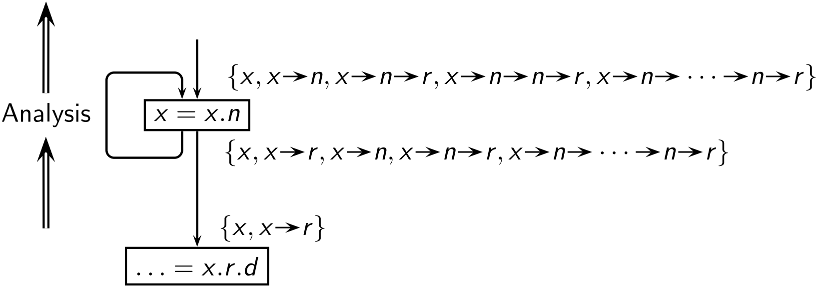
# Computing Explicit Liveness Using Sets of Access Paths

Liveness of Heap References: An *Any Path* problem



# Computing Explicit Liveness Using Sets of Access Paths

Liveness of Heap References: An *Any Path* problem



Analysis

$\{x, x\rightarrow n, x\rightarrow n\rightarrow r, x\rightarrow n\rightarrow n\rightarrow r, x\rightarrow n\rightarrow \cdots \rightarrow n\rightarrow r\}$

$x = x.n$

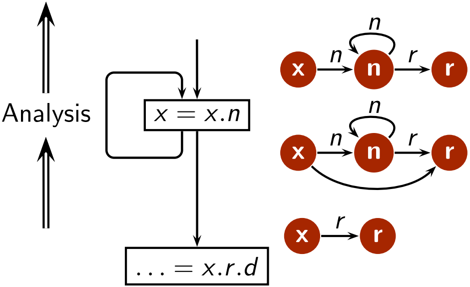
$\{x, x\rightarrow r, x\rightarrow n, x\rightarrow n\rightarrow r, x\rightarrow n\rightarrow \cdots \rightarrow n\rightarrow r\}$

$\{x, x\rightarrow r\}$

$\ldots = x.r.d$

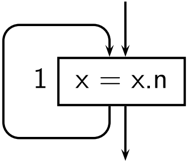
*Infinite Number of Unbounded Access Paths*

# Key Idea #5: Using Graphs as Data Flow Values



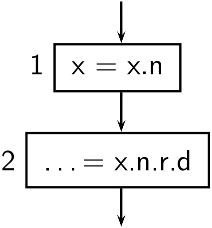
*Finite Number of Bounded Structures*

# Key Idea #6 : Include Program Point in Graphs



$\{x, x\rightarrow n, x\rightarrow n\rightarrow n, x\rightarrow n\rightarrow n\rightarrow n, \ldots\}$

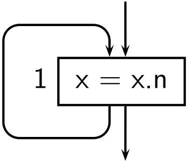
*Different occurrences of n's in an access path are* **Indistinguishable**



$\{x, x\rightarrow n, x\rightarrow n\rightarrow n, x\rightarrow n\rightarrow n\rightarrow r\}$

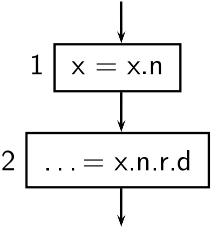
*Different occurrences of n's in an access path are* **Distinct**

# Key Idea #6 : Include Program Point in Graphs



$\{x, x\rightarrow n, x\rightarrow n\rightarrow n, x\rightarrow n\rightarrow n\rightarrow n, \ldots\}$

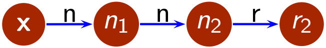
*Different occurrences of n's in an access path are* **Indistinguishable**



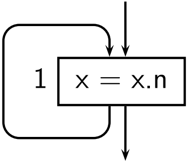
$\{x, x\rightarrow n, x\rightarrow n\rightarrow n, x\rightarrow n\rightarrow n\rightarrow r\}$

*Different occurrences of n's in an access path are* **Distinct**

Access Graph :



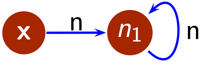
# Key Idea #6 : Include Program Point in Graphs

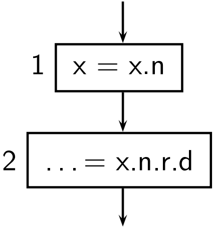


$\{x, x \rightarrow n, x \rightarrow n \rightarrow n, x \rightarrow n \rightarrow n \rightarrow n, \ldots\}$

*Different occurrences of n's in an access path are* **Indistinguishable**

Access Graph :

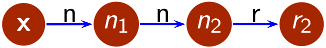




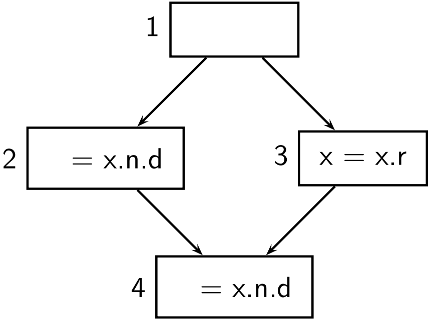
$\{x, x \rightarrow n, x \rightarrow n \rightarrow n, x \rightarrow n \rightarrow n \rightarrow r\}$

*Different occurrences of n's in an access path are* **Distinct**

Access Graph :



# Inclusion of Program Point Facilitates Summarization



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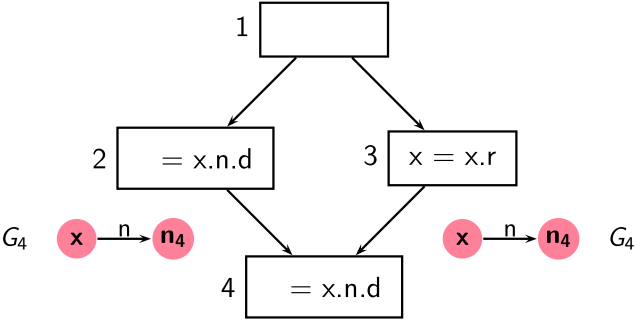

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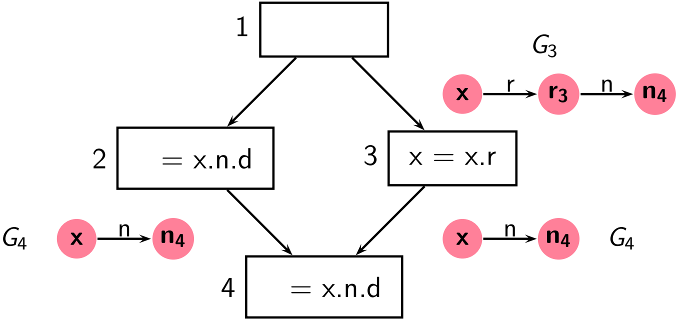

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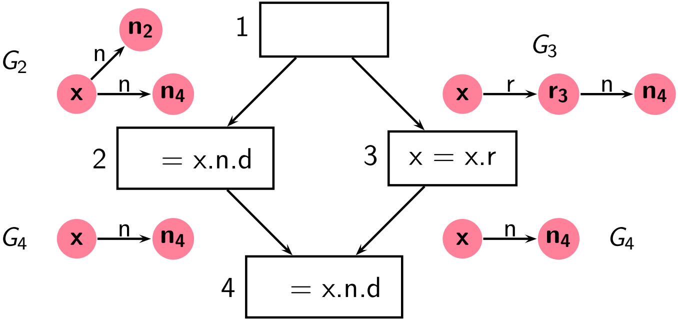

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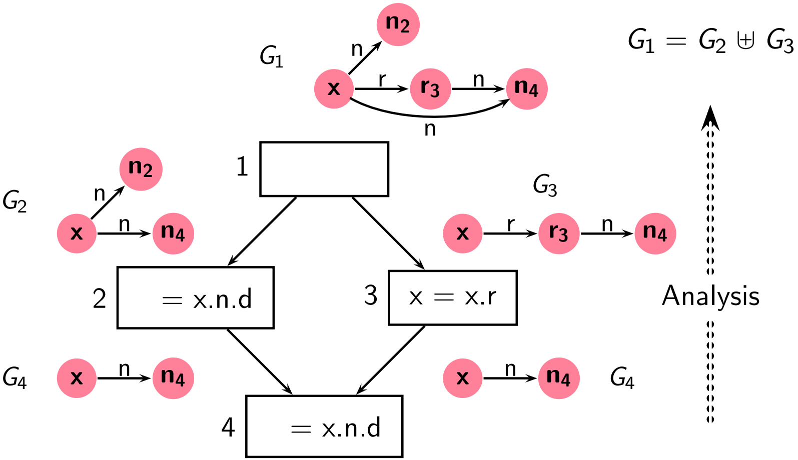
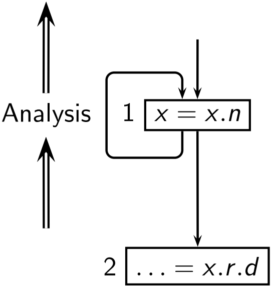
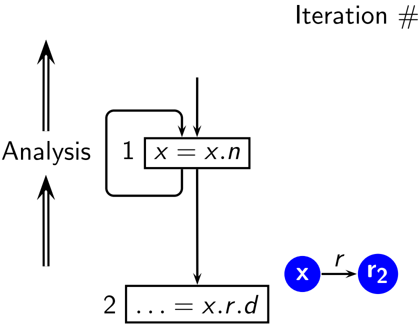


# Inclusion of Program Point Facilitates Summarization

Iteration #1

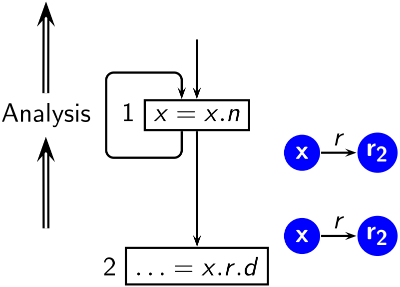


# Inclusion of Program Point Facilitates Summarization



# Inclusion of Program Point Facilitates Summarization

Iteration #1



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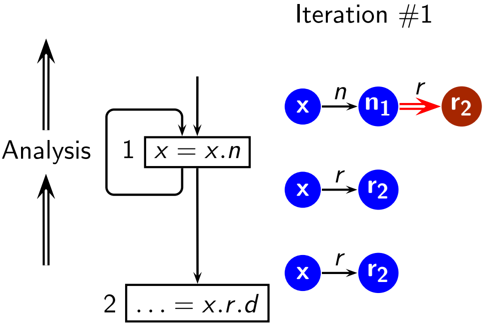

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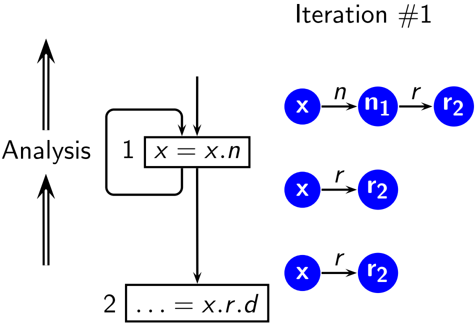
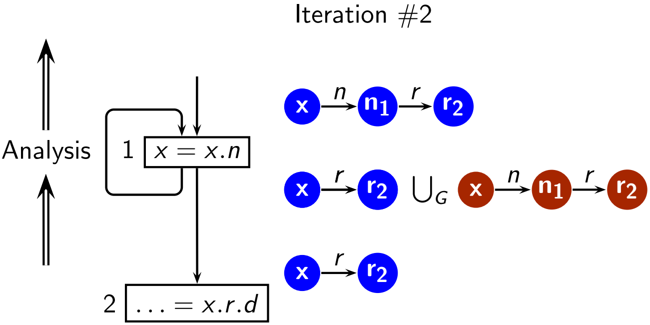
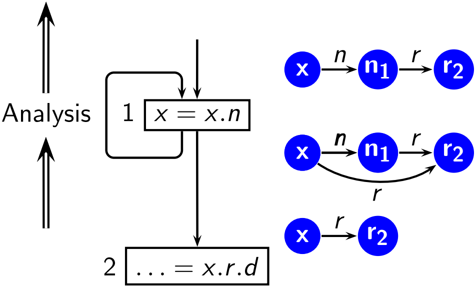


# Inclusion of Program Point Facilitates Summarization



# Inclusion of Program Point Facilitates Summarization

Iteration #2



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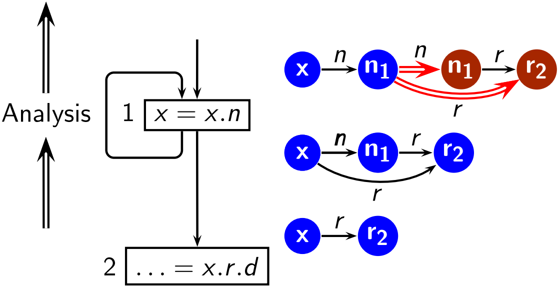

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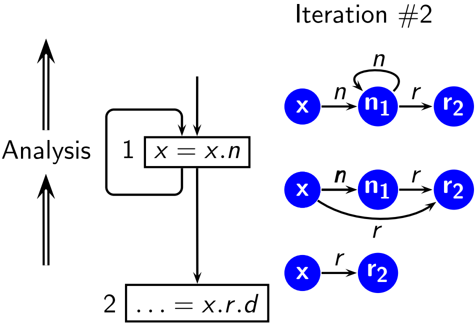

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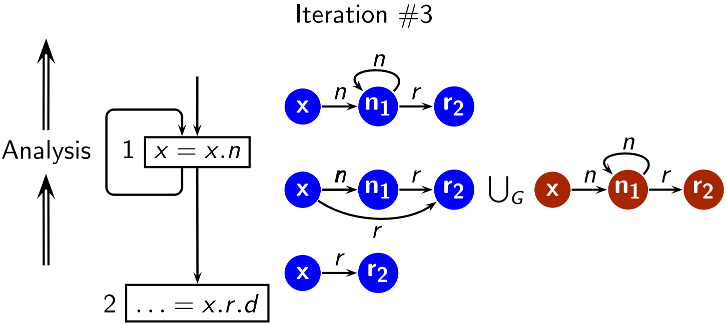

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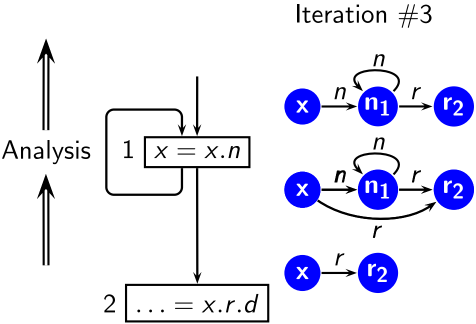

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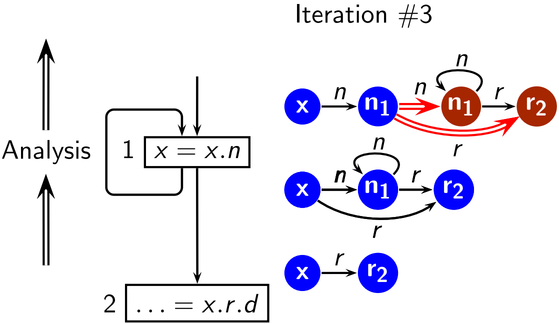

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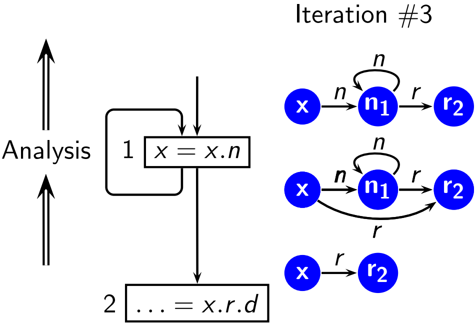
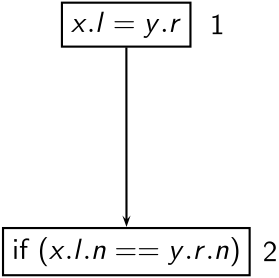


# Access Graph and Memory Graph

Program Fragment

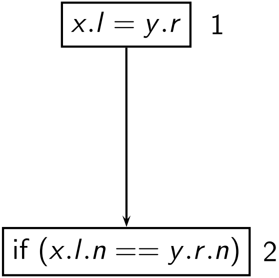


$x.l = y.r$ 1

if $(x.l.n == y.r.n)$ 2

# Access Graph and Memory Graph

Program Fragment



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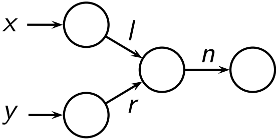
Memory Graph



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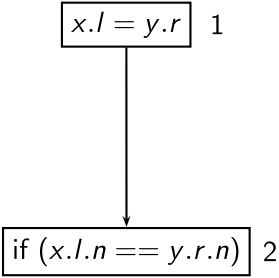

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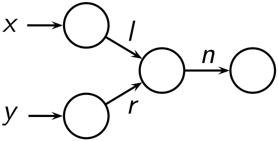
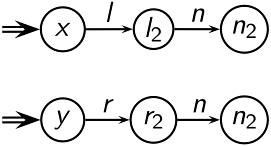


Access Graphs



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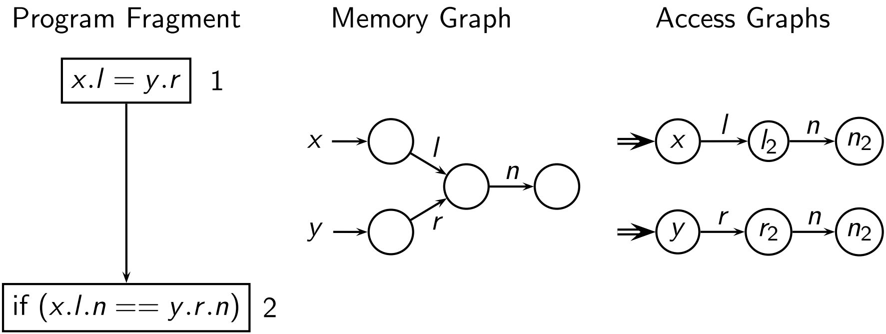
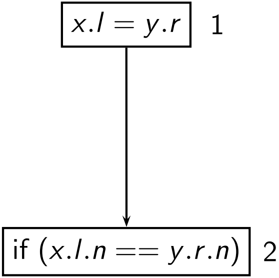


- Memory Graph: Nodes represent locations and edges represent links (i.e. pointers).

# Access Graph and Memory Graph

Program Fragment



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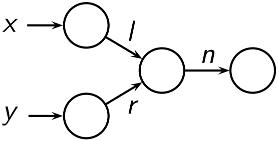
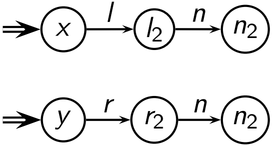


Access Graphs



- Memory Graph: Nodes represent locations and edges represent links (i.e. pointers).
- Access Graphs: Nodes represent dereference of links at particular statements. Memory locations are implicit.

# Lattice of Access Graphs

- Finite number of nodes in an access graph for a variable
- $\uplus$ induces a partial order on access graphs

  $\Rightarrow$ a finite (and hence complete) lattice

  $\Rightarrow$ All standard results of classical data flow analysis can be extended to this analysis.

  *Termination and boundedness, convergence on MFP, complexity etc.*

## Access Graph Operations

- *Union*. $G \uplus G'$
- *Path Removal*.
  $G \ominus \rho$ removes those access paths in $G$ which have $\rho$ as a prefix.
- *Factorization* $(/)$.
- *Extension*.

# Semantics of Access Graph Operations

- $P\,(G, M)$ is the set of paths in graph $G$ terminating on nodes in $M$. For graph $G_i$, $M_i$ is the set of all nodes in $G_i$.
- $S$ is the set of remainder graphs and $P\,(S, M_s)$ is the set of all paths in all remainder graphs in $S$.

| Operation | | Access Paths |
|---|---|---|
| Union | $G_3 = G_1 \uplus G_2$ | $P\,(G_3, M_3) \supseteq P\,(G_1, M_1) \cup P\,(G_2, M_2)$ |
| Path Removal | $G_2 = G_1 \ominus \rho$ | $P\,(G_2, M_2) \supseteq P\,(G_1, M_1) - \{\rho \rightarrow \sigma \mid \rho \rightarrow \sigma \in P\,(G_1, M_1)\}$ |
| Factorization | $S = G_1/(G_2, M)$ | $P\,(S, M_s) = \{\sigma \mid \rho' \rightarrow \sigma \in P\,(G_1, M_1),\ \rho' \in P\,(G_2, M)\}$ |
| Extension | $G_2 = (G_1, M) \# \emptyset$ | $P\,(G_2, M_2) = \emptyset$ |
| | $G_2 = (G_1, M) \# S$ | $P\,(G_2, M_2) \supseteq P\,(G_1, M_1) \cup \{\rho \rightarrow \sigma \mid \rho \in P\,(G_1, M),\ \sigma \in P\,(S, M_s)\}$ |

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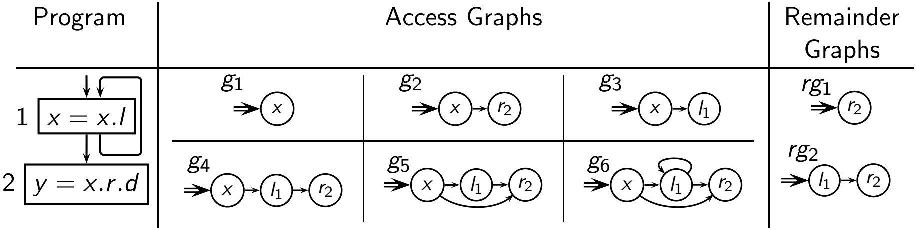
$\sigma$ represents remainder

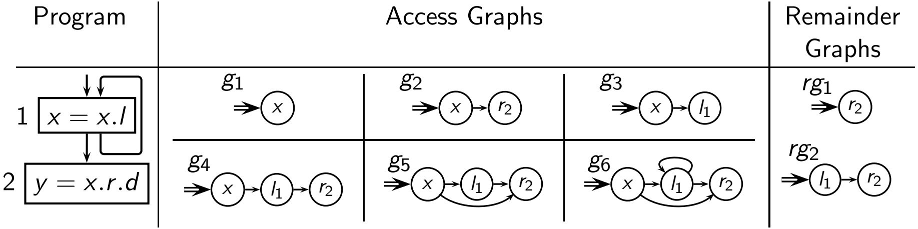
$\rho'$ represents quotient

# Access Graph Operations: Examples



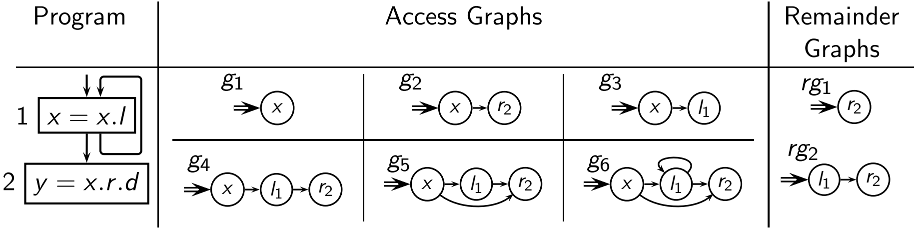
| Union | Path Removal | Factorisation | Extension |
|---|---|---|---|
| | | | |

# Access Graph Operations: Examples



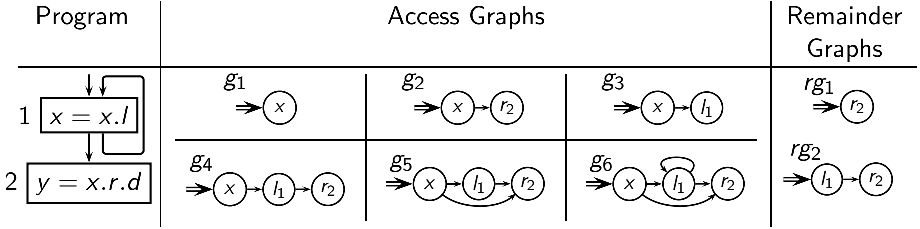
| Union | Path Removal | Factorisation | Extension |
|---|---|---|---|
| $g_3 \uplus g_4 = g_4$<br>$g_2 \uplus g_4 = g_5$<br>$g_5 \uplus g_4 = g_5$<br>$g_5 \uplus g_6 = g_6$ | | | |

# Access Graph Operations: Examples



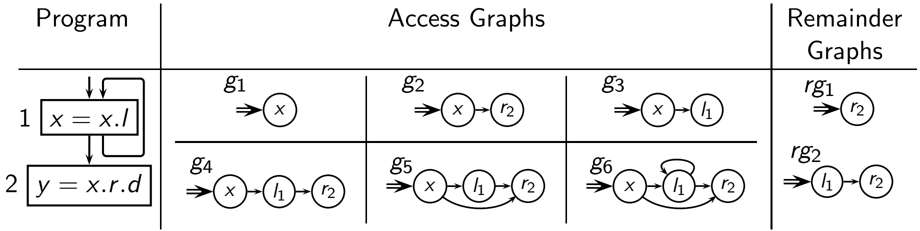
| Union | Path Removal | Factorisation | Extension |
|---|---|---|---|
| $g_3 \uplus g_4 = g_4$ | $g_6 \ominus x \rightarrow l = g_2$ | | |
| $g_2 \uplus g_4 = g_5$ | $g_5 \ominus x = \mathcal{E}_G$ | | |
| $g_5 \uplus g_4 = g_5$ | $g_4 \ominus x \rightarrow r = g_4$ | | |
| $g_5 \uplus g_6 = g_6$ | $g_4 \ominus x \rightarrow l = g_1$ | | |

# Access Graph Operations: Examples



| Union | Path Removal | Factorisation | Extension |
|---|---|---|---|
| $g_3 \uplus g_4 = g_4$ | $g_6 \ominus x \rightarrow l = g_2$ | $g_2 / (g_1, \{x\}) = \{rg_1\}$ | |
| $g_2 \uplus g_4 = g_5$ | $g_5 \ominus x = \mathcal{E}_G$ | $g_5 / (g_1, \{x\}) = \{rg_1, rg_2\}$ | |
| $g_5 \uplus g_4 = g_5$ | $g_4 \ominus x \rightarrow r = g_4$ | $g_5 / (g_2, \{r_2\}) = \{\epsilon_{RG}\}$ | |
| $g_5 \uplus g_6 = g_6$ | $g_4 \ominus x \rightarrow l = g_1$ | $g_4 / (g_2, \{r_2\}) = \emptyset$ | |

# Access Graph Operations: Examples



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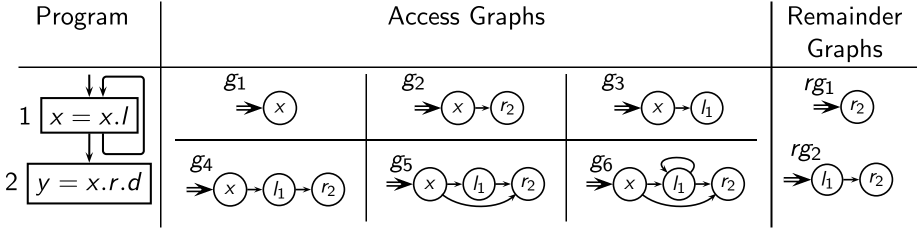
| $g_2 \uplus g_4 = g_5$ | $g_5 \ominus x = \mathcal{E}_G$ | $g_5 / (g_1, \{x\}) = \{rg_1, rg_2\}$ | $(g_3, \{x, l_1\}) \# \{rg_1, rg_2\} = g_6$ |
| $g_5 \uplus g_4 = g_5$ | $g_4 \ominus x \rightarrow r = g_4$ | $g_5 / (g_2, \{r_2\}) = \{\epsilon_{RG}\}$ | $(g_2, \{r_2\}) \# \{\epsilon_{RG}\} = g_2$ |
| $g_5 \uplus g_6 = g_6$ | $g_4 \ominus x \rightarrow l = g_1$ | $g_4 / (g_2, \{r_2\}) = \emptyset$ | $(g_2, \{r_2\}) \# \emptyset = \mathcal{E}_G$ |

# Access Graph Operations: Examples



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Remainder is empty

Quotient is empty

# Data Flow Equations for Heap Liveness Analysis

Computing Liveness Access Graph for variable $v$ by incorporating the effect of statement $n$.

$$\begin{aligned}
ELIn_n(v) &= (ELOut_n(v) \ominus ELKillPath_n(v)) \uplus ELGen_n(v) \\
ELOut_n(v) &= \begin{cases} makeGraph(v \rightarrow *) & n = End,\ v \in Globals \\ \mathcal{E}_G & n = End,\ v \notin Globals \\ \biguplus\limits_{s \in succ(n)} ELIn_s(v) & \text{otherwise} \end{cases} \\
ELGen_n(v) &= ELConstGen_n(v) \uplus ELDepGen_n(v)
\end{aligned}$$

(Note: This notation is slightly different from the notation in the book.)

# Flow Functions for Explicit Liveness Analysis

Access Paths $\Longrightarrow$

| | Use $\alpha_x.d$ | $\alpha_x = \alpha_y$ |
|---|---|---|
| $ConstKill_n$ | $\emptyset$ | $\{\rho_x \rightarrow *\}$ |
| $ConstGen_n$ | $prefixes(\rho_x)$ | $prefixes(base(\rho_x)) \cup$<br>$prefixes(base(\rho_y))$ |
| $DepGen_n(X)$ | $\emptyset$ | $\{\rho_y \rightarrow \sigma \mid \rho_x \rightarrow \sigma \in X\}$ |

Access Graphs $\Downarrow$

$$G_x = makeGraph(\rho_x) \quad G_x^B = makeGraph(base(\rho_x))$$
$$G_y = makeGraph(\rho_y) \quad G_y^B = makeGraph(base(\rho_y))$$

| | Use $\alpha_x.d$ | Use $\alpha_x$ | $\alpha_x = \alpha_y$ | $\alpha_x = Null$,<br>$\alpha_x = New$ |
|---|---|---|---|---|
| $ELKillPath_n(x)$ | $\mathcal{E}$ | $\mathcal{E}$ | $\rho_x$ | $\rho_x$ |
| $ELKillPath_n(y)$ | $\mathcal{E}$ | $\mathcal{E}$ | $\mathcal{E}$ | $\mathcal{E}$ |
| $ELConstGen_n(x)$ | $G_x$ | $G_x^B$ | $G_x^B$ | $G_x^B$ |
| $ELConstGen_n(y)$ | $\mathcal{E}_G$ | $\mathcal{E}_G$ | $G_y^B$ | $\mathcal{E}_G$ |
| $ELDepGen_n(x)(X)$ | $\mathcal{E}_G$ | $\mathcal{E}_G$ | $\mathcal{E}_G$ | $\mathcal{E}_G$ |
| $ELDepGen_n(y)(X)$ | $\mathcal{E}_G$ | $\mathcal{E}_G$ | $(G_y, M_y) \# (X / (G_x, M_x))$ | $\mathcal{E}_G$ |

# Flow Functions for Explicit Liveness Analysis

Access Paths ⟹

| | | |
|---|---|---|
| $ConstKill_n$ | | |
| $ConstGen_n$ | $prefixes(\rho_x)$ | $prefixes(base(\rho_x)) \cup$<br>$prefixes(base(\rho_y))$ |
| $DepGen_n(X)$ | $\emptyset$ | $\{\rho_y \rightarrow \sigma \mid \rho_x \rightarrow \sigma \in X\}$ |

The singleton set containing the last node corresponding to $\rho_x$

Access Graphs ⇓

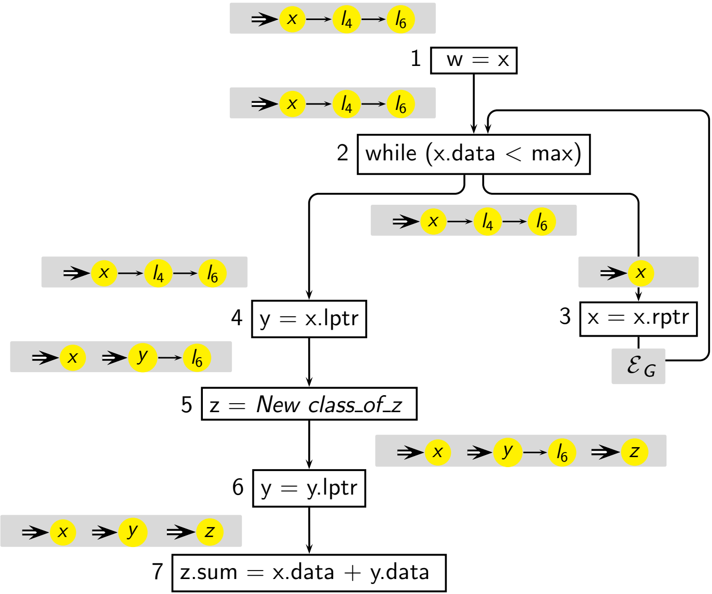
$G_x = makeGraph(\rho_x)$ $G_x^B = makeGraph(base(\rho_x))$

$G_y^B = makeGraph(base(\rho_y))$

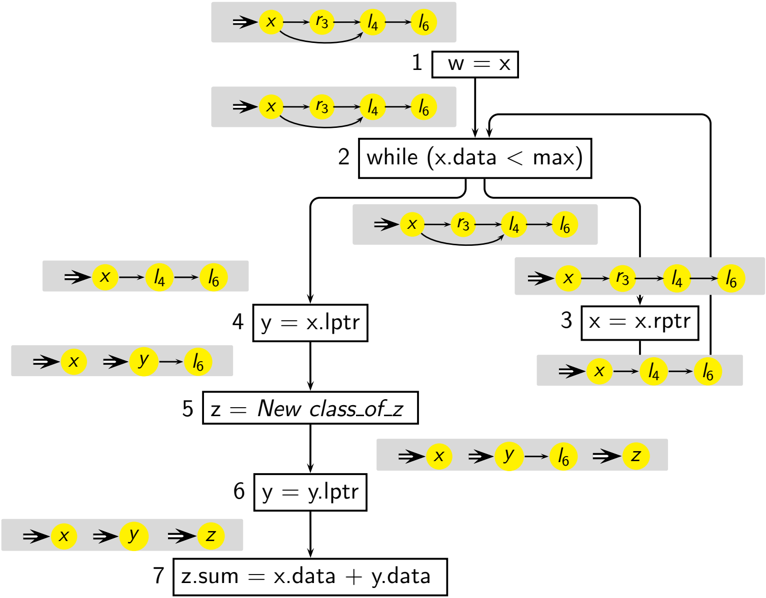
The singleton set containing the last node corresponding to $\rho_y$

| | | | $\alpha_x = \alpha_y$ | $\alpha_x = Null$,<br>$\alpha_x = New$ |
|---|---|---|---|---|
| $ELKillPath_n(x)$ | $\mathcal{E}$ | $\mathcal{E}$ | $\rho_x$ | $\rho_x$ |
| $ELKillPath_n(y)$ | $\mathcal{E}$ | $\mathcal{E}$ | $\mathcal{E}$ | $\mathcal{E}$ |
| $ELConstGen_n(x)$ | $G_x$ | $G_x^B$ | $G_x^B$ | $G_x^B$ |
| $ELConstGen_n(y)$ | $\mathcal{E}_G$ | $\mathcal{E}_G$ | $G_y^B$ | $\mathcal{E}_G$ |
| $ELDepGen_n(x)(X)$ | $\mathcal{E}_G$ | $\mathcal{E}_G$ | $\mathcal{E}_G$ | $\mathcal{E}_G$ |
| $ELDepGen_n(y)(X)$ | $\mathcal{E}_G$ | $\mathcal{E}_G$ | $(G_y, M_y)\#(X/(G_x, M_x))$ | $\mathcal{E}_G$ |

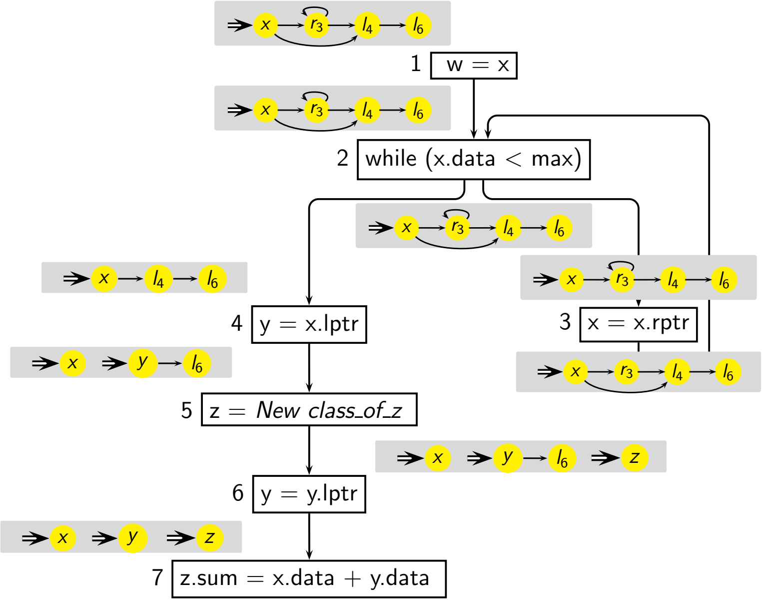
# Liveness Analysis of Example Program: Ist Iteration



# Liveness Analysis of Example Program: 2nd Iteration



# Liveness Analysis of Example Program: 3rd Iteration



1 w = x

2 while (x.data < max)

3 x = x.rptr

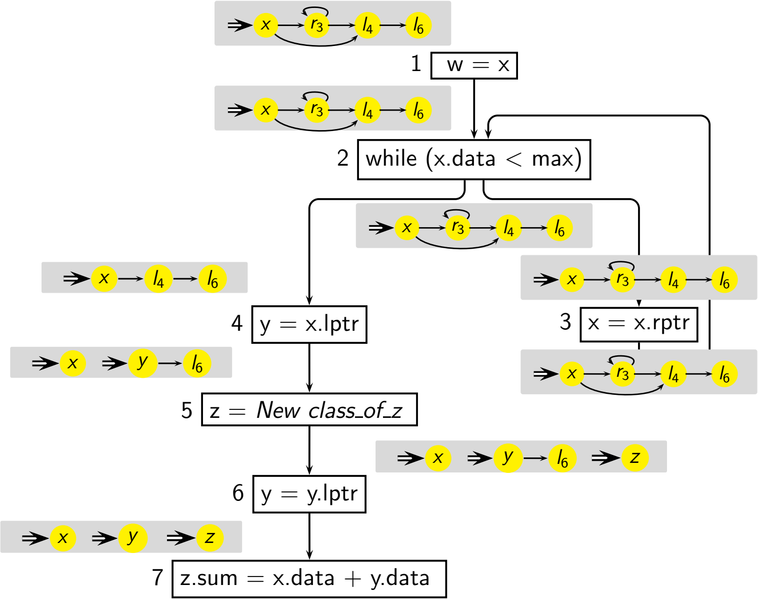
4 y = x.lptr

5 z = *New class_of_z*

6 y = y.lptr

7 z.sum = x.data + y.data

# Liveness Analysis of Example Program: 4th Iteration



# Which Access Paths Can be Nullified?

- Consider extensions of accessible paths for nullification.

```
Let $\rho$ be accessible at $p$ (i.e. available or anticipable)
for each reference field $f$ of the object pointed to by $\rho$
    if $\rho \rightarrowtail f$ is not live at $p$ then
        Insert $\rho \rightarrowtail f = \text{null}$ at $p$ subject to profitability
```

- For simple access paths, $\rho$ is empty and $f$ is the root variable name.

# Which Access Paths Can be Nullified?

Can be safely dereferenced

- Consider extensions of accessible paths for nullification.

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Can be safely dereferenced

Consider link aliases at $p$

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```

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Cannot be hoisted and is not redefined at $p$

# Availability and Anticipability Analyses

- $\rho$ is available at program point $p$ if the target of each prefix of $\rho$ is guaranteed to be created along every control flow path reaching $p$.
- $\rho$ is anticipable at program point $p$ if the target of each prefix of $\rho$ is guaranteed to be dereferenced along every control flow path starting at $p$.

# Availability and Anticipability Analyses

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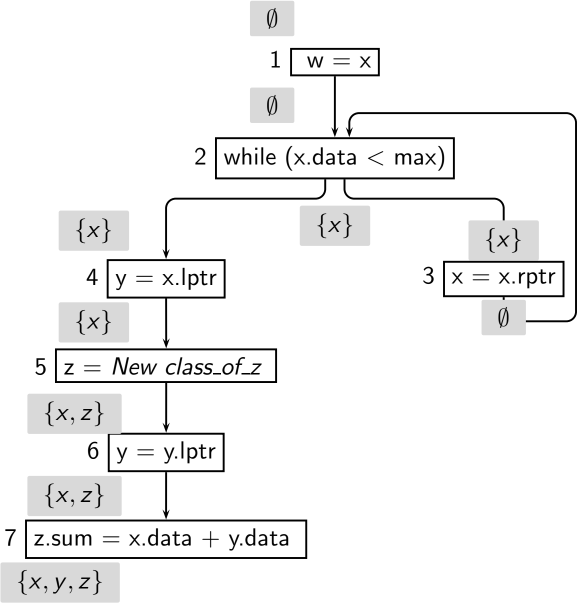
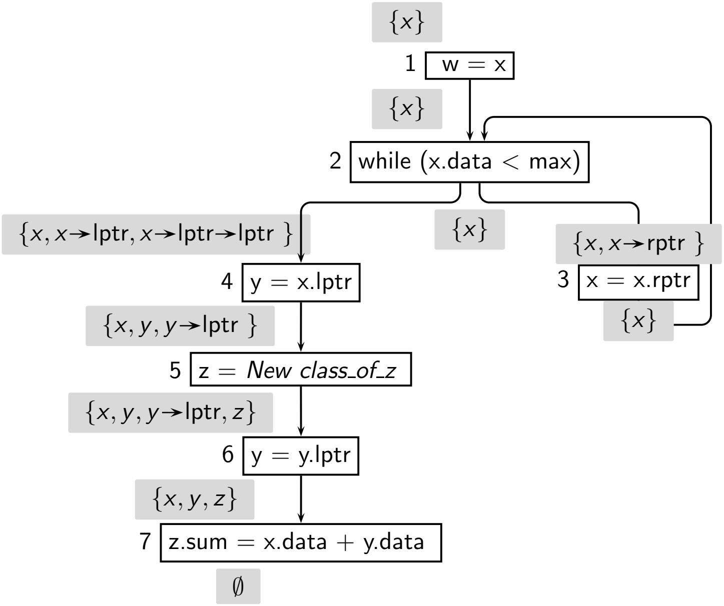
- $\rho$ is anticipable at program point *p* if the target of each prefix of $\rho$ is guaranteed to be dereferenced along every control flow path starting at *p*.
- Finiteness.
  - An anticipable (available) access path must be anticipable (available) along every paths. Thus unbounded paths arising out of loops cannot be anticipable (available).
  - Due to "every control flow path nature", computation of anticipable and available access paths uses $\cap$ as the confluence. Thus the sets are bounded.

$\Rightarrow$ No need ot access graphs.

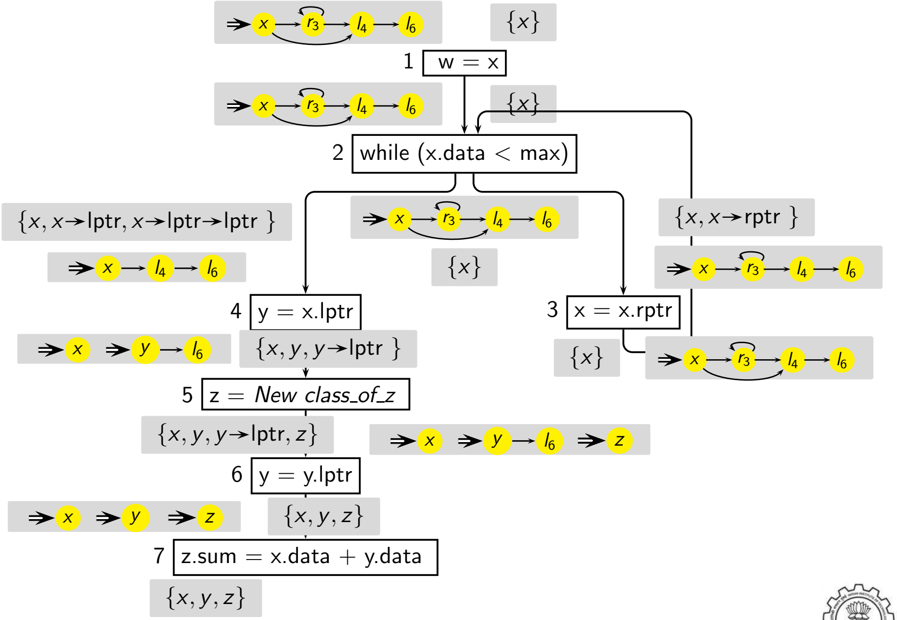
# Availability Analysis of Example Program



# Anticipability Analysis of Example Program



# Live and Accessible Paths



$\{x\}$

1 w = x

$\{x\}$

2 while (x.data < max)

$\{x, x\rightarrow \text{lptr}, x\rightarrow \text{lptr}\rightarrow \text{lptr}\}$

$\{x\}$

$\{x, x\rightarrow \text{rptr}\}$

3 x = x.rptr

$\{x\}$

4 y = x.lptr

$\{x, y, y\rightarrow \text{lptr}\}$

5 z = *New class_of_z*

$\{x, y, y\rightarrow \text{lptr}, z\}$

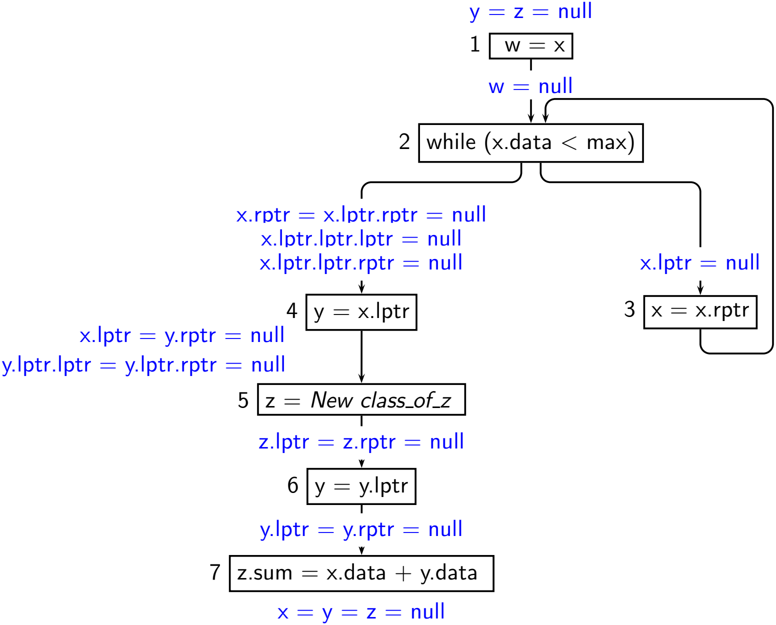
6 y = y.lptr

$\{x, y, z\}$

7 z.sum = x.data + y.data

$\{x, y, z\}$

# Creating null Assignments from Live and Accessible Paths



y = z = null

1 w = x

w = null

2 while (x.data < max)

x.rptr = x.lptr.rptr = null
x.lptr.lptr.lptr = null
x.lptr.lptr.rptr = null

4 y = x.lptr

x.lptr = null

3 x = x.rptr

x.lptr = y.rptr = null
y.lptr.lptr = y.lptr.rptr = null

5 z = *New class_of_z*

z.lptr = z.rptr = null

6 y = y.lptr

y.lptr = y.rptr = null

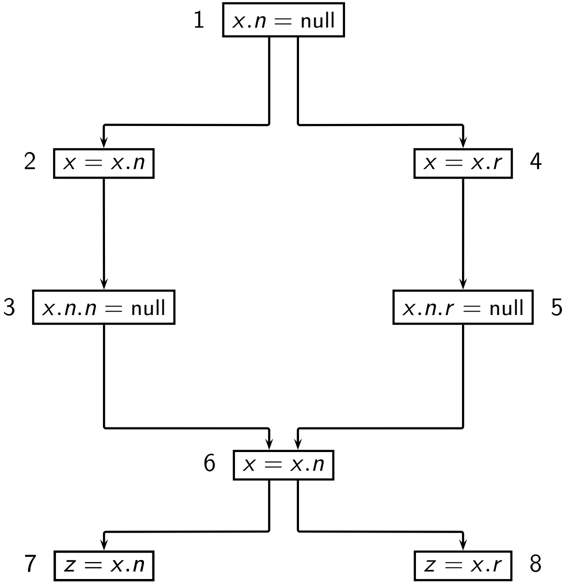
7 z.sum = x.data + y.data

x = y = z = null

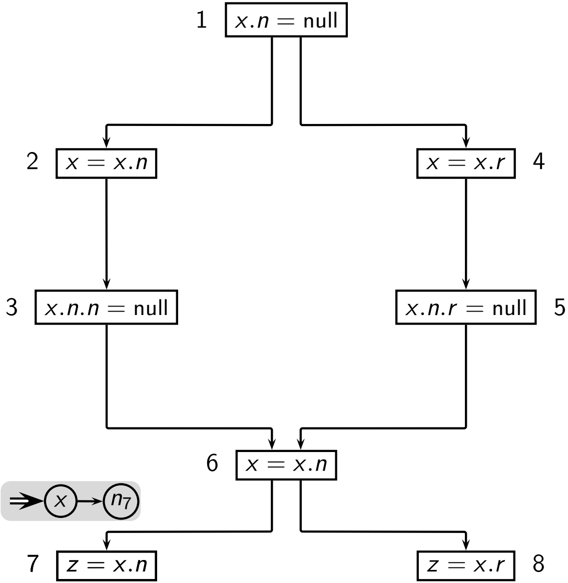
# The Resulting Program

```
                                    y = z = null
1  w = x
                                    w = null
2  while (x.data < max)
   {                                x.lptr = null
3       x = x.rptr     }
                                    x.rptr = x.lptr.rptr = null
                                    x.lptr.lptr.lptr = null
                                    x.lptr.lptr.rptr = null
4  y = x.lptr
                                    x.lptr = y.rptr = null
                                    y.lptr.lptr = y.lptr.rptr = null
5  z = New  class_of_z
                                    z.lptr = z.rptr = null
6  y = y.lptr
                                    y.lptr = y.rptr = null
7  z.sum = x.data + y.data
                                    x = y = z = null
```

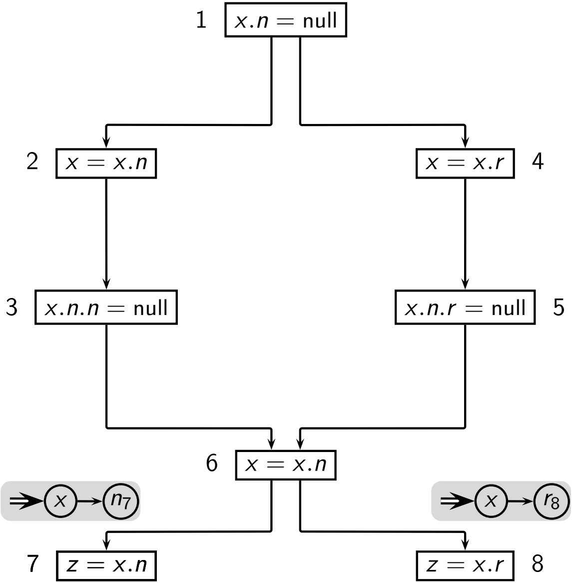
# Non-Distributivity of Explicit Liveness Analysis



# Non-Distributivity of Explicit Liveness Analysis



# Non-Distributivity of Explicit Liveness Analysis



1 $x.n = \text{null}$

2 $x = x.n$

4 $x = x.r$

3 $x.n.n = \text{null}$

5 $x.n.r = \text{null}$

6 $x = x.n$

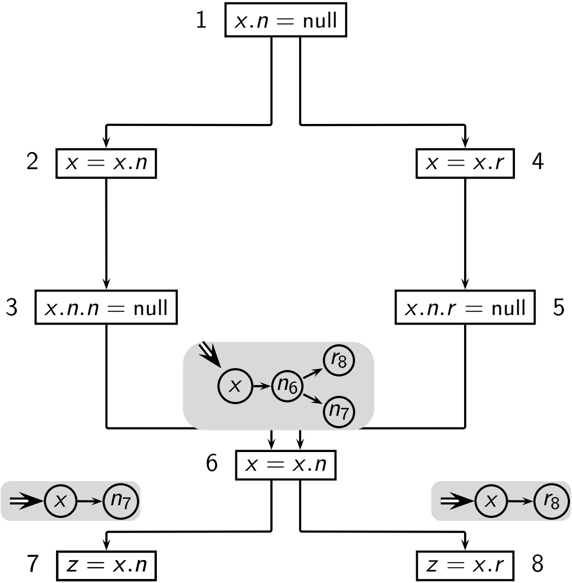
$\Rightarrow x \rightarrow n_7$

$\Rightarrow x \rightarrow r_8$

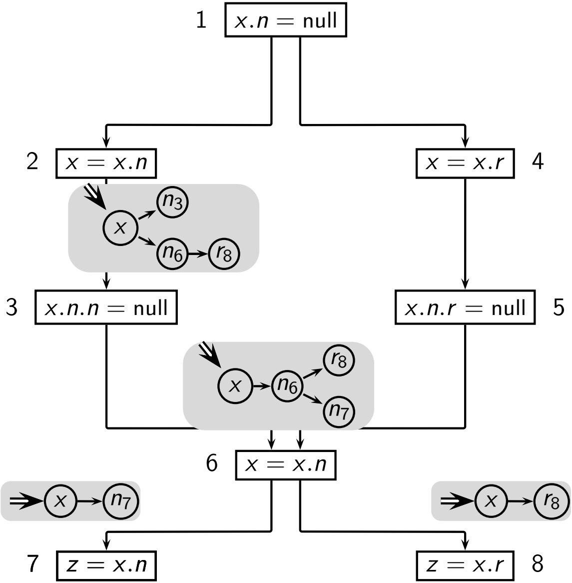
7 $z = x.n$

8 $z = x.r$

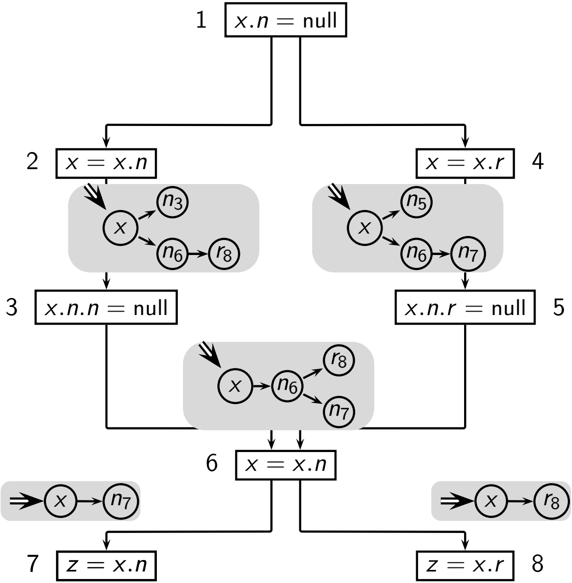
# Non-Distributivity of Explicit Liveness Analysis



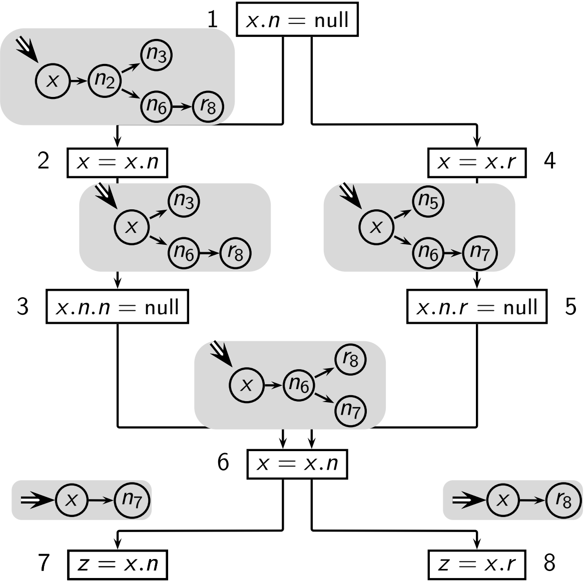
# Non-Distributivity of Explicit Liveness Analysis



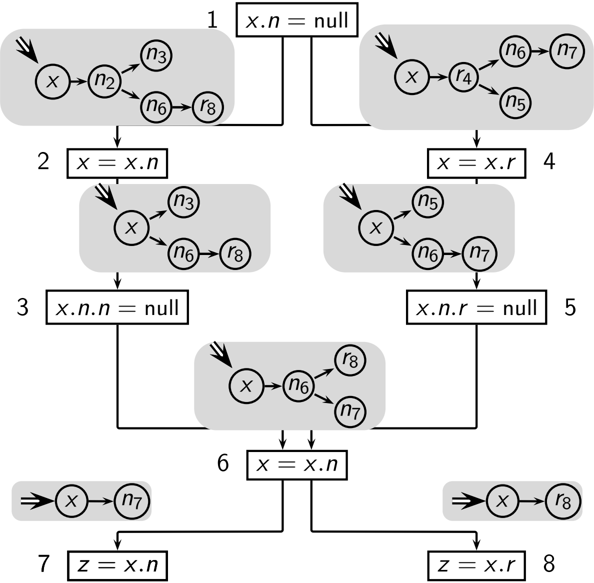
# Non-Distributivity of Explicit Liveness Analysis



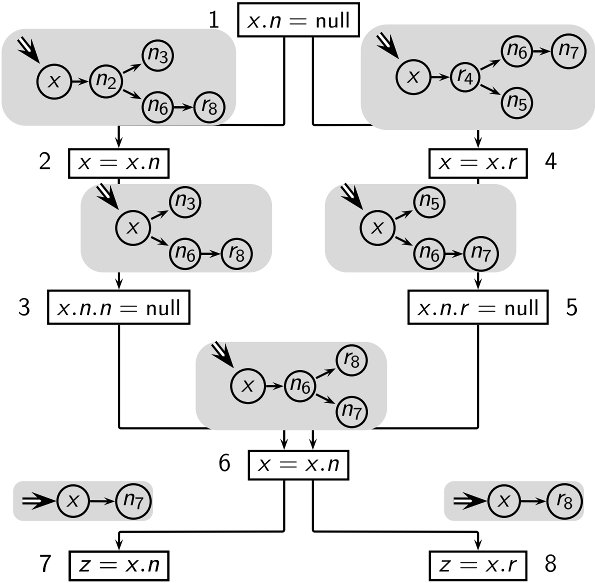
# Non-Distributivity of Explicit Liveness Analysis



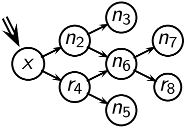
# Non-Distributivity of Explicit Liveness Analysis



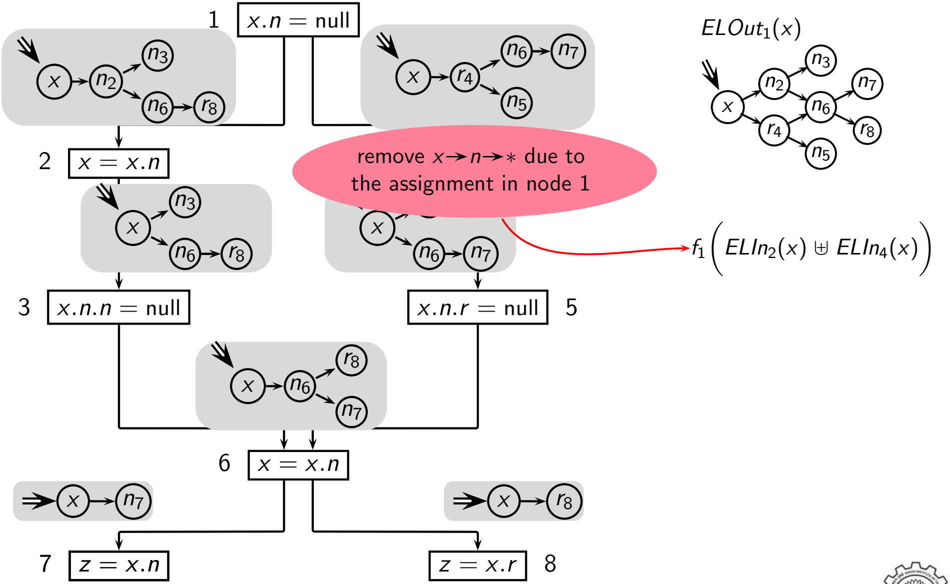
# Non-Distributivity of Explicit Liveness Analysis



$ELOut_1(x)$



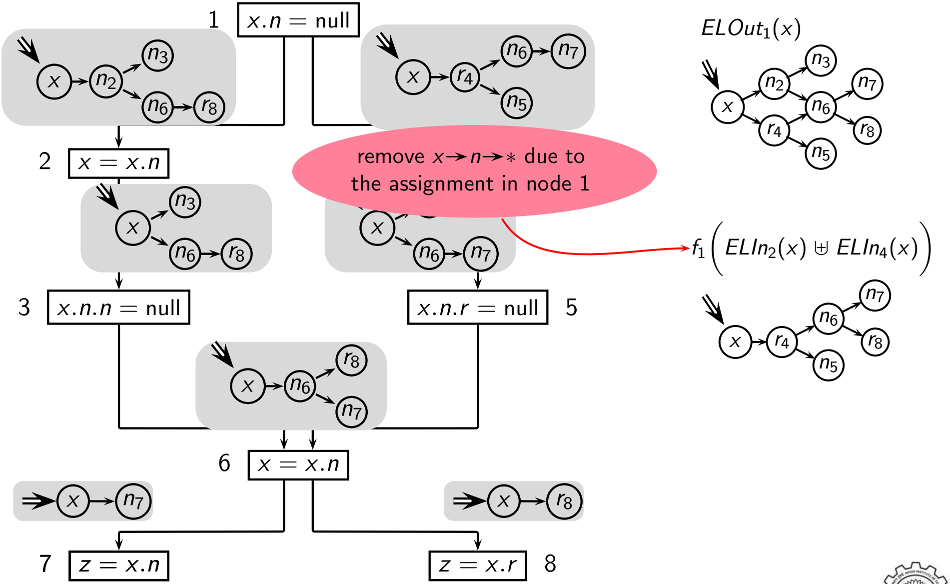
# Non-Distributivity of Explicit Liveness Analysis



remove $x \rightarrow n \rightarrow *$ due to the assignment in node 1

$$f_1\left(ELIn_2(x) \uplus ELIn_4(x)\right)$$

# Non-Distributivity of Explicit Liveness Analysis

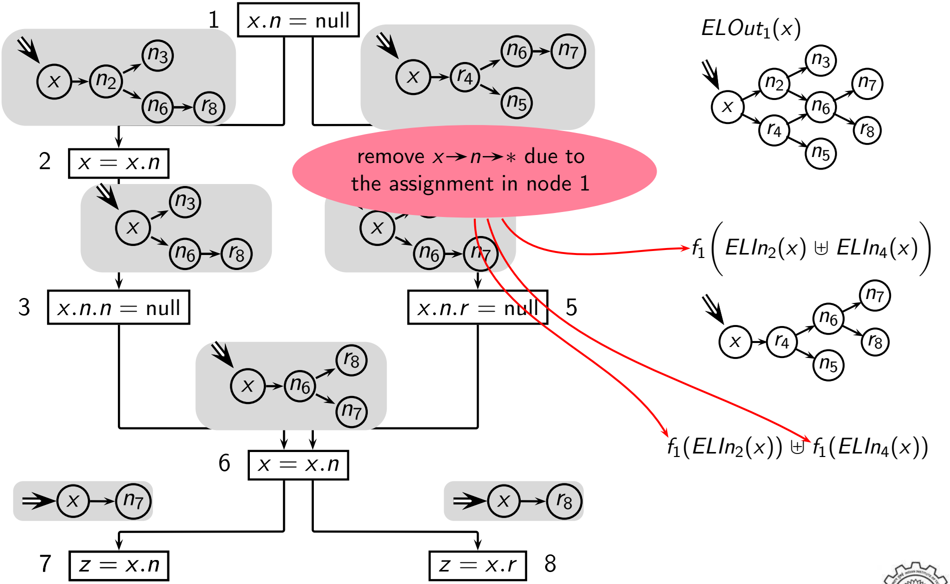


remove $x \rightarrow n \rightarrow *$ due to the assignment in node 1

$ELOut_1(x)$

$f_1\left(ELIn_2(x) \uplus ELIn_4(x)\right)$

# Non-Distributivity of Explicit Liveness Analysis



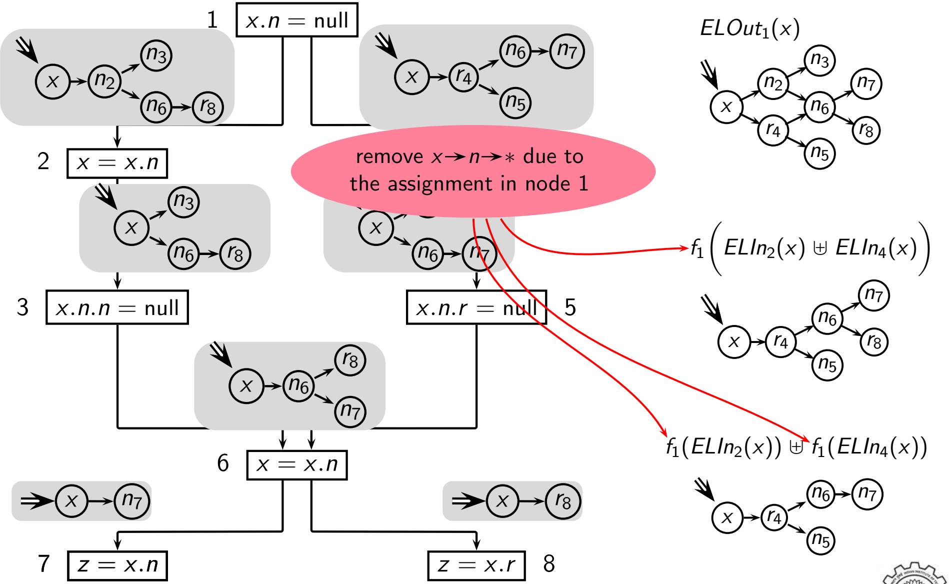
$ELOut_1(x)$

remove $x \rightarrow n \rightarrow *$ due to the assignment in node 1

$$f_1\left(ELIn_2(x) \uplus ELIn_4(x)\right)$$

$$f_1(ELIn_2(x)) \uplus f_1(ELIn_4(x))$$

# Non-Distributivity of Explicit Liveness Analysis



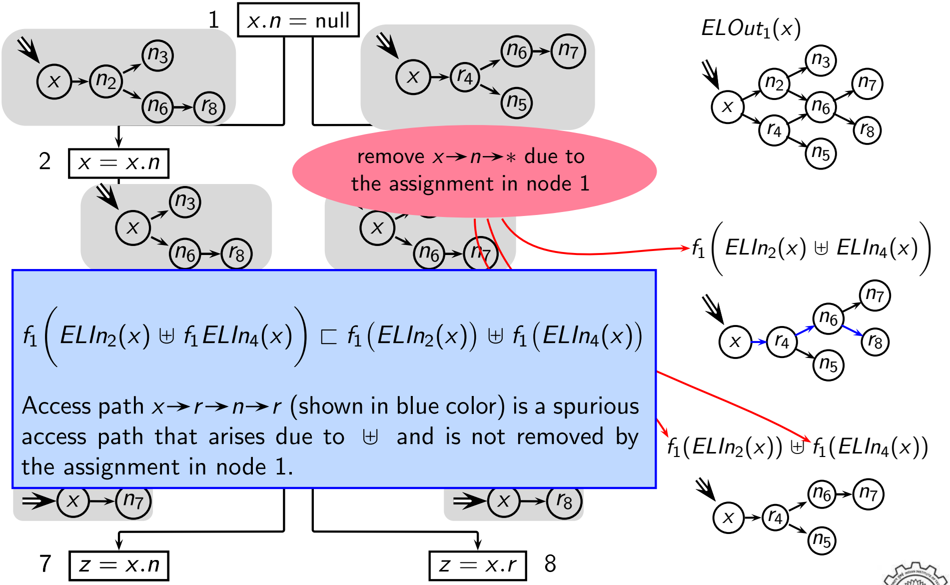
remove $x \rightarrow n \rightarrow *$ due to the assignment in node 1

$ELOut_1(x)$

$f_1\left(ELIn_2(x) \uplus ELIn_4(x)\right)$

$f_1(ELIn_2(x)) \uplus f_1(ELIn_4(x))$

# Non-Distributivity of Explicit Liveness Analysis



remove $x \rightarrow n \rightarrow *$ due to the assignment in node 1

$$f_1\left(ELIn_2(x) \uplus f_1\, ELIn_4(x)\right) \sqsubset f_1\left(ELIn_2(x)\right) \uplus f_1\left(ELIn_4(x)\right)$$

Access path $x \rightarrow r \rightarrow n \rightarrow r$ (shown in blue color) is a spurious access path that arises due to $\uplus$ and is not removed by the assignment in node 1.

# Issues Not Covered in These Slides

- Precision of information
  - Cyclic Data Structures
  - Eliminating Redundant null Assignments
- Properties of Data Flow Analysis:
  Monotonicity, Boundedness, Complexity
- Interprocedural Analysis
- Extensions for C/C++