# *Bit Vector Data Flow Frameworks*

Uday Khedker
(www.cse.iitb.ac.in/~uday)

Department of Computer Science and Engineering,
Indian Institute of Technology, Bombay

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*Part 1*

# *About These Slides*



## Copyright

These slides constitute the lecture notes for CS618 Program Analysis course at IIT Bombay and have been made available as teaching material accompanying the book:

- Uday Khedker, Amitabha Sanyal, and Bageshri Karkare. *Data Flow Analysis: Theory and Practice*. CRC Press (Taylor and Francis Group). 2009.

  (Indian edition published by Ane Books in 2013)

Apart from the above book, some slides are based on the material from the following books

- M. S. Hecht. *Flow Analysis of Computer Programs*. Elsevier North-Holland Inc. 1977.
- F. Nielson, H. R. Nielson, and C. Hankin. *Principles of Program Analysis*. Springer-Verlag. 1998.







## Outline

- Live Variables Analysis
- Observations about Data Flow Analysis
- Available Expressions Analysis
- Anticipable Expressions Analysis
- Reaching Definitions Analysis
- Common Features of Bit Vector Frameworks
- Partial Redundancy Elimination

*Part 2*

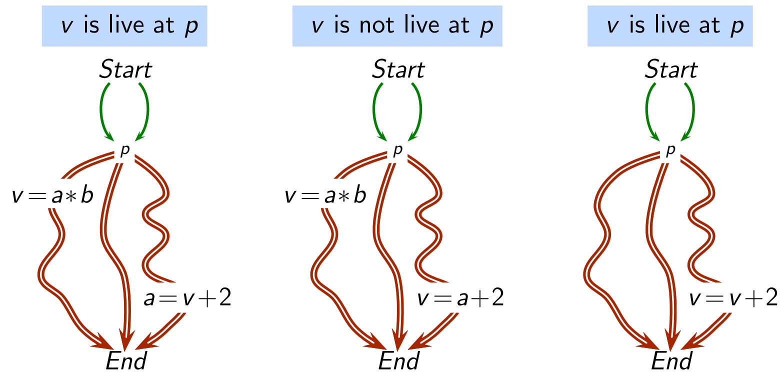
# *Live Variables Analysis*



## Defining Live Variables Analysis

A variable $v$ is live at a program point $p$, if some path from $p$ to program exit contains an r-value occurrence of $v$ which is not preceded by an l-value occurrence of $v$.

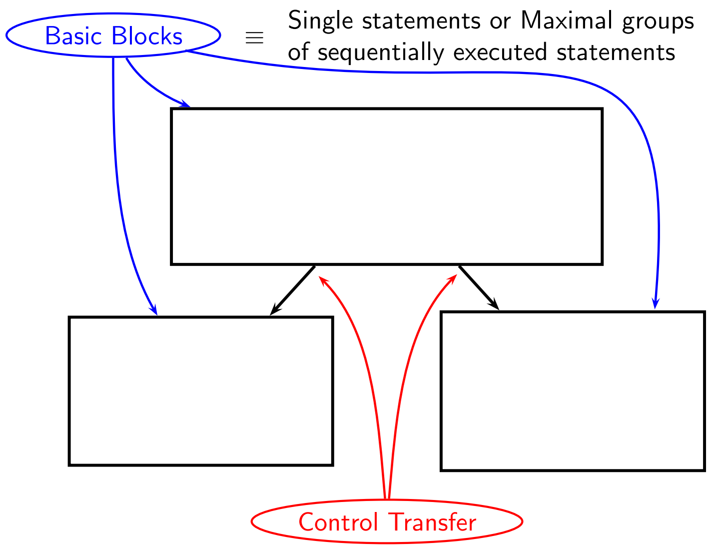
Path based specification







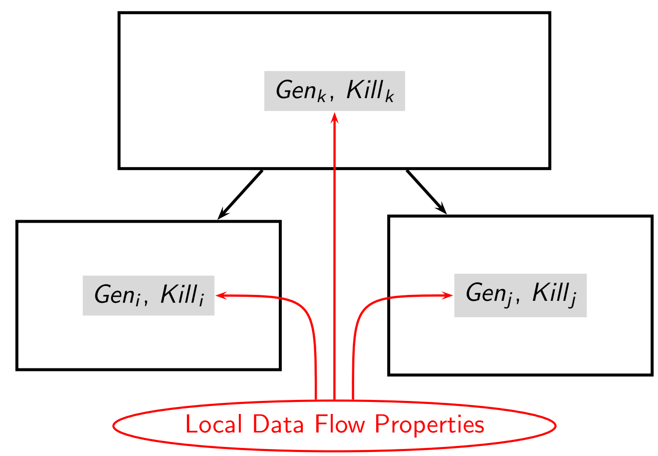

## Defining Data Flow Analysis for Live Variables Analysis








## Defining Data Flow Analysis for Live Variables Analysis



## Local Data Flow Properties for Live Variables Analysis

r-value occurrence
Value is only read, e.g. x,y,z in
x.sum = y.data + z.data

l-value occurrence
Value is modified e.g. y in
y = x.lptr

$$
\begin{aligned}
Gen_n &= \{\, v \mid \text{variable } v \text{ is used in basic block } n \text{ and} \\
&\qquad \text{is not preceded by a definition of } v \,\} \\
Kill_n &= \{\, v \mid \text{basic block } n \text{ contains a definition of } v \,\}
\end{aligned}
$$

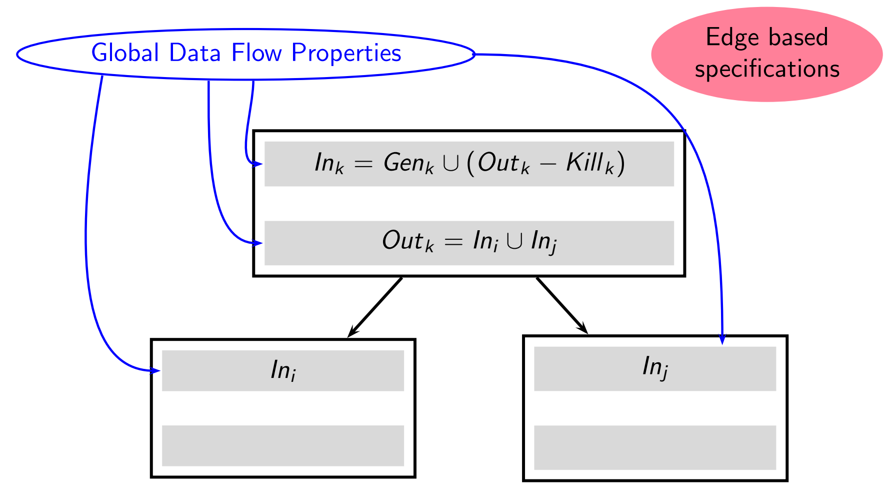
within $n$

anywhere in $n$





## Defining Data Flow Analysis for Live Variables Analysis








## Data Flow Equations For Live Variables Analysis

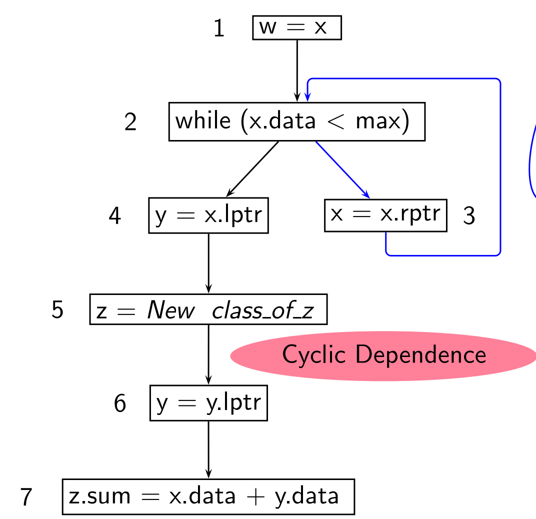
$$
\begin{aligned}
In_n &= (Out_n - Kill_n) \cup Gen_n \\
Out_n &= \begin{cases} BI & n \text{ is } End \text{ block} \\ \bigcup_{s \in succ(n)} In_s & \text{otherwise} \end{cases}
\end{aligned}
$$

- $In_n$ and $Out_n$ are sets of variables
- $BI$ is boundary information representing the effect of calling contexts
  - $\emptyset$ for local variables except for the values being returned
  - set of global variables used further in any calling context (can be safely approximated by the set of all global variables)



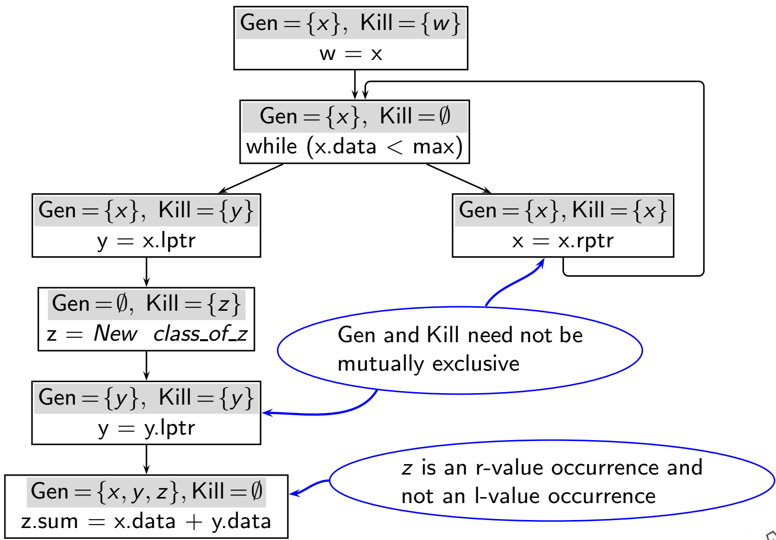


## Data Flow Equations for Our Example




$$
\begin{aligned}
In_1 &= (Out_1 - Kill_1) \cup Gen_1 \\
Out_1 &= In_2 \\
In_2 &= (Out_2 - Kill_2) \cup Gen_2 \\
Out_2 &= In_3 \cup In_4 \\
In_3 &= (Out_3 - Kill_3) \cup Gen_3 \\
Out_3 &= In_2 \\
In_4 &= (Out_4 - Kill_4) \cup Gen_4 \\
Out_4 &= In_5 \\
In_5 &= (Out_5 - Kill_5) \cup Gen_5 \\
Out_5 &= In_6 \\
In_6 &= (Out_6 - Kill_6) \cup Gen_6 \\
Out_6 &= In_7 \\
In_7 &= (Out_7 - Kill_7) \cup Gen_7 \\
Out_7 &= \emptyset
\end{aligned}
$$

# Performing Live Variables Analysis



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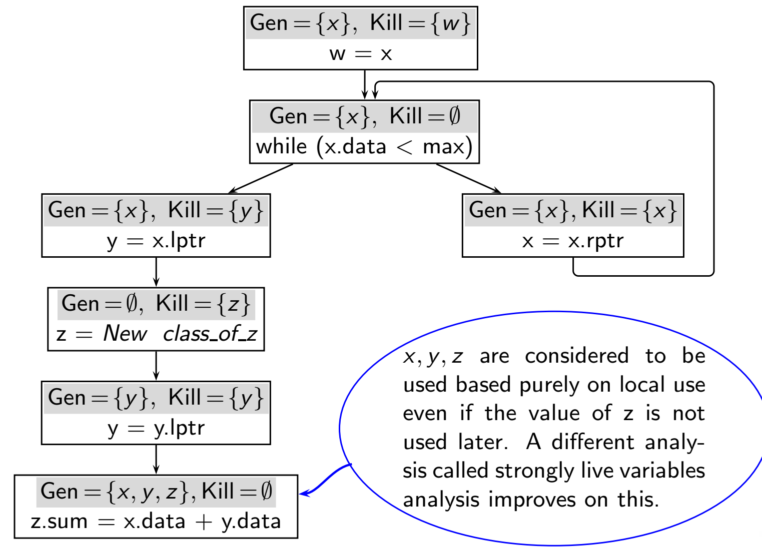
# Performing Live Variables Analysis

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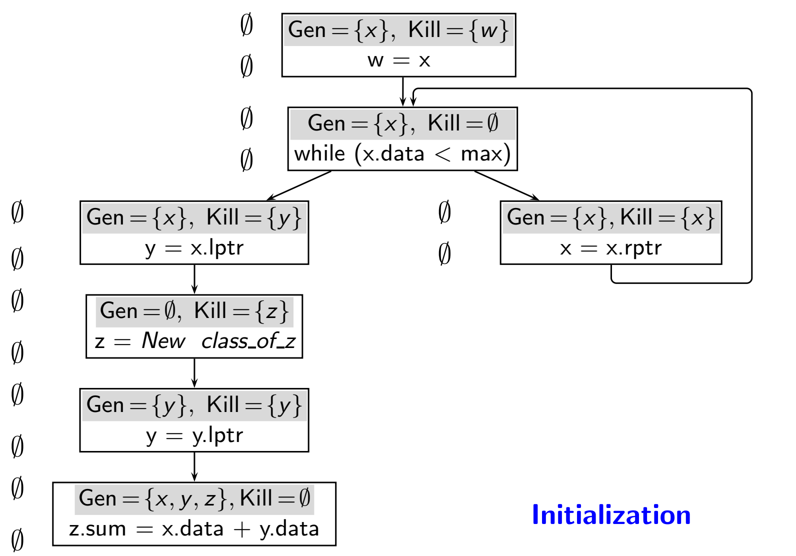
# Performing Live Variables Analysis

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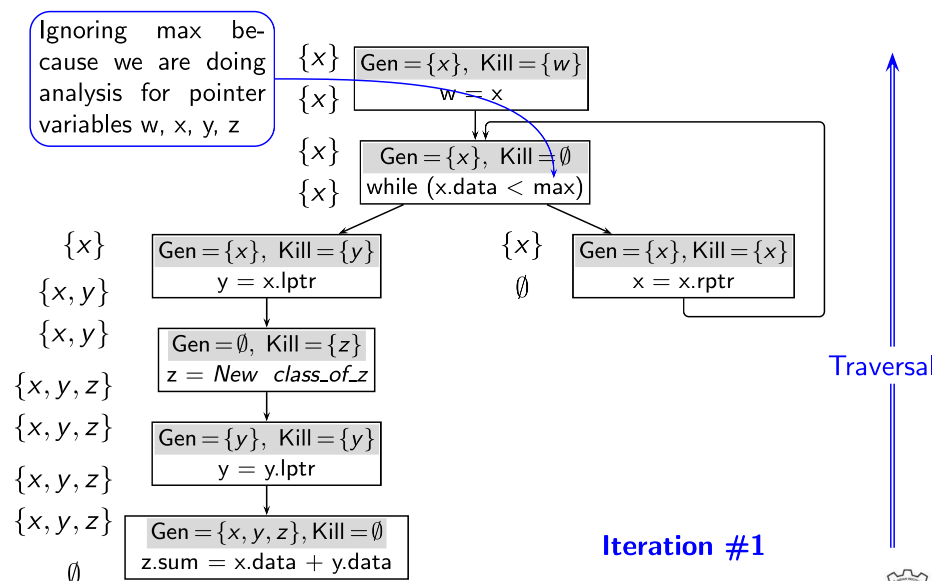
# Performing Live Variables Analysis

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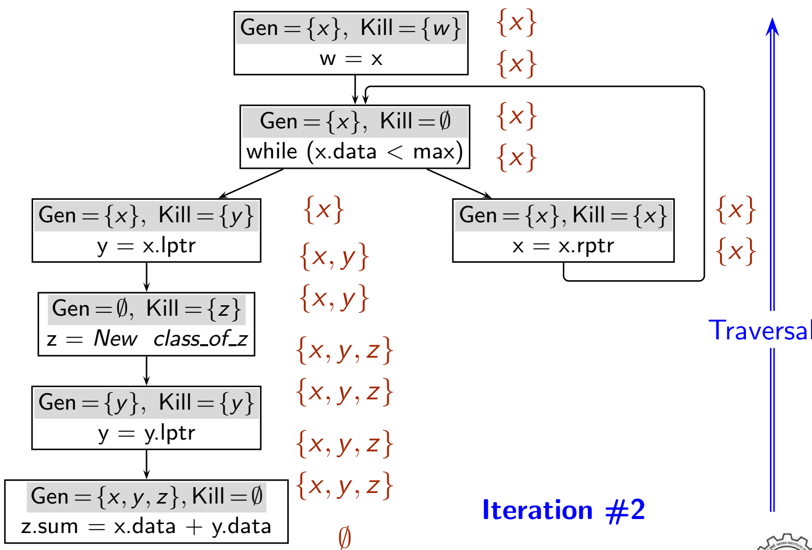
# Performing Live Variables Analysis

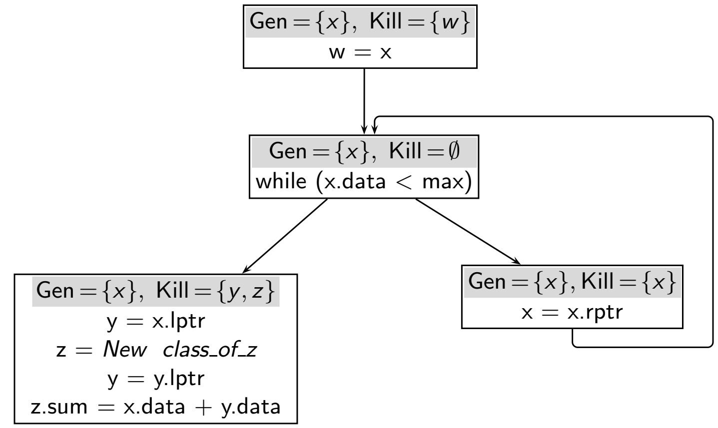







# Performing Live Variables Analysis

Local data flow properties when basic blocks contain multiple statements








# Local Data Flow Properties for Live Variables Analysis

$$In_n = Gen_n \cup (Out_n - Kill_n)$$

- $Gen_n$ : Use not preceded by definition

  Upwards exposed use

- $Kill_n$ : Definition anywhere in a block

  Stop the effect from being propagated across a block





# Local Data Flow Properties for Live Variables Analysis

| Case | Local Information | | Example | Explanation |
|---|---|---|---|---|
| 1 | $v \notin Gen_n$ | $v \notin Kill_n$ | $a = b + c$<br>$b = c * d$ | liveness of $v$ is unaffected by the basic block |
| 2 | $v \in Gen_n$ | $v \notin Kill_n$ | $a = b + c$<br>$b = v * d$ | $v$ becomes live before the basic block |
| 3 | $v \notin Gen_n$ | $v \in Kill_n$ | $a = b + c$<br>$v = c * d$ | $v$ ceases to be live before the basic block |
| 4 | $v \in Gen_n$ | $v \in Kill_n$ | $a = v + c$<br>$v = c * d$ | liveness of $v$ is killed but $v$ becomes live before the basic block |

# Using Data Flow Information of Live Variables Analysis

- Used for register allocation

  If variable $x$ is live in a basic block $b$, it is a potential candidate for register allocation

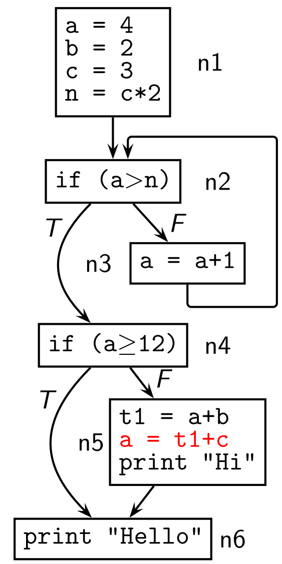
- Used for dead code elimination

  If variable $x$ is not live after an assignment $x = \ldots$, then the assignment is redundant and can be deleted as dead code





# Tutorial Problem 1: Perform Dead Code Elimination



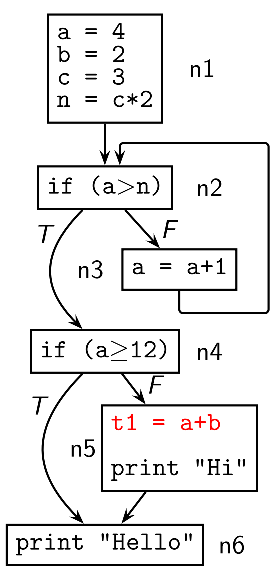

| Local Data Flow Information | | |
|---|---|---|
| | *Gen* | *Kill* |
| n1 | $\emptyset$ | $\{a, b, c, n\}$ |
| n2 | $\{a, n\}$ | $\emptyset$ |
| n3 | $\{a\}$ | $\{a\}$ |
| n4 | $\{a\}$ | $\emptyset$ |
| n5 | $\{a, b, c\}$ | $\{a, t1\}$ |
| n6 | $\emptyset$ | $\emptyset$ |

| Global Data Flow Information | | | | |
|---|---|---|---|---|
| | Iteration #1 | | Iteration #2 | |
| | *Out* | *In* | *Out* | *In* |
| n6 | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| n5 | $\emptyset$ | $\{a, b, c\}$ | $\emptyset$ | $\{a, b, c\}$ |
| n4 | $\{a, b, c\}$ | $\{a, b, c\}$ | $\{a, b, c\}$ | $\{a, b, c\}$ |
| n3 | $\emptyset$ | $\{a\}$ | $\{a, b, c, n\}$ | $\{a, b, c, n\}$ |
| n2 | $\{a, b, c\}$ | $\{a, b, c, n\}$ | $\{a, b, c, n\}$ | $\{a, b, c, n\}$ |
| n1 | $\{a, b, c, n\}$ | $\emptyset$ | $\{a, b, c, n\}$ | $\emptyset$ |





# Tutorial Problem 1: Round #2 of Dead Code Elimination



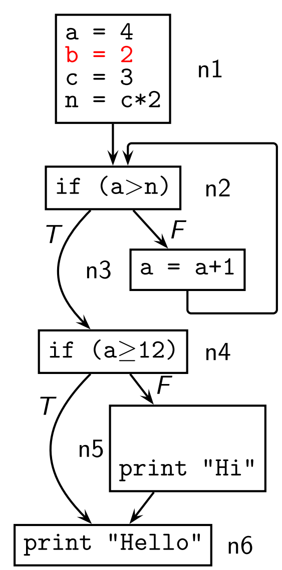

| Local Data Flow Information | | |
|---|---|---|
| | *Gen* | *Kill* |
| n1 | $\emptyset$ | $\{a, b, c, n\}$ |
| n2 | $\{a, n\}$ | $\emptyset$ |
| n3 | $\{a\}$ | $\{a\}$ |
| n4 | $\{a\}$ | $\emptyset$ |
| n5 | $\{a, b\}$ | $\{t1\}$ |
| n6 | $\emptyset$ | $\emptyset$ |

| Global Data Flow Information | | | | |
|---|---|---|---|---|
| | Iteration #1 | | Iteration #2 | |
| | *Out* | *In* | *Out* | *In* |
| n6 | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| n5 | $\emptyset$ | $\{a, b\}$ | $\emptyset$ | $\{a, b\}$ |
| n4 | $\{a, b\}$ | $\{a, b\}$ | $\{a, b\}$ | $\{a, b\}$ |
| n3 | $\emptyset$ | $\{a\}$ | $\{a, b, n\}$ | $\{a, b, n\}$ |
| n2 | $\{a, b\}$ | $\{a, b, n\}$ | $\{a, b, n\}$ | $\{a, b, n\}$ |
| n1 | $\{a, b, n\}$ | $\emptyset$ | $\{a, b, n\}$ | $\emptyset$ |





# Tutorial Problem 1: Round #3 of Dead Code Elimination




| Local Data Flow Information | | |
|---|---|---|
| | *Gen* | *Kill* |
| n1 | $\emptyset$ | $\{a, b, c, n\}$ |
| n2 | $\{a, n\}$ | $\emptyset$ |
| n3 | $\{a\}$ | $\{a\}$ |
| n4 | $\{a\}$ | $\emptyset$ |
| n5 | $\emptyset$ | $\emptyset$ |
| n6 | $\emptyset$ | $\emptyset$ |

| Global Data Flow Information | | | | |
|---|---|---|---|---|
| | Iteration #1 | | Iteration #2 | |
| | *Out* | *In* | *Out* | *In* |
| n6 | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| n5 | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| n4 | $\emptyset$ | $\{a\}$ | $\emptyset$ | $\{a\}$ |
| n3 | $\emptyset$ | $\{a\}$ | $\{a, n\}$ | $\{a, n\}$ |
| n2 | $\{a\}$ | $\{a, n\}$ | $\{a, n\}$ | $\{a, n\}$ |
| n1 | $\{a, n\}$ | $\emptyset$ | $\{a, n\}$ | $\emptyset$ |

*Part 3*

# *Some Observations*



## What Does Data Flow Analysis Involve?

- Defining the analysis. Define the properties of execution paths
- Formulating the analysis. Define data flow equations
  - Linear simultaneous equations on sets rather than numbers
  - Later we will generalize the domain of values
- Performing the analysis. Solve data flow equations for the given program flow graph
- Many unanswered questions
  Initial value? Termination? Complexity? Properties of Solutions?





## A Digression: Iterative Solution of Linear Simultaneous Equations

- Simultaneous equations represented in the form of the product of a matrix of coefficients (**A**) with the vector of unknowns (**x**)

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

- Start with approximate values
- Compute new values repeatedly from old values
- Two classical methods
  - Gauss-Seidel Method (Gauss: 1823, 1826), (Seidel: 1874)
  - Jacobi Method (Jacobi: 1845)





## A Digression: An Example of Iterative Solution of Linear Simultaneous Equations

| Equations | Solution |
|---|---|
| $4w = x + y + 32$<br>$4x = y + z + 32$<br>$4y = z + w + 32$<br>$4z = w + x + 32$ | $w = x = y = z = 16$ |

- Rewrite the equations to define $w, x, y$, and $z$

$$\begin{aligned} w &= 0.25x + 0.25y + 8 \\ x &= 0.25y + 0.25z + 8 \\ y &= 0.25z + 0.25w + 8 \\ z &= 0.25w + 0.25x + 8 \end{aligned}$$

- Assume some initial values of $w_0, x_0, y_0$, and $z_0$
- Compute $w_i, x_i, y_i$, and $z_i$ within some margin of error

# A Digression: Gauss-Seidel Method

| Equations | Initial Values | Error Margin |
|---|---|---|
| $w = 0.25x + 0.25y + 8$ | $w_0 = 24$ | $w_{i+1} - w_i \leq 0.35$ |
| $x = 0.25y + 0.25z + 8$ | $x_0 = 24$ | $x_{i+1} - x_i \leq 0.35$ |
| $y = 0.25z + 0.25w + 8$ | $y_0 = 24$ | $y_{i+1} - y_i \leq 0.35$ |
| $z = 0.25w + 0.25x + 8$ | $z_0 = 24$ | $z_{i+1} - z_i \leq 0.35$ |

| Iteration 1 | Iteration 2 | Iteration 3 |
|---|---|---|
| $w_1 = 6 + 6 + 8 = 20$ | $w_2 = 5 + 5 + 8 = 18$ | $w_3 = 4.5 + 4.5 + 8 = 17$ |
| $x_1 = 6 + 6 + 8 = 20$ | $x_2 = 5 + 5 + 8 = 18$ | $x_3 = 4.5 + 4.5 + 8 = 17$ |
| $y_1 = 6 + 6 + 8 = 20$ | $y_2 = 5 + 5 + 8 = 18$ | $y_3 = 4.5 + 4.5 + 8 = 17$ |
| $z_1 = 6 + 6 + 8 = 20$ | $z_2 = 5 + 5 + 8 = 18$ | $z_3 = 4.5 + 4.5 + 8 = 17$ |

| Iteration 4 | Iteration 5 |
|---|---|
| $w_4 = 4.25 + 4.25 + 8 = 16.5$ | $w_5 = 4.125 + 4.125 + 8 = 16.25$ |
| $x_4 = 4.25 + 4.25 + 8 = 16.5$ | $x_5 = 4.125 + 4.125 + 8 = 16.25$ |
| $y_4 = 4.25 + 4.25 + 8 = 16.5$ | $y_5 = 4.125 + 4.125 + 8 = 16.25$ |
| $z_4 = 4.25 + 4.25 + 8 = 16.5$ | $z_5 = 4.125 + 4.125 + 8 = 16.25$ |





# A Digression: Jacobi Method

Use values from the current iteration wherever possible

| Equations | Initial Values | Error Margin |
|---|---|---|
| $w = 0.25x + 0.25y + 8$ | $w_0 = 24$ | $w_{i+1} - w_i \leq 0.35$ |
| $x = 0.25y + 0.25z + 8$ | $x_0 = 24$ | $x_{i+1} - x_i \leq 0.35$ |
| $y = 0.25z + 0.25w + 8$ | $y_0 = 24$ | $y_{i+1} - y_i \leq 0.35$ |
| $z = 0.25w + 0.25x + 8$ | $z_0 = 24$ | $z_{i+1} - z_i \leq 0.35$ |

| Iteration 1 | Iteration 2 |
|---|---|
| $w_1 = 6 + 6 + 8 = 20$ | $w_2 = 5 + 4.75 + 8 = 17.75$ |
| $x_1 = 6 + 6 + 8 = 20$ | $x_2 = 4.75 + 4.5 + 8 = 17.25$ |
| $y_1 = 6 + 5 + 8 = 19$ | $y_2 = 4.5 + 4.4375 + 8 = 16.935$ |
| $z_1 = 5 + 5 + 8 = 18$ | $z_2 = 4.4375 + 4.375 + 8 = 16.8125$ |

| Iteration 3 | Iteration 4 |
|---|---|
| $w_3 = 4.3125 + 4.23375 + 8 = 16.54625$ | $w_4 = 16.20172$ |
| $x_3 = 4.23375 + 4.23375 + 8 = 16.436875$ | $x_4 = 16.17844$ |
| $y_3 = 4.23375 + 4.1365625 + 8 = 16.370$ | $y_4 = 16.13637$ |
| $z_3 = 4.1365625 + 4.11 + 8 = 16.34375$ | $z_4 = 16.09504$ |





# Our Method of Performing Data Flow Analysis

- Round robin iteration
- Essentially Jacobi method
- Unknowns are the data flow variables $In_i$ and $Out_i$
- Domain of values is not numbers
- Computation in a fixed order
  - either forward (reverse post order) traversal, or
  - backward (post order) traversal

  over the control flow graph



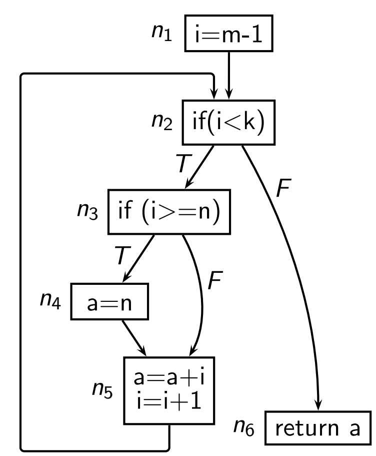


# Tutorial Problem 2 for Liveness Analysis

Draw the control flow graph and perform live variables analysis

```
int f(int m, int n, int k)
{
  int a,i;

  for (i=m-1; i<k; i++)
  {   if (i>=n)
         a = n;
      a = a+i;
  }
  return a;
}
```



# The Semantics of Return Statement for Live Variables Analysis

"return a" is modelled by the statement "return_value_in_stack = a"

- If we assume that the statement is executed *within* the block
  $\Longrightarrow$ $BI$ can be $\emptyset$
- If we assume that the statement is executed *outside of* the block and along the edge connecting the procedure to its caller
  $\Longrightarrow$ $a \in BI$





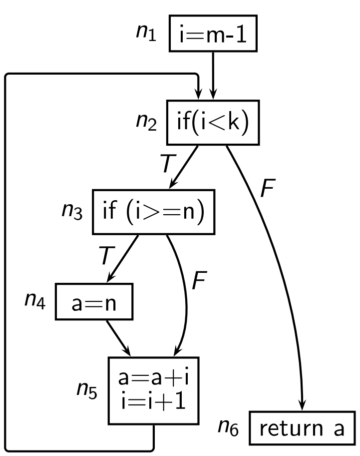
# Solution of Tutorial Problem 2

| Block | Local Information | | Global Information | | | |
|---|---|---|---|---|---|---|
| | | | Iteration # 1 | | Change in iteration # 2 | |
| | $Gen_n$ | $Kill_n$ | $Out_n$ | $In_n$ | $Out_n$ | $In_n$ |
| $n_6$ | $\{a\}$ | $\emptyset$ | $\emptyset$ | $\{a\}$ | | |
| $n_5$ | $\{a, i\}$ | $\{a, i\}$ | $\emptyset$ | $\{a, i\}$ | $\{a, i, k, n\}$ | $\{a, i, k, n\}$ |
| $n_4$ | $\{n\}$ | $\{a\}$ | $\{a, i\}$ | $\{i, n\}$ | $\{a, i, k, n\}$ | $\{i, k, n\}$ |
| $n_3$ | $\{i, n\}$ | $\emptyset$ | $\{a, i, n\}$ | $\{a, i, n\}$ | $\{a, i, k, n\}$ | $\{a, i, k, n\}$ |
| $n_2$ | $\{i, k\}$ | $\emptyset$ | $\{a, i, n\}$ | $\{a, i, k, n\}$ | $\{a, i, k, n\}$ | |
| $n_1$ | $\{m\}$ | $\{i\}$ | $\{a, i, k, n\}$ | $\{a, k, m, n\}$ | | |





# Interpreting the Result of Liveness Analysis for Tutorial Problem 2



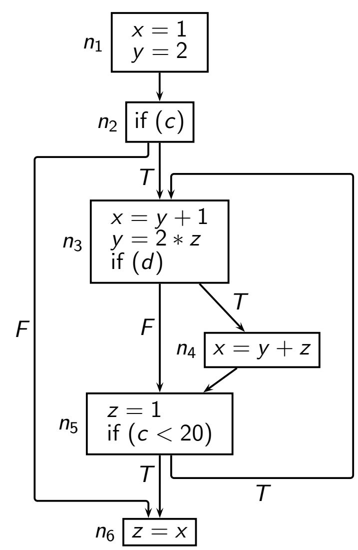

- Is a live at the exit of $n_5$ at the end of iteration 1? Why?
  (We have used post order traversal)
- Is a live at the exit of $n_5$ at the end of iteration 2? Why?
  (We have used post order traversal)
- Show an execution path along which a is live at the exit of $n_5$
- Show an execution path along which a is live at the exit of $n_3$
  $n_1 \rightarrow n_2 \rightarrow n_3 \rightarrow n_5 \rightarrow n_2 \rightarrow \ldots$
- Show an execution path along which a is not live at the exit of $n_3$
  $n_1 \rightarrow n_2 \rightarrow n_3 \rightarrow n_4 \rightarrow n_2 \rightarrow \ldots$





# Tutorial Problem 3 for Liveness Analysis

Also write a C program for this CFG without using goto or break




```
void f()
{   int x, y, z;
    int c, d;
    x = 1;
    y = 2;
    if (c)
    {   do
        {   x = y+1;
            y = 2*z;
            if (d)
                x = y+z;
            z = 1;
        } while (c < 20);
    }
    z = x;
}
```

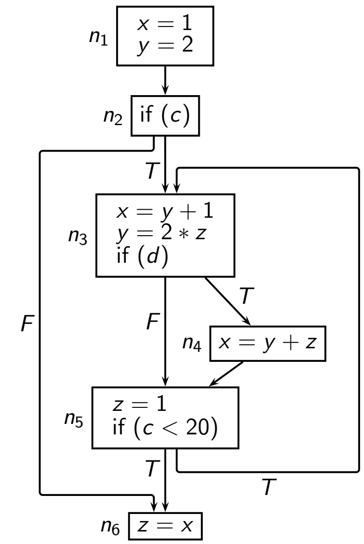
# Solution of Tutorial Problem 3

| Block | Local Information | | Global Information | | | |
|---|---|---|---|---|---|---|
| | | | Iteration # 1 | | Change in iteration # 2 | |
| | $Gen_n$ | $Kill_n$ | $Out_n$ | $In_n$ | $Out_n$ | $In_n$ |
| $n_6$ | $\{x\}$ | $\{z\}$ | $\emptyset$ | $\{x\}$ | | |
| $n_5$ | $\{c\}$ | $\{z\}$ | $\{x\}$ | $\{x, c\}$ | $\{x, y, z, c, d\}$ | $\{x, y, c, d\}$ |
| $n_4$ | $\{y, z\}$ | $\{x\}$ | $\{x, c\}$ | $\{y, z, c\}$ | $\{x, y, c, d\}$ | $\{y, z, c, d\}$ |
| $n_3$ | $\{y, z, d\}$ | $\{x, y\}$ | $\{x, y, z, c\}$ | $\{y, z, c, d\}$ | $\{x, y, z, c, d\}$ | |
| $n_2$ | $\{c\}$ | $\emptyset$ | $\{x, y, z, c, d\}$ | $\{x, y, z, c, d\}$ | | |
| $n_1$ | $\emptyset$ | $\{x, y\}$ | $\{x, y, z, c, d\}$ | $\{z, c, d\}$ | | |





# Interpreting the Result of Liveness Analysis for Tutorial Problem 3




- Why is z live at the exit of $n_5$?
- Why is z not live at the entry of $n_5$?
- Why is x live at the exit of $n_3$ inspite of being killed in $n_4$?
- Identify the instance of dead code elimination $z = x$ in $n_6$
- Would the first round of dead code elimination cause liveness information to change? Yes
- Would the second round of liveness analysis lead to further dead code elimination? Yes





# Choice of Initialization

What should be the initial value of internal nodes?

- Confluence is $\cup$
- Identity of $\cup$ is $\emptyset$
- We begin with $\emptyset$ and let the sets at each program point grow

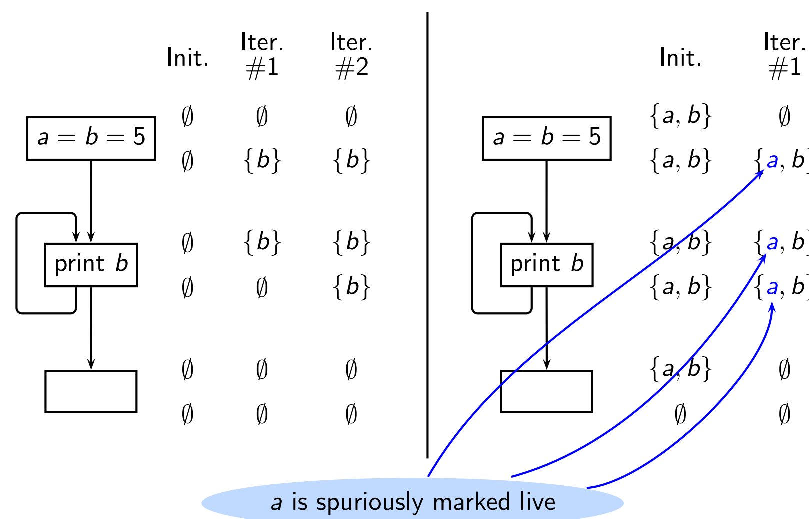
  A revisit to a program point
  - may consider a new execution path
  - more variables may be found to be live
  - a variable found to be live earlier does not become dead

The role of boundary info *BI* explained later in the context of available expressions analysis





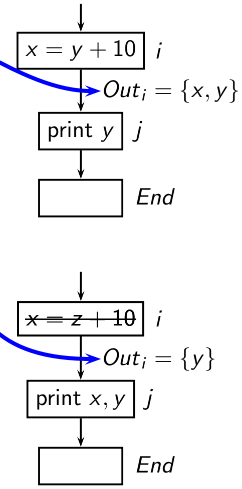
# How Does the Initialization Affect the Solution?



## Soundness and Precision of Live Variables Analysis

Consider dead code elimination based on liveness information

- Spurious inclusion of a non-live variable
  - A dead assignment may not be eliminated
  - Solution is sound but may be imprecise
- Spurious exclusion of a live variable
  - A useful assignment may be eliminated
  - Solution is unsound
- Given $L_2 \supseteq L_1$ representing liveness information
  - Using $L_2$ in place of $L_1$ is sound
  - Using $L_1$ in place of $L_2$ may not be sound
- The smallest set of all live variables is most precise
  - Since liveness sets grow (confluence is $\cup$), we choose $\emptyset$ as the initial conservative value








## Termination, Convergence, and Complexity

- For live variables analysis,
  - The set of all variables is finite, and
  - the confluence operation (i.e. meet) is union, hence
  - the set associated with a data flow variable can only grow

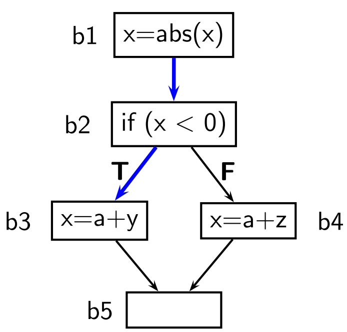
  $\Longrightarrow$ Termination is guaranteed
- Since initial value is $\emptyset$, live variables analysis converges on the smallest set
- How many iterations do we need for reaching the convergence?
- Going beyond live variables analysis
  - Do the sets always grow for other data flow frameworks?
  - What is the complexity of round robin analysis for other analyses?

  Answered formally in module 2 (Theoretical Abstractions)





## Conservative Nature of Analysis (1)




- abs(n) returns the absolute value of n
- Is y live on entry to block b2?
- By execution semantics, NO

  Path b1→b2→b3 is an infeasible execution path
- A compiler makes conservative assumptions:

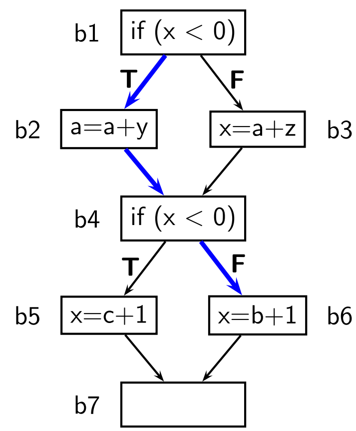
  *All branch outcomes are possible*

  ⇒ Consider every path in CFG as a potential execution path
- Our analysis concludes that y is live on entry to block b2





## Conservative Nature of Analysis (2)




- Is b live on entry to block b2?
- By execution semantics, NO

  Path b1→b2→b4→b6 is an infeasible execution path
- Is c live on entry to block b3?

  Path b1→b3→b4→b6 is a feasible execution path
- A compiler make conservative assumptions

  ⇒ our analysis is *path insensitive*

  Note: It is *flow sensitive* (i.e. information is computed for every control flow points)
- Our analysis concludes that b is live at the entry of b2
- Is c live at the entry of b3?

## Conservative Nature of Analysis at Intraprocedural Level

- We assume that all paths are potentially executable
- Our analysis is path insensitive
  - The data flow information at a program point $p$ is path insensitive
    - information at $p$ is merged along all paths reaching $p$
  - The data flow information reaching $p$ is computed path insensitively
    - information is merged at all shared points in paths reaching $p$
    - may generate spurious information due to non-distributive flow functions

More about it in module 2





## Conservative Nature of Analysis at Interprocedural Level

- Context insensitivity
  - Merges of information across all calling contexts
- Flow insensitivity
  - Disregards the control flow

More about it in module 4





## What About Soundness of Analysis Results?

- No compromises
- We will study it in module 2

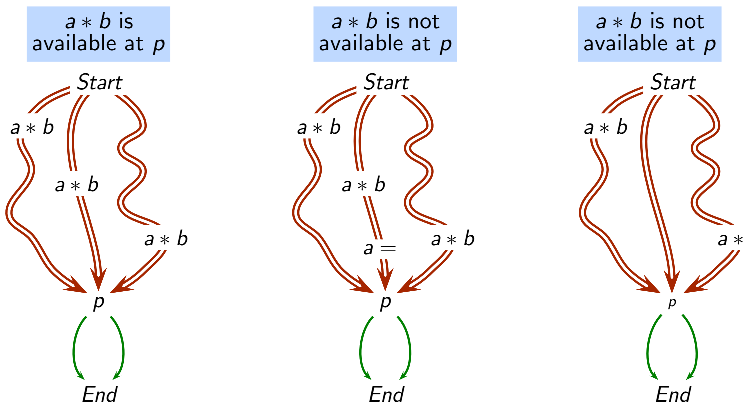


*Part 4*

# *Available Expressions Analysis*

## Defining Available Expressions Analysis

An expression $e$ is available at a program point $p$, if every path from program entry to $p$ contains an evaluation of $e$ which is not followed by a definition of any operand of $e$.








## Local Data Flow Properties for Available Expressions Analysis

$Gen_n$ = { $e$ | expression $e$ is evaluated in basic block $n$ and this evaluation is not followed by a definition of any operand of $e$ }

$Kill_n$ = { $e$ | basic block $n$ contains a definition of an operand of $e$ }

| | Entity | Manipulation | Exposition |
|---|---|---|---|
| $Gen_n$ | Expression | Use | Downwards |
| $Kill_n$ | Expression | Modification | Anywhere |





## Data Flow Equations For Available Expressions Analysis

$$In_n = \begin{cases} BI & n \text{ is } Start \text{ block} \\ \bigcap_{p \in pred(n)} Out_p & \text{otherwise} \end{cases}$$

$$Out_n = Gen_n \cup (In_n - Kill_n)$$

Alternatively,

$$Out_n = f_n(In_n), \quad \text{where}$$

$$f_n(X) = Gen_n \cup (X - Kill_n)$$

- $In_n$ and $Out_n$ are sets of expressions
- $BI$ is $\emptyset$ for expressions involving a local variable





## Using Data Flow Information of Available Expressions Analysis

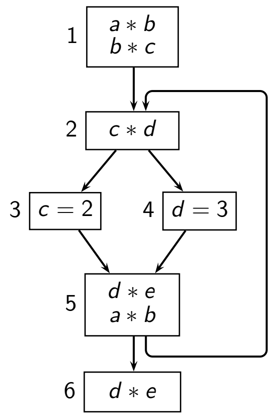
- Common subexpression elimination
  - If an expression is available at the entry of a block $n$ ($In_n$) **and**
  - a computation of the expression exists in $n$ **such that**
  - it is not preceded by a definition of any of its operands ($AntGen_n$)

  Then the expression is redundant

  $$Redundant_n = In_n \cap AntGen_n$$

- A redundant expression is upwards exposed whereas the expressions in $Gen_n$ are downwards exposed

# An Example of Available Expressions Analysis




Let $e_1 \equiv a * b$, $e_2 \equiv b * c$, $e_3 \equiv c * d$, $e_4 \equiv d * e$

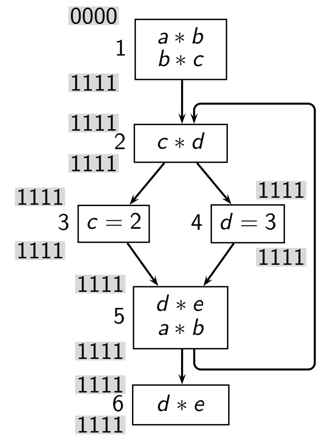
| Node | *Gen* | | *Kill* | | Available | | Redund. | |
|---|---|---|---|---|---|---|---|---|
| 1 | $\{e_1, e_2\}$ | 1100 | $\emptyset$ | 0000 | $\emptyset$ | 0000 | $\emptyset$ | 0000 |
| 2 | $\{e_3\}$ | 0010 | $\emptyset$ | 0000 | $\{e_1\}$ | 1000 | $\emptyset$ | 0000 |
| 3 | $\emptyset$ | 0000 | $\{e_2, e_3\}$ | 0110 | $\{e_1, e_3\}$ | 1010 | $\emptyset$ | 0000 |
| 4 | $\emptyset$ | 0000 | $\{e_3, e_4\}$ | 0011 | $\{e_1, e_3\}$ | 1010 | $\emptyset$ | 0000 |
| 5 | $\{e_1, e_4\}$ | 1001 | $\emptyset$ | 0000 | $\{e_1\}$ | 1000 | $\{e_1\}$ | 1000 |
| 6 | $\{e_4\}$ | 0001 | $\emptyset$ | 0000 | $\{e_1, e_4\}$ | 1001 | $\{e_4\}$ | 0001 |





# An Example of Available Expressions Analysis

**Initialisation**




Let $e_1 \equiv a * b$, $e_2 \equiv b * c$, $e_3 \equiv c * d$, $e_4 \equiv d * e$

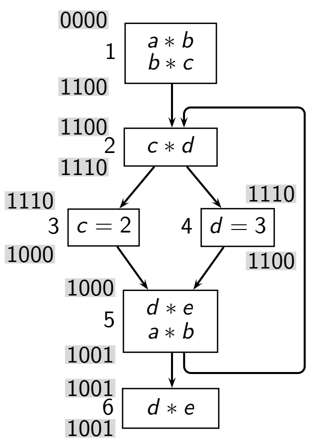
| Node | *Gen* | | *Kill* | | Available | | Redund. | |
|---|---|---|---|---|---|---|---|---|
| 1 | $\{e_1, e_2\}$ | 1100 | $\emptyset$ | 0000 | $\emptyset$ | 0000 | $\emptyset$ | 0000 |
| 2 | $\{e_3\}$ | 0010 | $\emptyset$ | 0000 | $\{e_1\}$ | 1000 | $\emptyset$ | 0000 |
| 3 | $\emptyset$ | 0000 | $\{e_2, e_3\}$ | 0110 | $\{e_1, e_3\}$ | 1010 | $\emptyset$ | 0000 |
| 4 | $\emptyset$ | 0000 | $\{e_3, e_4\}$ | 0011 | $\{e_1, e_3\}$ | 1010 | $\emptyset$ | 0000 |
| 5 | $\{e_1, e_4\}$ | 1001 | $\emptyset$ | 0000 | $\{e_1\}$ | 1000 | $\{e_1\}$ | 1000 |
| 6 | $\{e_4\}$ | 0001 | $\emptyset$ | 0000 | $\{e_1, e_4\}$ | 1001 | $\{e_4\}$ | 0001 |





# An Example of Available Expressions Analysis

**Iteration #1**




Let $e_1 \equiv a * b$, $e_2 \equiv b * c$, $e_3 \equiv c * d$, $e_4 \equiv d * e$

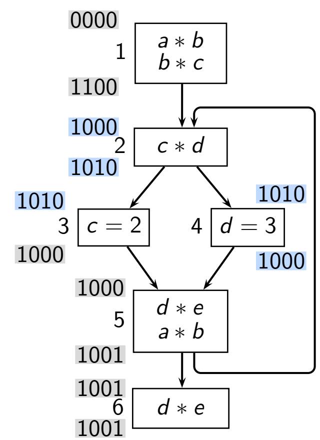
| Node | *Gen* | | *Kill* | | Available | | Redund. | |
|---|---|---|---|---|---|---|---|---|
| 1 | $\{e_1, e_2\}$ | 1100 | $\emptyset$ | 0000 | $\emptyset$ | 0000 | $\emptyset$ | 0000 |
| 2 | $\{e_3\}$ | 0010 | $\emptyset$ | 0000 | $\{e_1\}$ | 1000 | $\emptyset$ | 0000 |
| 3 | $\emptyset$ | 0000 | $\{e_2, e_3\}$ | 0110 | $\{e_1, e_3\}$ | 1010 | $\emptyset$ | 0000 |
| 4 | $\emptyset$ | 0000 | $\{e_3, e_4\}$ | 0011 | $\{e_1, e_3\}$ | 1010 | $\emptyset$ | 0000 |
| 5 | $\{e_1, e_4\}$ | 1001 | $\emptyset$ | 0000 | $\{e_1\}$ | 1000 | $\{e_1\}$ | 1000 |
| 6 | $\{e_4\}$ | 0001 | $\emptyset$ | 0000 | $\{e_1, e_4\}$ | 1001 | $\{e_4\}$ | 0001 |





# An Example of Available Expressions Analysis

**Iteration #2**

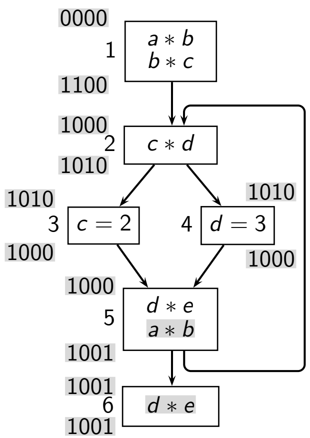



Let $e_1 \equiv a * b$, $e_2 \equiv b * c$, $e_3 \equiv c * d$, $e_4 \equiv d * e$

| Node | *Gen* | | *Kill* | | Available | | Redund. | |
|---|---|---|---|---|---|---|---|---|
| 1 | $\{e_1, e_2\}$ | 1100 | $\emptyset$ | 0000 | $\emptyset$ | 0000 | $\emptyset$ | 0000 |
| 2 | $\{e_3\}$ | 0010 | $\emptyset$ | 0000 | $\{e_1\}$ | 1000 | $\emptyset$ | 0000 |
| 3 | $\emptyset$ | 0000 | $\{e_2, e_3\}$ | 0110 | $\{e_1, e_3\}$ | 1010 | $\emptyset$ | 0000 |
| 4 | $\emptyset$ | 0000 | $\{e_3, e_4\}$ | 0011 | $\{e_1, e_3\}$ | 1010 | $\emptyset$ | 0000 |
| 5 | $\{e_1, e_4\}$ | 1001 | $\emptyset$ | 0000 | $\{e_1\}$ | 1000 | $\{e_1\}$ | 1000 |
| 6 | $\{e_4\}$ | 0001 | $\emptyset$ | 0000 | $\{e_1, e_4\}$ | 1001 | $\{e_4\}$ | 0001 |

# An Example of Available Expressions Analysis

**Final Result**



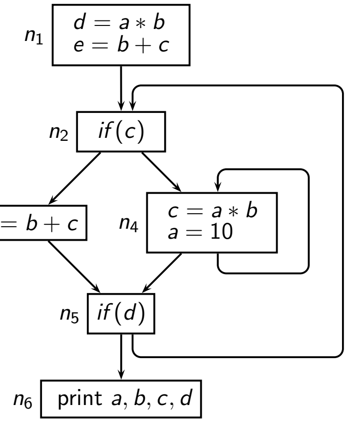

Let $e_1 \equiv a * b$, $e_2 \equiv b * c$, $e_3 \equiv c * d$, $e_4 \equiv d * e$

| Node | *Gen* | | *Kill* | | Available | | Redund. | |
|---|---|---|---|---|---|---|---|---|
| 1 | $\{e_1, e_2\}$ | 1100 | $\emptyset$ | 0000 | $\emptyset$ | 0000 | $\emptyset$ | 0000 |
| 2 | $\{e_3\}$ | 0010 | $\emptyset$ | 0000 | $\{e_1\}$ | 1000 | $\emptyset$ | 0000 |
| 3 | $\emptyset$ | 0000 | $\{e_2, e_3\}$ | 0110 | $\{e_1, e_3\}$ | 1010 | $\emptyset$ | 0000 |
| 4 | $\emptyset$ | 0000 | $\{e_3, e_4\}$ | 0011 | $\{e_1, e_3\}$ | 1010 | $\emptyset$ | 0000 |
| 5 | $\{e_1, e_4\}$ | 1001 | $\emptyset$ | 0000 | $\{e_1\}$ | 1000 | $\{e_1\}$ | 1000 |
| 6 | $\{e_4\}$ | 0001 | $\emptyset$ | 0000 | $\{e_1, e_4\}$ | 1001 | $\{e_4\}$ | 0001 |





# Tutorial Problem 2 for Available Expressions Analysis




$\mathbb{E}$xpr = { $a * b$, $b + c$ }





# Solution of the Tutorial Problem 2

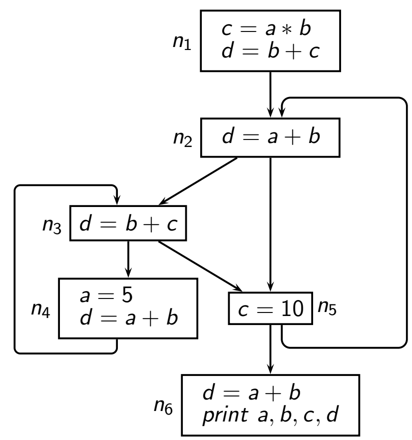
Bit vector | $a * b$ | $b + c$ |

| Node | Local Information | | | Global Information | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | Iteration # 1 | | Changes in iteration # 2 | | *Redundant$_n$* |
| | *Gen$_n$* | *Kill$_n$* | *AntGen$_n$* | *In$_n$* | *Out$_n$* | *In$_n$* | *Out$_n$* | |
| $n_1$ | 11 | 00 | 11 | 00 | 11 | | | 00 |
| $n_2$ | 00 | 00 | 00 | 11 | 11 | 00 | 00 | 00 |
| $n_3$ | 01 | 10 | 01 | 11 | 01 | 00 | | 00 |
| $n_4$ | 00 | 11 | 10 | 11 | 00 | 00 | | 00 |
| $n_5$ | 00 | 00 | 00 | 00 | 00 | | | 00 |
| $n_6$ | 00 | 00 | 00 | 00 | 00 | | | 00 |





# Tutorial Problem 3 for Available Expressions Analysis




$\mathbb{E}$xpr = { $a * b$, $b + c$, $a + b$ }

# Solution of the Tutorial Problem 3

Bit vector | $a * b$ | $b + c$ | $a + b$

| Node | Local Information | | | Global Information | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Iteration # 1 | | Changes in Iteration # 2 | | Changes in Iteration # 3 | | $Redundant_n$ |
| | $Gen_n$ | $Kill_n$ | $AntGen_n$ | $In_n$ | $Out_n$ | $In_n$ | $Out_n$ | $In_n$ | $Out_n$ | |
| $n_1$ | 110 | 010 | 100 | 000 | 110 | | | | | 000 |
| $n_2$ | 001 | 000 | 001 | 110 | 111 | 100 | 101 | 000 | 001 | 000 |
| $n_3$ | 010 | 000 | 010 | 111 | 111 | 001 | 011 | | | 000 |
| $n_4$ | 001 | 101 | 000 | 111 | 011 | 011 | | | | 000 |
| $n_5$ | 000 | 010 | 000 | 111 | 101 | 001 | 001 | | | 000 |
| $n_6$ | 001 | 000 | 001 | 101 | 101 | 001 | 001 | | | 001 |

Why do we need 3 iterations as against 2 for previous problems?

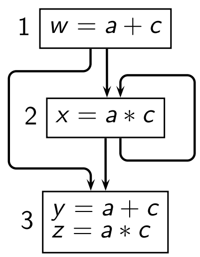




# The Effect of *BI* and Initialization on a Solution

Bit Vector

| $a + c$ | $a * c$ |
|---|---|




| *BI* | Node | Initialization $\mathbb{U}$ | | Initialization $\emptyset$ | |
|---|---|---|---|---|---|
| | | $In_n$ | $Out_n$ | $In_n$ | $Out_n$ |
| $\emptyset$ | 1 | 00 | 10 | 00 | 10 |
| | 2 | 10 | 11 | 00 | 01 |
| | 3 | 10 | 11 | 01 | 11 |
| $\mathbb{U}$ | 1 | 11 | 11 | 11 | 11 |
| | 2 | 11 | 11 | 00 | 01 |
| | 3 | 11 | 11 | 01 | 11 |

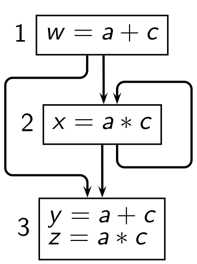




# The Effect of *BI* and Initialization on a Solution

Bit Vector

| $a + c$ | $a * c$ |
|---|---|




| *BI* | Node | Initialization $\mathbb{U}$ | | Initialization $\emptyset$ | |
|---|---|---|---|---|---|
| | | $In_n$ | $Out_n$ | $In_n$ | $Out_n$ |
| $\emptyset$ | 1 | 00 | 10 | 00 | 10 |
| | 2 | 10 | 11 | 00 | 01 |
| | 3 | 10 | 11 | 01 | 11 |
| $\mathbb{U}$ | 1 | 11 | 11 | 11 | 11 |
| | 2 | 11 | 11 | 00 | 01 |
| | 3 | 11 | 11 | 01 | 11 |

This represents the expected availability information leading to elimination of $a + c$ in node 3 ($a * c$ is not redundant in node 3)

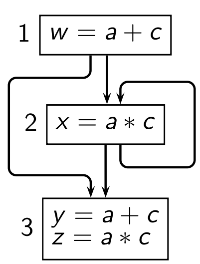




# The Effect of *BI* and Initialization on a Solution

Bit Vector

| $a + c$ | $a * c$ |
|---|---|




| *BI* | Node | Initialization $\mathbb{U}$ | | Initialization $\emptyset$ | |
|---|---|---|---|---|---|
| | | $In_n$ | $Out_n$ | $In_n$ | $Out_n$ |
| $\emptyset$ | 1 | 00 | 10 | 00 | 10 |
| | 2 | 10 | 11 | 00 | 01 |
| | 3 | 10 | 11 | 01 | 11 |
| $\mathbb{U}$ | 1 | 11 | 11 | 11 | 11 |
| | 2 | 11 | 11 | 00 | 01 |
| | 3 | 11 | 11 | 01 | 11 |

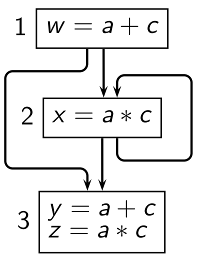
This misses the availability of $a + c$ in node 3

## The Effect of *BI* and Initialization on a Solution

This makes $a * c$ available in node 3 although its computation in node 3 is not redundant

Bit Vector

| | |
|---|---|
| $a + c$ | $a * c$ |




| *BI* | Node | Initialization $\mathbb{U}$ | | Initialization $\emptyset$ | |
|---|---|---|---|---|---|
| | | $In_n$ | $Out_n$ | $In_n$ | $Out_n$ |
| $\emptyset$ | 1 | 00 | 10 | 00 | 10 |
| | 2 | 10 | 11 | 00 | 01 |
| | 3 | 10 | 11 | 01 | 11 |
| $\mathbb{U}$ | 1 | 11 | 11 | 11 | 11 |
| | 2 | 11 | 11 | 00 | 01 |
| | 3 | 11 | 11 | 01 | 11 |



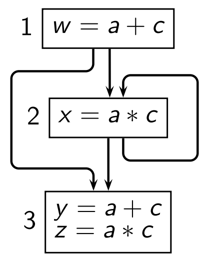


## The Effect of *BI* and Initialization on a Solution

This make $a * c$ available in node 3 and but misses the availability of $a + c$ in node 3

Bit Vector

| | |
|---|---|
| $a + c$ | $a * c$ |




| *BI* | Node | Initialization $\mathbb{U}$ | | Initialization $\emptyset$ | |
|---|---|---|---|---|---|
| | | $In_n$ | $Out_n$ | $In_n$ | $Out_n$ |
| $\emptyset$ | 1 | 00 | 10 | 00 | 10 |
| | 2 | 10 | 11 | 00 | 01 |
| | 3 | 10 | 11 | 01 | 11 |
| $\mathbb{U}$ | 1 | 11 | 11 | 11 | 11 |
| | 2 | 11 | 11 | 00 | 01 |
| | 3 | 11 | 11 | 01 | 11 |

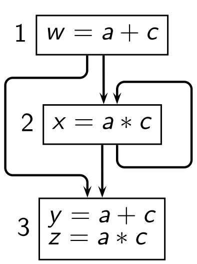




## The Effect of *BI* and Initialization on a Solution

Bit Vector

| | |
|---|---|
| $a + c$ | $a * c$ |




| *BI* | Node | Initialization $\mathbb{U}$ | | Initialization $\emptyset$ | |
|---|---|---|---|---|---|
| | | $In_n$ | $Out_n$ | $In_n$ | $Out_n$ |
| $\emptyset$ | 1 | | | | |
| | 2 | Sound & Precise | | Sound & Imprecise | |
| | 3 | | | | |
| $\mathbb{U}$ | 1 | | | | |
| | 2 | Unsound | | Unsound | |
| | 3 | | | | |





## Some Observations

- Data flow equations do not require a particular order of computation
  - Specification. Data flow equations define what needs to be computed and not how it is to be computed
  - Implementation. Round robin iterations perform the actual computation
  - Specification and implementation are distinct
- Initialization governs the quality of solution found
  - Only precision is affected, soundness is guaranteed
  - Associated with "internal" nodes
- *BI* depends on the semantics of the calling context
  - May cause unsoundness
  - Associated with "boundary" node (specified by data flow equations) Does not vary with the method or order of traversal

## Still More Tutorial Problems ☺

A New Data Flow Framework: Partially available expressions analysis

- Expressions that are computed and remain unmodified along some path reaching $p$
- The data flow equations are same as that of available expressions analysis except that the confluence is changed to $\cup$

Perform partially available expressions analysis for the example program used for available expressions analysis





## Solution of the Tutorial Problem 2 for Partial Availability Analysis

Bit vector | $a * b$ | $b + c$ |

| Node | Local Information | | | Global Information | | |
|---|---|---|---|---|---|---|
| | | | | Iteration # 1 | | $ParRedund_n$ |
| | $Gen_n$ | $Kill_n$ | $AntGen_n$ | $PavIn_n$ | $PavOut_n$ | |
| $n_1$ | 11 | 00 | 11 | 00 | 11 | 00 |
| $n_2$ | 00 | 00 | 00 | 11 | 11 | 00 |
| $n_3$ | 01 | 10 | 01 | 11 | 01 | 01 |
| $n_4$ | 00 | 11 | 10 | 11 | 00 | 10 |
| $n_5$ | 00 | 00 | 00 | 01 | 01 | 00 |
| $n_6$ | 00 | 00 | 00 | 01 | 01 | 00 |





## Solution of the Tutorial Problem 3 for Partial Availability Analysis

Bit vector | $a * b$ | $b + c$ | $a + b$ |

| Node | Local Information | | | Global Information | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | Iteration # 1 | | Changes in iteration # 2 | | $ParRedund_n$ |
| | $Gen_n$ | $Kill_n$ | $AntGen_n$ | $PavIn_n$ | $PavOut_n$ | $In_n$ | $Out_n$ | |
| $n_1$ | 110 | 010 | 100 | 000 | 110 | | | 000 |
| $n_2$ | 001 | 000 | 001 | 110 | 111 | 111 | | 001 |
| $n_3$ | 010 | 000 | 010 | 111 | 111 | | | 010 |
| $n_4$ | 001 | 101 | 000 | 111 | 011 | | | 000 |
| $n_5$ | 000 | 010 | 000 | 111 | 101 | | | 000 |
| $n_6$ | 001 | 000 | 001 | 101 | 101 | | | 001 |



*Part 5*

# *Reaching Definitions Analysis*

# Defining Reaching Definitions Analysis

- A definition $d_x : x = e$ reaches a program point $p$ if it appears (without a redefinition of $x$) on some path from program entry to $p$

  ($x$ is a variable and $e$ is an expression)

- Application : Copy Propagation

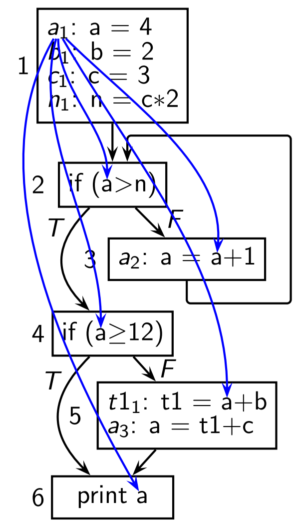
  A use of a variable $x$ at a program point $p$ can be replaced by $y$ if $d_x : x = y$ is the only definition which reaches $p$ and $y$ is not modified between the point of $d_x$ and $p$.





# Using Reaching Definitions for Def-Use and Use-Def Chains

*Def-Use Chains*

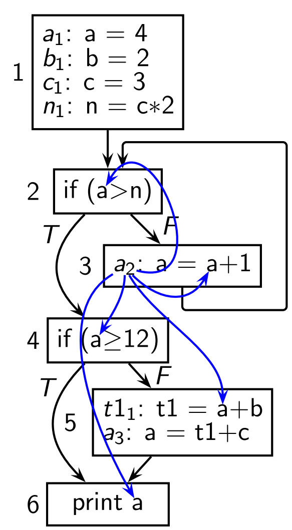







# Using Reaching Definitions for Def-Use and Use-Def Chains

*Def-Use Chains*

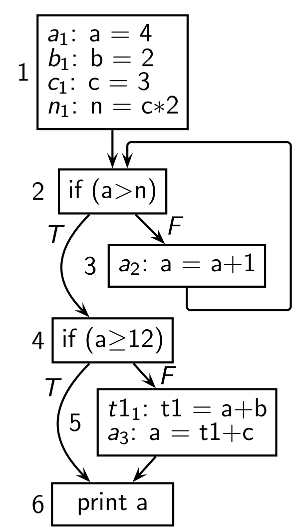




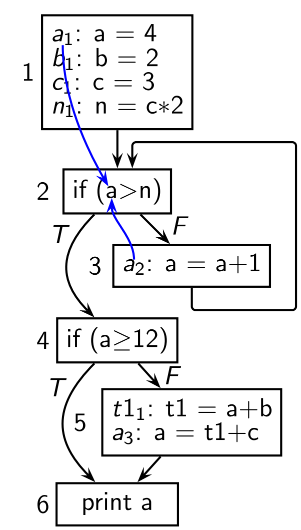



# Using Reaching Definitions for Def-Use and Use-Def Chains

*Def-Use Chains*

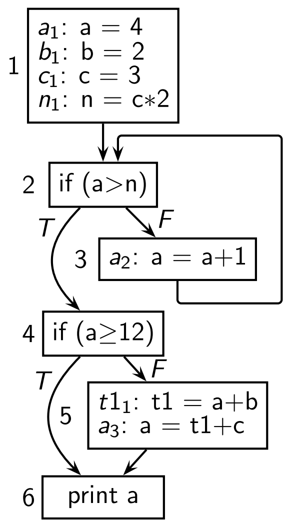



*Use-Def Chains*

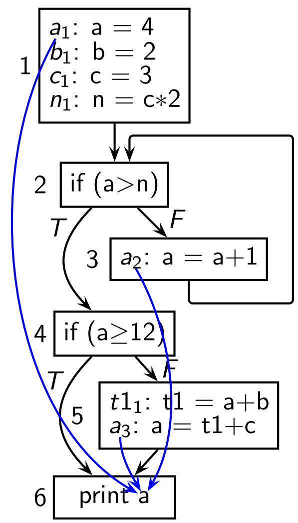


## Using Reaching Definitions for Def-Use and Use-Def Chains

*Def-Use Chains*




*Use-Def Chains*

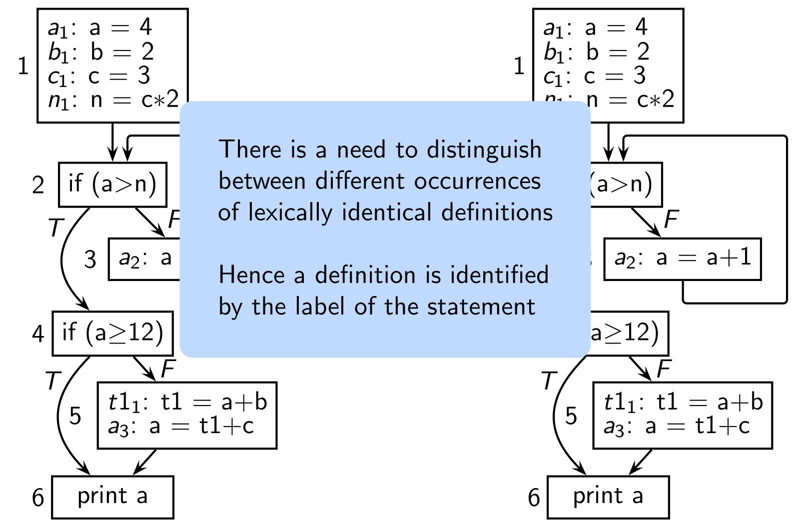







## Using Reaching Definitions for Def-Use and Use-Def Chains

*Def-Use Chains* *Use-Def Chains*



There is a need to distinguish between different occurrences of lexically identical definitions

Hence a definition is identified by the label of the statement





## Defining Data Flow Analysis for Reaching Definitions Analysis

Let $d_v$ be a definition of variable $v$

$$Gen_n = \{\, d_v \mid \text{variable } v \text{ is defined in basic block } n \text{ and this definition is not followed (within } n\text{) by a definition of } v \}$$

$$Kill_n = \{\, d_v \mid \text{basic block } n \text{ contains a definition of } v \}$$

| | Entity | Manipulation | Exposition |
|---|---|---|---|
| $Gen_n$ | Definition | Occurrence | Downwards |
| $Kill_n$ | Definition | Occurrence | Anywhere |





## Data Flow Equations for Reaching Definitions Analysis

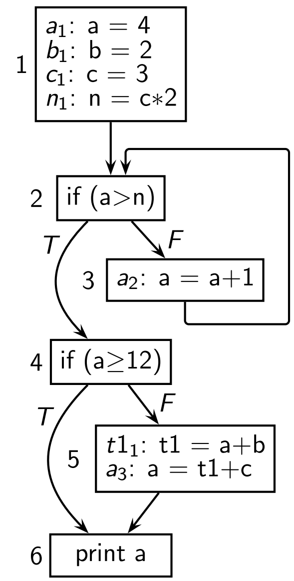
$$In_n = \begin{cases} BI & n \text{ is } Start \text{ block} \\ \bigcup_{p \in pred(n)} Out_p & \text{otherwise} \end{cases}$$

$$Out_n = Gen_n \cup (In_n - Kill_n)$$

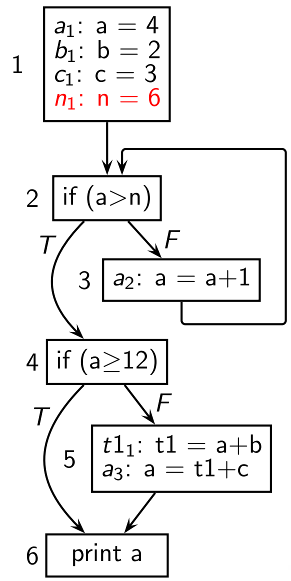
$$BI = \{ d_x : x = undef \mid x \in \mathbb{V}\text{ar} \}$$

$In_n$ and $Out_n$ are sets of definitions

# Tutorial Problem for Copy Propagation



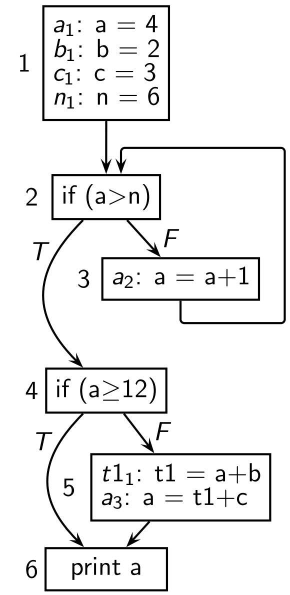

Local copy propagation and constant folding








# Tutorial Problem for Copy Propagation




| | *Gen* | *Kill* |
|---|---|---|
| n1 | $\{a_1, b_1, c_1, n_1\}$ | $\{a_0, a_1, a_2, a_3, b_0, b_1, c_0, c_1, n_0, n_1\}$ |
| n2 | $\emptyset$ | $\emptyset$ |
| n3 | $\{a_2\}$ | $\{a_0, a_1, a_2, a_3\}$ |
| n4 | $\emptyset$ | $\emptyset$ |
| n5 | $\{a_3\}$ | $\{a_0, a_1, a_2, a_3\}$ |
| n6 | $\emptyset$ | $\emptyset$ |

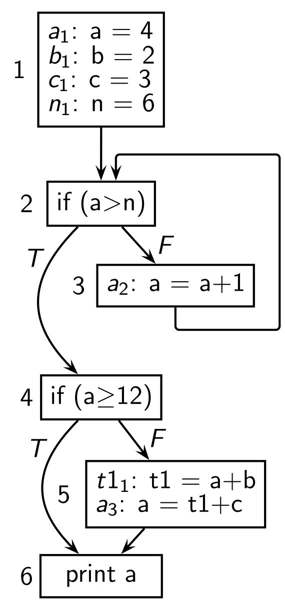
- Temporary variable t1 is ignored
- For variable $v$, $v_0$ denotes the definition $v$ = ?

  This is used for defining *BI*





# Tutorial Problem for Copy Propagation



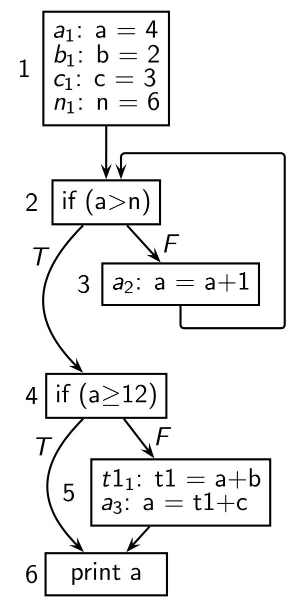

| | *Gen* | *Kill* |
|---|---|---|
| n1 | $\{a_1, b_1, c_1, n_1\}$ | $\{a_0, a_1, a_2, a_3, b_0, b_1, c_0, c_1, n_0, n_1\}$ |
| n2 | $\emptyset$ | $\emptyset$ |
| n3 | $\{a_2\}$ | $\{a_0, a_1, a_2, a_3\}$ |
| n4 | $\emptyset$ | $\emptyset$ |
| n5 | $\{a_3\}$ | $\{a_0, a_1, a_2, a_3\}$ |
| n6 | $\emptyset$ | $\emptyset$ |

| | Iteration #1 | |
|---|---|---|
| | *In* | *Out* |
| n1 | $\{a_0, b_0, c_0, n_0\}$ | $\{a_1, b_1, c_1, n_1\}$ |
| n2 | $\{a_1, b_1, c_1, n_1\}$ | $\{a_1, b_1, c_1, n_1\}$ |
| n3 | $\{a_1, b_1, c_1, n_1\}$ | $\{a_2, b_1, c_1, n_1\}$ |
| n4 | $\{a_1, b_1, c_1, n_1\}$ | $\{a_1, b_1, c_1, n_1\}$ |
| n5 | $\{a_1, b_1, c_1, n_1\}$ | $\{a_3, b_1, c_1, n_1\}$ |
| n6 | $\{a_1, a_3, b_1, c_1, n_1\}$ | $\{a_1, a_3, b_1, c_1, n_1\}$ |





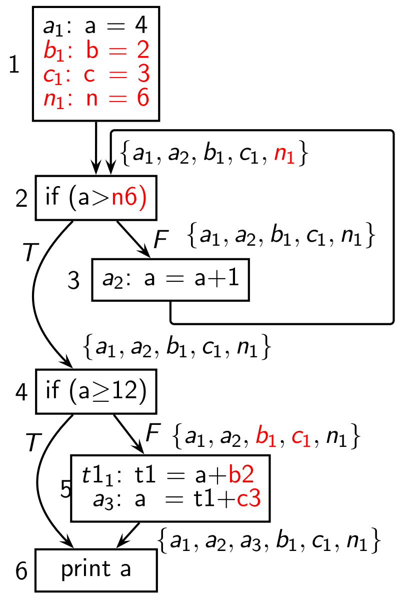
# Tutorial Problem for Copy Propagation




| | *Gen* | *Kill* |
|---|---|---|
| n1 | $\{a_1, b_1, c_1, n_1\}$ | $\{a_0, a_1, a_2, a_3, b_0, b_1, c_0, c_1, n_0, n_1\}$ |
| n2 | $\emptyset$ | $\emptyset$ |
| n3 | $\{a_2\}$ | $\{a_0, a_1, a_2, a_3\}$ |
| n4 | $\emptyset$ | $\emptyset$ |
| n5 | $\{a_3\}$ | $\{a_0, a_1, a_2, a_3\}$ |
| n6 | $\emptyset$ | $\emptyset$ |

| | Iteration #2 | |
|---|---|---|
| | *In* | *Out* |
| n1 | $\{a_0, b_0, c_0, n_0\}$ | $\{a_1, b_1, c_1, n_1\}$ |
| n2 | $\{a_1, a_2, b_1, c_1, n_1\}$ | $\{a_1, a_2, b_1, c_1, n_1\}$ |
| n3 | $\{a_1, a_2, b_1, c_1, n_1\}$ | $\{a_2, b_1, c_1, n_1\}$ |
| n4 | $\{a_1, a_2, b_1, c_1, n_1\}$ | $\{a_1, a_2, b_1, c_1, n_1\}$ |
| n5 | $\{a_1, a_2, b_1, c_1, n_1\}$ | $\{a_3, b_1, c_1, n_1\}$ |
| n6 | $\{a_1, a_2, a_3, b_1, c_1, n_1\}$ | $\{a_1, a_2, a_3, b_1, c_1, n_1\}$ |

## Tutorial Problem for Copy Propagation

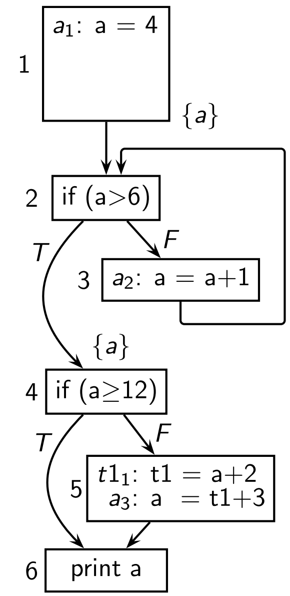



- RHS of $n_1$ is constant and hence cannot change
- In block 2, $n$ can be replaced by 6
- RHS of $b_1$ and $c_1$ are constant and hence cannot change
- In block 5, $b$ can be replaced by 2 and $c$ can be replaced by 3





## Tutorial Problem for Copy Propagation




So what is the advantage?

Dead Code Elimination

- Only $a$ is live at the exit of 1
- Assignments of $b$, $c$, and $n$ are dead code
- Can be deleted



*Part 6*

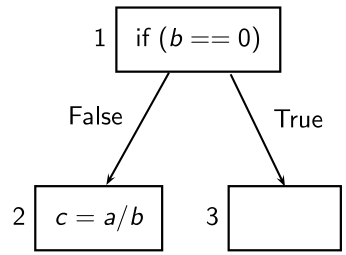
# *Anticipable Expressions Analysis*

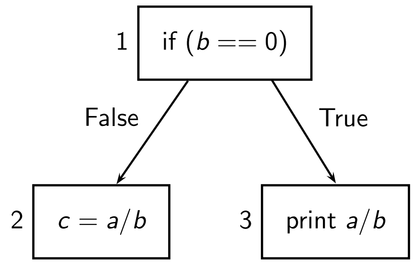


## Defining Anticipable Expressions Analysis

- An expression $e$ is anticipable at a program point $p$, if every path from $p$ to the program exit contains an evaluation of $e$ which is not preceded by a redefinition of any operand of $e$.
- Application : Safety of Code Placement

# Safety of Code Placement




Placing $a/b$ at the exit of 1 is unsafe
($\equiv$ can change the behaviour of the optimized program)

??

A guarded computation of an expression should not be converted to an unguarded computation





# Defining Data Flow Analysis for Anticipable Expressions Analysis

$Gen_n$ = { $e$ | expression $e$ is evaluated in basic block $n$ and this evaluation is not preceded (within $n$) by a definition of any operand of $e$}

$Kill_n$ = { $e$ | basic block $n$ contains a definition of an operand of $e$}

| | Entity | Manipulation | Exposition |
|---|---|---|---|
| $Gen_n$ | Expression | Use | Upwards |
| $Kill_n$ | Expression | Modification | Anywhere |





# Data Flow Equations for Anticipable Expressions Analysis

$$In_n = Gen_n \cup (Out_n - Kill_n)$$

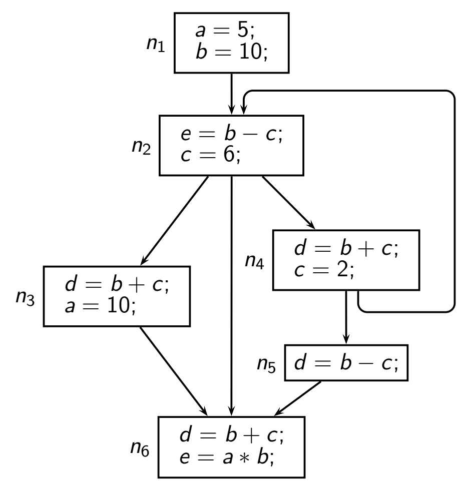
$$Out_n = \begin{cases} BI & n \text{ is } End \text{ block} \\ \bigcap_{s \in succ(n)} In_s & \text{otherwise} \end{cases}$$

$In_n$ and $Out_n$ are sets of expressions





# Tutorial Problem 1 for Anticipable Expressions Analysis




$\mathbb{E}$xpr = { $a * b$, $b + c$, $b - c$ }

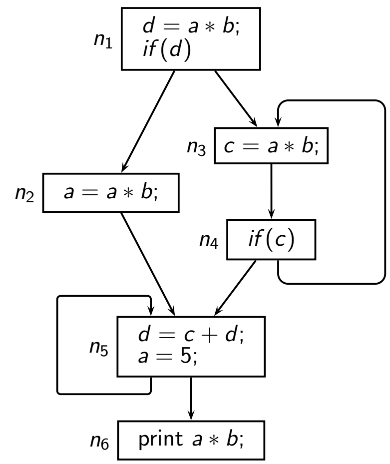
## Solution of Tutorial Problem 1

| Block | Local Information | | Global Information | | | |
|---|---|---|---|---|---|---|
| | | | Iteration # 1 | | Change in iteration # 2 | |
| | $Gen_n$ | $Kill_n$ | $Out_n$ | $In_n$ | $Out_n$ | $In_n$ |
| $n_6$ | 110 | 000 | 000 | 110 | | |
| $n_5$ | 001 | 000 | 110 | 111 | | |
| $n_4$ | 010 | 011 | 111 | 110 | 001 | 010 |
| $n_3$ | 010 | 100 | 110 | 010 | | |
| $n_2$ | 001 | 011 | 010 | 001 | | |
| $n_1$ | 000 | 111 | 001 | 000 | | |





## Tutorial Problem 2 for Anticipable Expressions Analysis




$\mathbb{E}$xpr = { $a * b$, $c + d$ }





## Solution of Tutorial Problem 2

| Block | Local Information | | Global Information | | | |
|---|---|---|---|---|---|---|
| | | | Iteration # 1 | | Change in iteration # 2 | |
| | $Gen_n$ | $Kill_n$ | $Out_n$ | $In_n$ | $Out_n$ | $In_n$ |
| $n_6$ | 10 | 00 | 00 | 10 | | |
| $n_5$ | 01 | 11 | 10 | 01 | 00 | |
| $n_4$ | 00 | 00 | 01 | 01 | 00 | 00 |
| $n_3$ | 10 | 01 | 01 | 10 | 00 | |
| $n_2$ | 10 | 10 | 01 | 11 | | |
| $n_1$ | 10 | 01 | 10 | 10 | | |



*Part 7*

# *Common Features of Bit Vector Data Flow Frameworks*

## Defining Local Data Flow Properties

- Live variables analysis

| | Entity | Manipulation | Exposition |
|---|---|---|---|
| $Gen_n$ | Variable | Use | Upwards |
| $Kill_n$ | Variable | Modification | Anywhere |

- Analysis of expressions

| | Entity | Manipulation | Exposition | |
|---|---|---|---|---|
| | | | Availability | Anticipability |
| $Gen_n$ | Expression | Use | Downwards | Upwards |
| $Kill_n$ | Expression | Modification | Anywhere | Anywhere |





## Common Form of Data Flow Equations

Data Flow Information: So far we have seen sets (or bit vectors). Could be entities other than sets.

Flow Function: So far we have seen constant *Gen* and *Kill*. Could be dependent *Gen* and *Kill*.

$$X_i = f(Y_i)$$

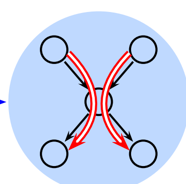
$$Y_i = \sqcap X_j$$

Confluence: So far we have seen ∪ and ∩. Could be other operations.



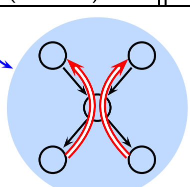


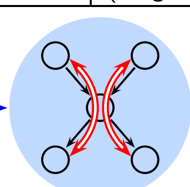
## A Taxonomy of Bit Vector Data Flow Frameworks



Any Path → Union; All Paths → Intersection

| | Confluence | |
|---|---|---|
| | Union | Intersection |
| Forward | Reaching Definitions | Available Expressions |
| Backward | Live Variables | Anticipable Expressions |
| Bidirectional (limited) | | Partial Redundancy Elimination (Original M-R Formulation) |

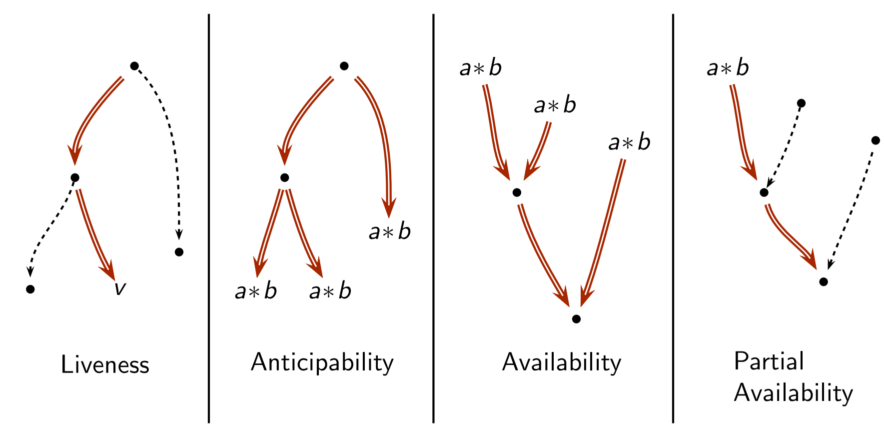




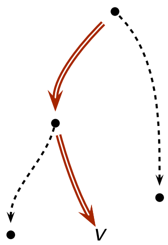




## Data Flow Paths Discovered by Data Flow Analysis



## Data Flow Paths Discovered by Data Flow Analysis




Liveness

Sequence of blocks $(n_1, n_2, \ldots, n_k)$ which is a prefix of some potential execution path starting at $n_1$ such that:

- $n_k$ contains an upwards exposed use of $v$, **and**
- no other block on the path contains an assignment to $v$.



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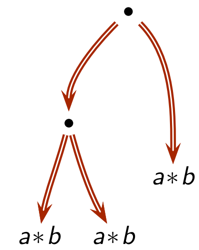
Data Flow Paths Discovered by Data Flow Analysis




Anticipability

Sequence of blocks $(n_1, n_2, \ldots, n_k)$ which is a prefix of some potential execution path starting at $n_1$ such that:

- $n_k$ contains an upwards exposed use of $a * b$, **and**
- no other block on the path contains an assignment to $a$ or $b$, **and**
- every path starting at $n_1$ is an anticipability path of $a * b$.

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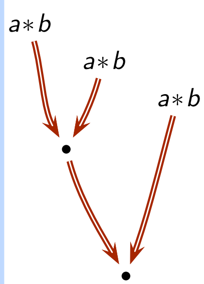
Data Flow Paths Discovered by Data Flow Analysis

Sequence of blocks $(n_1, n_2, \ldots, n_k)$ which is a prefix of some potential execution path starting at $n_1$ such that:

- $n_1$ contains a downwards exposed use of $a * b$, **and**
- no other block on the path contains an assignment to $a$ or $b$, **and**
- every path ending at $n_k$ is an availability path of $a * b$.




Availability

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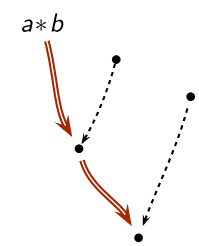
Data Flow Paths Discovered by Data Flow Analysis

Sequence of blocks $(n_1, n_2, \ldots, n_k)$ which is a prefix of some potential execution path starting at $n_1$ such that:

- $n_1$ contains a downwards exposed use of $a * b$, **and**
- no other block on the path contains an assignment to $a$ or $b$.




Partial Availability

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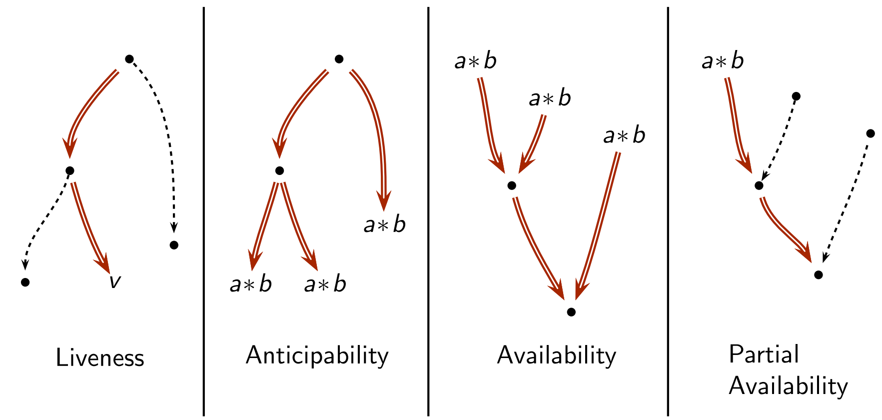
# Data Flow Paths Discovered by Data Flow Analysis






*Part 9*

# *Partial Redundancy Elimination*

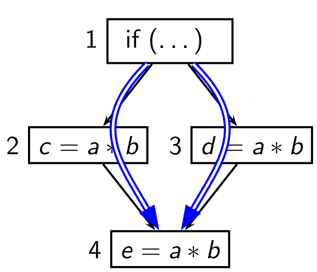


## Precursor: Common Subexpression Elimination

**Code Fragment**

```
if (...)
    c = a*b;
else
    d = a*b;
e = a*b;
```

**Flow Graph**




**Remarks**

- $a$ and $b$ are not modified along paths $1 \rightarrow 2 \rightarrow 4$ and $1 \rightarrow 3 \rightarrow 4$
- Computation of $a * b$ in 4 is redundant
- Previous value can be used



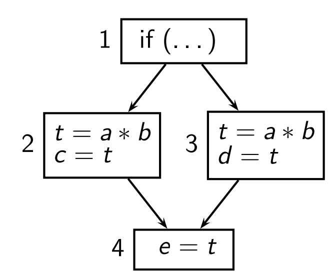


## Precursor: Common Subexpression Elimination

**Code Fragment**

```
if (...)
    c = a*b;
else
    d = a*b;
e = a*b;
```

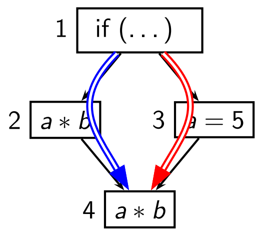
**Flow Graph**




**Remarks**

- $a$ and $b$ are not modified along paths $1 \rightarrow 2 \rightarrow 4$ and $1 \rightarrow 3 \rightarrow 4$
- Computation of $a * b$ in 4 is redundant
- Previous value can be used

## Partial Redundancy Elimination

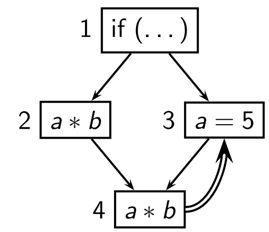



- Computation of $a * b$ in 4 is
  - redundant along path $1 \rightarrow 2 \rightarrow 4$, but ...
  - not redundant along path $1 \rightarrow 3 \rightarrow 4$





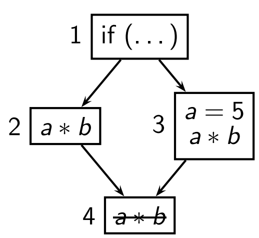
## Code Hoisting for Partial Redundancy Elimination








## Code Hoisting for Partial Redundancy Elimination



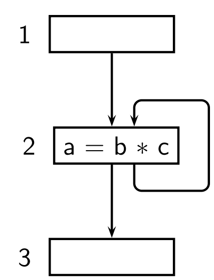

- Computation of $a * b$ in 3 becomes totally redundant
- Can be deleted



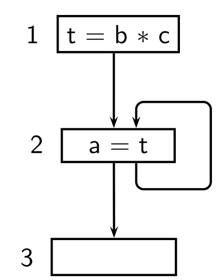


## PRE Subsumes Loop Invariant Movement

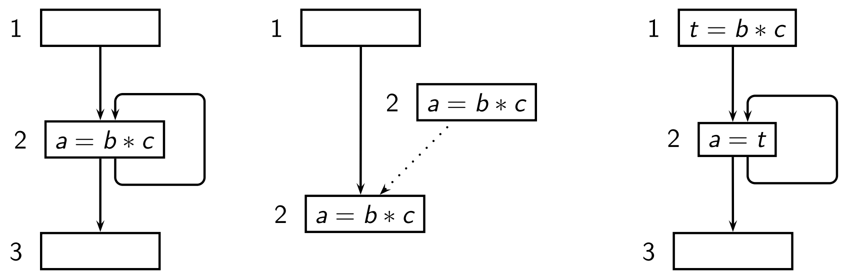
What's that?




Translate to



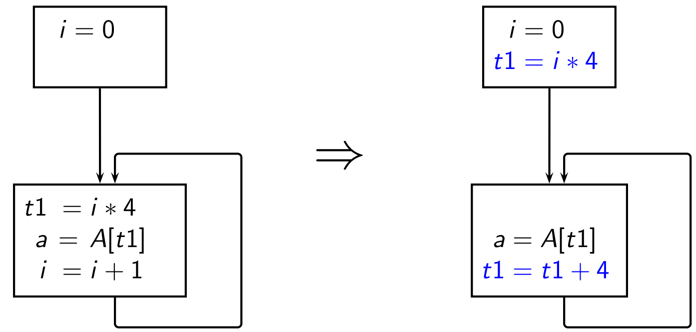
# PRE Subsumes Loop Invariant Movement








# PRE Can be Used for Strength Reduction

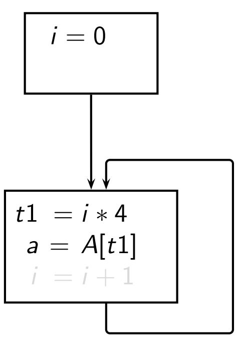



- $*$ in the loop has been replaced by $+$
- $i = i + 1$ in the loop has been eliminated





# PRE Can be Used for Strength Reduction

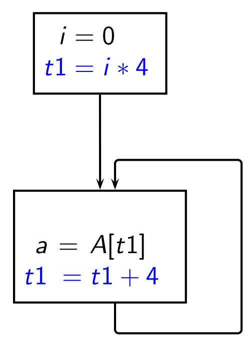



- Delete $i = i + 1$
- Expression $i * 4$ becomes loop invariant



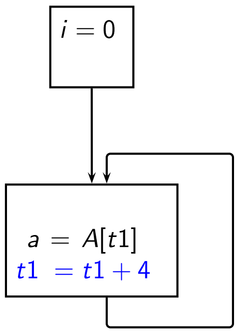


# PRE Can be Used for Strength Reduction




- Delete $i = i + 1$
- Expression $i * 4$ becomes loop invariant
- Hoist it and increment $t1$ in the loop

## PRE Can be Used for Strength Reduction




- $*$ in the loop has been replaced by $+$
- $i = i + 1$ in the loop has been eliminated





## Performing Partial Redundancy Elimination

1. Identify partial redundancies
2. Identify program points where computations can be inserted
3. Insert expressions
4. Partial redundancies become total redundancies
   $\Longrightarrow$ Delete them.

Morel-Renvoise Algorithm (*CACM*, 1979.)

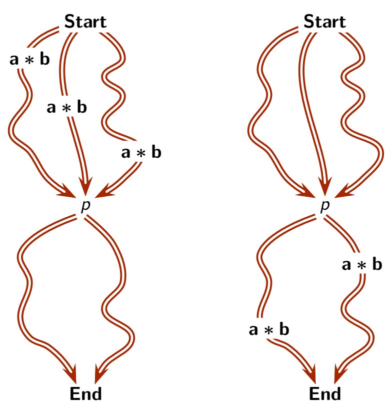




## Defining Hoisting Criteria

- An expression can be safely inserted at a program point $p$ if it is

Available at $p$ Anticipable at $p$

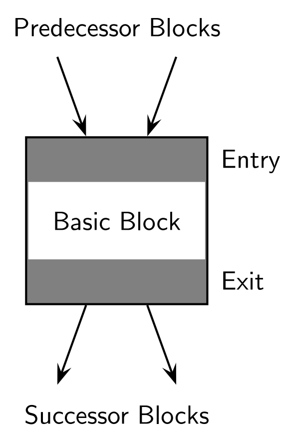



- If it is available at $p$, then there is no need to insert it at $p$.
- If it is anticipable at $p$ then all such occurrences should be hoisted to $p$.
- *An expression should be hoisted to $p$ provided it can be hoisted to $p$ along all paths from $p$ to exit.*

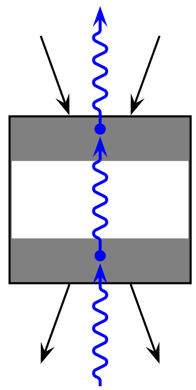




## Safety of Hoisting an Expression



## Safety of Hoisting an Expression

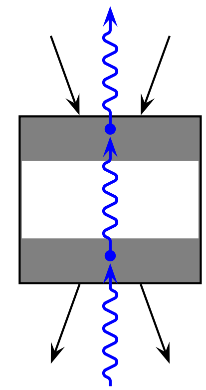


- *Safety of hoisting to the exit of a block*
  - S.1 Hoist only if it can be hoisted out of the entries of all successor blocks
- *Safety of hoisting to the entry of a block*
  - S.2 Hoist only if
    - S.2.a it is upwards exposed, or
    - S.2.b it can be hoisted to its exit and is transparent in the block
- *Safety of hoisting out of the entry of a block*
  - S.3 Hoist only if for each predecessor
    - S.3.a it can be hoisted to its exit, or
    - S.3.b it is available at its exit.





## Safety of Hoisting an Expression



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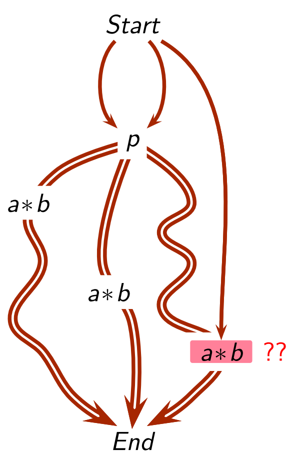
**Anticipability**

**??**





## Anticipability and Code Hoisting



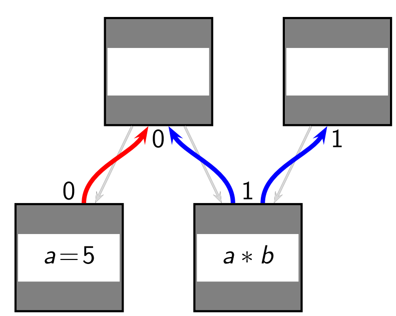

- What is the meaning of the assertion
  *"$a * b$ is anticipable at program point p"*
  - $a * b$ is computed along every path from p to *End* before a or b are modified
  - The value computed at p would be same as the next value computed on any path
  - $a * b$ can be safely inserted at p
- It does not say that the subsequent computations of $a * b$ can be deleted
  (Expression may not be available at the subsequent points)
- Hoisting involves
  - making the expressions available and
  - deleting their subsequent computations





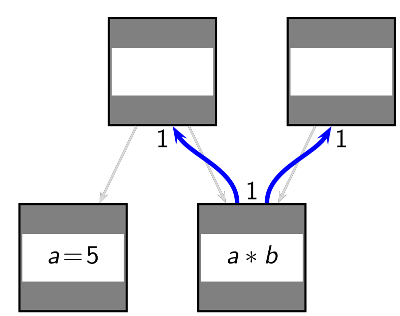
## A Comparison of Anticipability and Hoistability

Anticipability



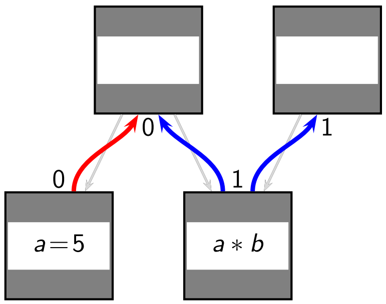

Characterises safety of placement but not safety of hoisting

Hoistability



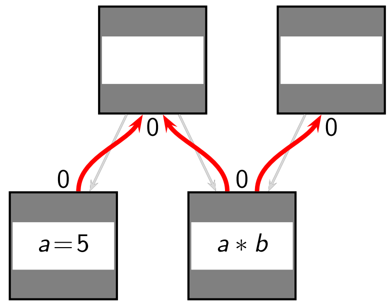
## A Comparison of Anticipability and Hoistability

Anticipability




Characterises safety of placement but not safety of hoisting

Hoistability



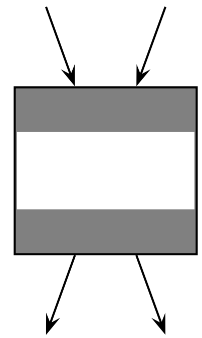

Characterises safety of hoisting

*Hoist an expression to the entry of a block only if it can be hoisted out of the block into all predecessor blocks*





## Revised Safety Criteria of Hoisting an Expression

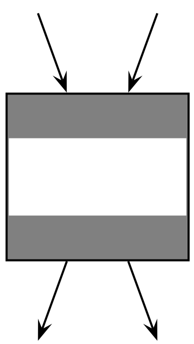


- *Safety of hoisting to the exit of a block*
  - S.1 Hoist only if it can be hoisted out of the entries of all successor blocks
- *Safety of hoisting to the entry of a block*
  - S.2 Hoist only if
    - S.2.a it is upwards exposed, or
    - S.2.b it can be hoisted to its exit and is transparent in the block
- *Safety of hoisting out of the entry of a block*
  - S.3 Hoist only if for each predecessor
    - S.3.a it can be hoisted to its exit, or
    - S.3.b it is available at its exit.





## Revised Safety Criteria of Hoisting an Expression



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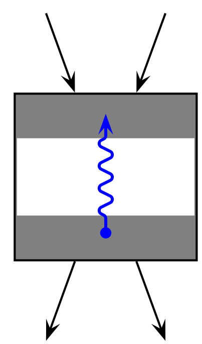
  - S.3 Hoist only if for each predecessor
    - S.3.a it can be hoisted to its exit, or
    - S.3.b it is available at its exit.





## Desirability of Hoisting an Expression



- *Desirability of hoisting to the entry of a block*
  - D.1 Hoist only if it is partially available

## Final Hoisting Criteria

- *Safety of hoisting to the exit of a block*
  - S.1 Hoist only if it can be hoisted out of the entries of all successor blocks
- *Safety of hoisting to the entry of a block*
  - S.2 Hoist only if
    - S.2.a it is upwards exposed, or
    - S.2.b it can be hoisted to its exit and is transparent in the block
  - S.3 Hoist only if for each predecessor
    - S.3.a it can be hoisted to its exit, or
    - S.3.b it is available at its exit.
- *Desirability of hoisting to the entry of a block*
  - D.1 Hoist only if it is partially available





## From Hoisting Criteria to Data Flow Equations (1)

First Level Global Data Flow Properties in PRE

- Partial Availability.

$$PavIn_n = \begin{cases} BI & n \text{ is } Start \text{ block} \\ \bigcup_{p \in pred(n)} PavOut_p & \text{otherwise} \end{cases}$$

$$PavOut_n = Gen_n \cup (PavIn_n - Kill_n)$$

- Total Availability.

$$AvIn_n = \begin{cases} BI & n \text{ is } Start \text{ block} \\ \bigcap_{p \in pred(n)} AvOut_p & \text{otherwise} \end{cases}$$

$$AvOut_n = Gen_n \cup (AvIn_n - Kill_n)$$





## From Hoisting Criteria to Data Flow Equations (2)

- *Safety of hoisting to the exit of a block*
  - S.1 Hoist only if it can be hoisted out of the entries of all successor blocks

$\forall s \in succ(n),\ Out_n \subseteq In_s$

- *Safety of hoisting to the entry of a block*
  - S.2 Hoist only if
    - S.2.a it is upwards exposed, or
    - S.2.b it can be hoisted to its exit and is transparent in the block

$In_n \subseteq AntGen_n \cup (Out_n - Kill_n)$

  - S.3 Hoist only if for each predecessor
    - S.3.a it can be hoisted to its exit, or
    - S.3.b it is available at its exit.

$\forall p \in pred(n),\ In_n \subseteq AvOut_p \cup Out_p$

- *Desirability of hoisting to the entry of a block*
  - D.1 Hoist only if it is partially available

$In_n \subseteq PavIn_n$





## From Hoisting Criteria to Data Flow Equations (3)

$\forall s \in succ(n),\ Out_n \subseteq In_s$

$In_n \subseteq AntGen_n \cup (Out_n - Kill_n)$

$\forall p \in pred(n),\ In_n \subseteq AvOut_p \cup Out_p$

$In_n \subseteq PavIn_n$

Find out the largest such set

# From Hoisting Criteria to Data Flow Equations (3)

$\forall s \in succ(n),$
$Out_n \subseteq In_s$

$In_n \subseteq AntGen_n \cup (Out_n - Kill_n)$

$\forall p \in pred(n),$
$In_n \subseteq AvOut_p \cup Out_p$

$In_n \subseteq PavIn_n$

Desirability: D.1

$In_n \;=\; PavIn_n$

Expressions should be partially available, and





# From Hoisting Criteria to Data Flow Equations (3)

$\forall s \in succ(n),$
$Out_n \subseteq In_s$

$In_n \subseteq AntGen_n \cup (Out_n - Kill_n)$

$\forall p \in pred(n),$
$In_n \subseteq AvOut_p \cup Out_p$

$In_n \subseteq PavIn_n$

Safety: S.2.a

$In_n \;=\; PavIn_n \cap \Big( AntGen_n \cup \qquad \Big)$

Expressions should be upwards exposed, or





# From Hoisting Criteria to Data Flow Equations (3)

$\forall s \in succ(n),$
$Out_n \subseteq In_s$

$In_n \subseteq AntGen_n \cup (Out_n - Kill_n)$

$\forall p \in pred(n),$
$In_n \subseteq AvOut_p \cup Out_p$

$In_n \subseteq PavIn_n$

Safety: S.2.b

$In_n \;=\; PavIn_n \cap \Big( AntGen_n \cup (Out_n - Kill_n) \Big)$

Expressions can be hoisted to the exit and are transparent in the block





# From Hoisting Criteria to Data Flow Equations (3)

$\forall s \in succ(n),$
$Out_n \subseteq In_s$

$In_n \subseteq AntGen_n \cup (Out_n - Kill_n)$

$\forall p \in pred(n),$
$In_n \subseteq AvOut_p \cup Out_p$

$In_n \subseteq PavIn_n$

Safety: S.3.b

$$In_n \;=\; PavIn_n \cap \Big( AntGen_n \cup (Out_n - Kill_n) \Big) \bigcap_{p \in pred(n)} \Big( Out_p \cup \qquad \Big)$$

For every predecessor, expressions can be hoisted to its exit, or

## From Hoisting Criteria to Data Flow Equations (3)

$\forall s \in succ(n),$
$Out_n \subseteq In_s$

$In_n \subseteq AntGen_n \cup (Out_n - Kill_n)$

$\forall p \in pred(n),$
$In_n \subseteq AvOut_p \cup Out_p$

$In_n \subseteq PavIn_n$

Safety: S.3.a

$$In_n = PavIn_n \cap \Big( AntGen_n \cup (Out_n - Kill_n) \Big) \bigcap_{p \in pred(n)} \Big( Out_p \cup AvOut_p \Big)$$

. . . expressions are available at the exit of the same predecessor



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## From Hoisting Criteria to Data Flow Equations (3)

$\forall s \in succ(n),$
$Out_n \subseteq In_s$

$In_n \subseteq AntGen_n \cup (Out_n - Kill_n)$

$\forall p \in pred(n),$
$In_n \subseteq AvOut_p \cup Out_p$

$In_n \subseteq PavIn_n$

$$In_n = PavIn_n \cap \Big( AntGen_n \cup (Out_n - Kill_n) \Big) \bigcap_{p \in pred(n)} \Big( Out_p \cup AvOut_p \Big)$$

$$Out_n = \begin{cases} BI & n \text{ is } End \text{ block} \\ & \text{otherwise} \end{cases}$$

Boundary condition

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## From Hoisting Criteria to Data Flow Equations (3)

$\forall s \in succ(n),$
$Out_n \subseteq In_s$

$In_n \subseteq AntGen_n \cup (Out_n - Kill_n)$

$\forall p \in pred(n),$
$In_n \subseteq AvOut_p \cup Out_p$

$In_n \subseteq PavIn_n$

Safety: S.1

$$In_n = PavIn_n \cap \Big( AntGen_n \cup (Out_n - Kill_n) \Big) \bigcap_{p \in pred(n)} \Big( Out_p \cup AvOut_p \Big)$$

$$Out_n = \begin{cases} BI & n \text{ is } End \text{ block} \\ \bigcap_{s \in succ(n)} In_s & \text{otherwise} \end{cases}$$

Expressions should be hoisted to the exit of a block if they can be hoisted to the entry of all successors

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## From Hoisting Criteria to Data Flow Equations (3)

$\forall s \in succ(n),$
$Out_n \subseteq In_s$

$In_n \subseteq AntGen_n \cup (Out_n - Kill_n)$

$\forall p \in pred(n),$
$In_n \subseteq AvOut_p \cup Out_p$

$In_n \subseteq PavIn_n$

$$In_n = PavIn_n \cap \Big( AntGen_n \cup (Out_n - Kill_n) \Big) \bigcap_{p \in pred(n)} \Big( Out_p \cup AvOut_p \Big)$$

$$Out_n = \begin{cases} BI & n \text{ is } End \text{ block} \\ \bigcap_{s \in succ(n)} In_s & \text{otherwise} \end{cases}$$

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## Anticipability and PRE (Hoistability) Data Flow Equations

| PRE Hoistability | Anticipability |
|---|---|
| $In_n = PavIn_n \cap (AntGen_n \cup (Out_n - Kill_n)) \bigcap_{p \in pred(n)} (Out_p \cup AvOut_p)$ | $In_n = AntGen_n \cup (Out_n - Kill_n)$ |
| $Out_n = \begin{cases} BI & n \text{ is } End \text{ block} \\ \bigcap_{s \in succ(n)} In_s & \text{otherwise} \end{cases}$ | $Out_n = \begin{cases} BI & n \text{ is } End \text{ block} \\ \bigcap_{s \in succ(n)} In_s & \text{otherwise} \end{cases}$ |

*PRE Hoistability is anticipability restricted by*
- *safety of hoisting and*
- *partial availability*





## Deletion Criteria in PRE

- An expression is redundant in node $n$ if
  - it can be placed at the entry (i.e. can be "hoisted" out) of $n$, AND
  - it is upwards exposed in node $n$.

$$Redundant_n = In_n \cap AntGen_n$$

- A hoisting path for an expression $e$ begins at $n$ if $e \in Redundant_n$
- This hoisting path extends against the control flow.





## Insertion Criteria in PRE

- An expression is inserted at the exit of node $n$ is
  - it can be placed at the exit of $n$, AND
  - it is not available at the exit of $n$, AND
  - it cannot be hoisted out of $n$, OR it is modified in $n$.

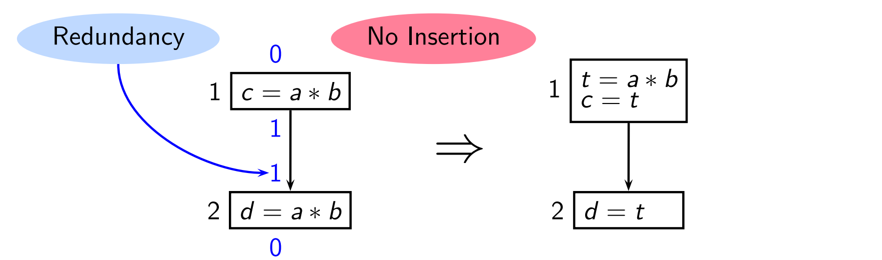
$$Insert_n = Out_n \cap (\neg AvOut_n) \cap (\neg In_n \cup Kill_n)$$

- A hoisting path for an expression $e$ ends at $n$ if $e \in Insert_n$



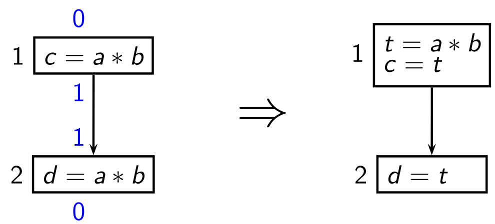


## Performing PRE by Computing *In*/*Out*: Simple Cases (1)




| Node | First Level Values | | | | Init. | | Iter. 1 | | Iter. 2 | | Redund. | Insert |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *AntGen* | *Kill* | *PavIn* | *AvOut* | *Out* | *In* | *Out* | *In* | *Out* | *In* | | |
| 2 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |

## Performing PRE by Computing *In*/*Out*: Simple Cases (1)

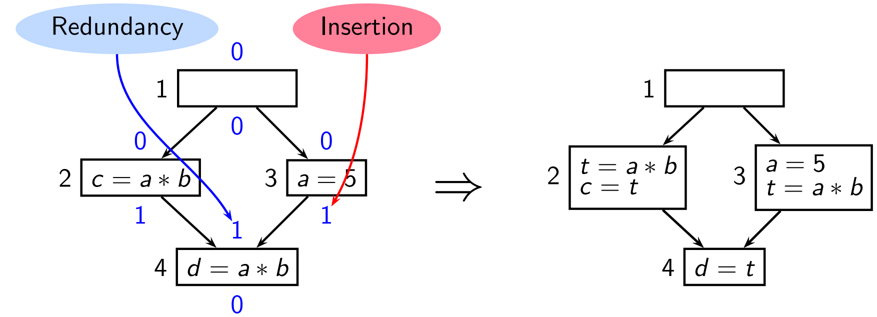



*This is an instance of Common Subexpression Elimination*





## Performing PRE by Computing *In*/*Out*: Simple Cases (2)

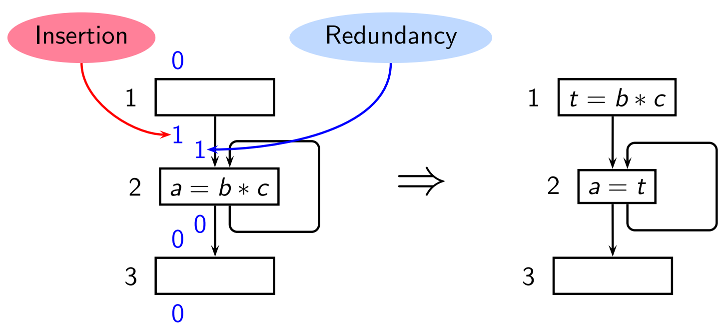



| Node | First Level Values | | | | Init. | | Iter. 1 | | Iter. 2 | | Redund. | Insert |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *AntGen* | *Kill* | *PavIn* | *AvOut* | *Out* | *In* | *Out* | *In* | *Out* | *In* | | |
| 4 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 3 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 2 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |





## Performing PRE by Computing *In*/*Out*: Simple Cases (3)

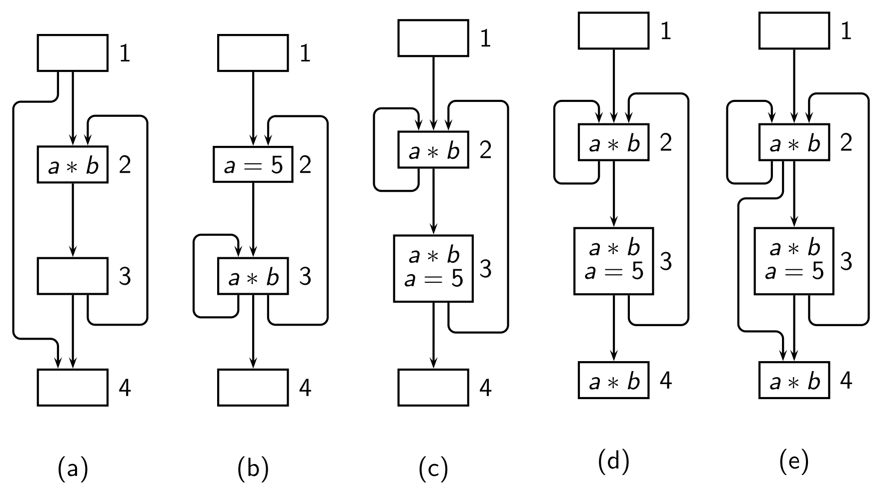



| Node | First Level Values | | | | Init. | | Iter. 1 | | Iter. 2 | | Redund. | Insert |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *AntGen* | *Kill* | *PavIn* | *AvOut* | *Out* | *In* | *Out* | *In* | *Out* | *In* | | |
| 3 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |

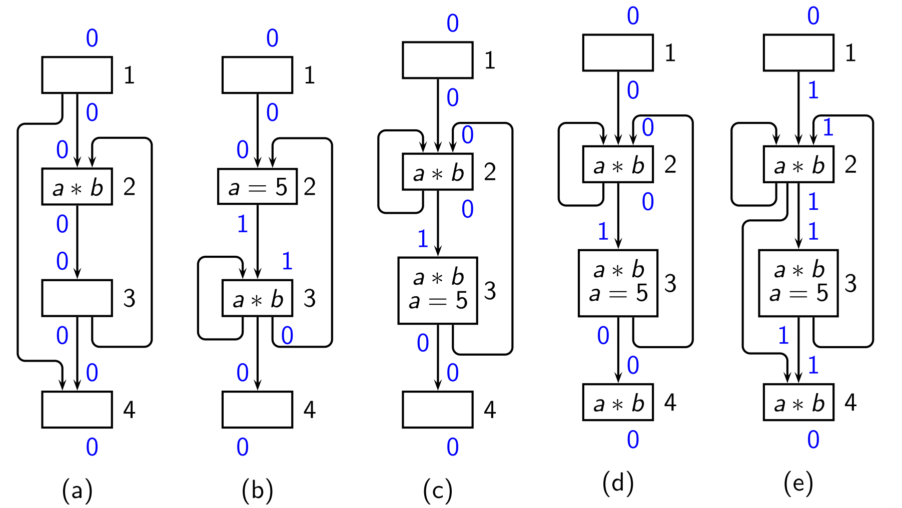




## Tutorial Problems for PRE



# Tutorial Problems for PRE



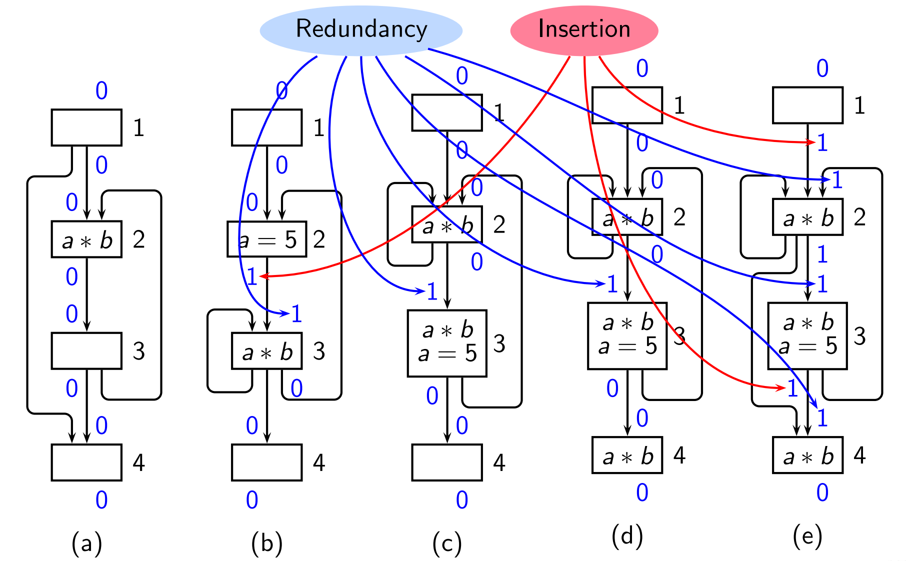







# Tutorial Problems for PRE

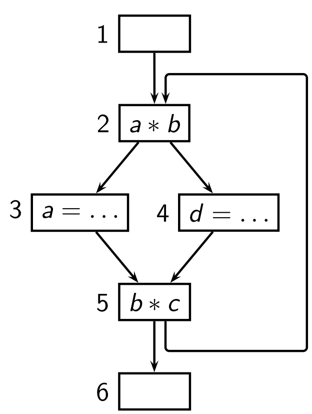









# Further Tutorial Problem for PRE




Let $\{a * b, b * c\} \equiv$ bit string 11

| Node $n$ | $Kill_n$ | $AntGen_n$ | $PavIn_n$ | $AvOut_n$ |
|---|---|---|---|---|
| 1 | 00 | 00 | 00 | 00 |
| 2 | 00 | 10 | 11 | 10 |
| 3 | 10 | 00 | 11 | 00 |
| 4 | 00 | 00 | 11 | 10 |
| 5 | 00 | 01 | 11 | 01 |
| 6 | 00 | 00 | 11 | 01 |

- Compute $In_n / Out_n / Redundant_n / Insert_n$
- Identify hoisting paths



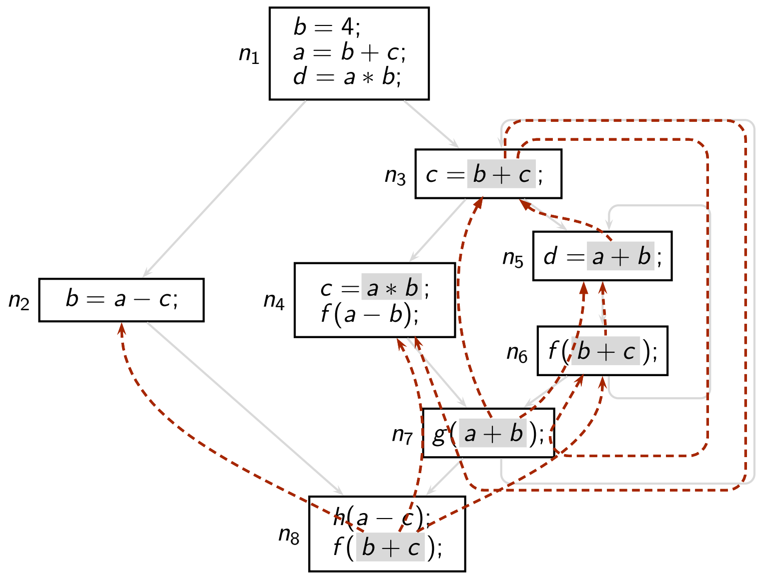


# Result of PRE Data Flow Analysis of the Running Example

| Bit vector | $a * b$ | $a + b$ | $a - b$ | $a - c$ | $b + c$ |
|---|---|---|---|---|---|

| Block | Global Information | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Constant information | | Iteration # 1 | | Changes in iteration # 2 | | Changes in iteration # 3 | |
| | $PavIn_n$ | $AvOut_n$ | $Out_n$ | $In_n$ | $Out_n$ | $In_n$ | $Out_n$ | $In_n$ |
| $n_8$ | 11111 | 00011 | 00000 | 00011 | | | | 00001 |
| $n_7$ | 11101 | 11000 | 00011 | 01001 | 00001 | | | |
| $n_6$ | 11101 | 11001 | 01001 | 01001 | | | 01000 | |
| $n_5$ | 11101 | 11000 | 01001 | 01001 | | 01000 | | |
| $n_4$ | 11100 | 10100 | 01001 | 11100 | | 11000 | | |
| $n_3$ | 11101 | 10000 | 01000 | 01001 | | 00001 | | |
| $n_2$ | 10001 | 00010 | 00011 | 00000 | | | 00001 | |
| $n_1$ | 00000 | 10001 | 00000 | 00000 | | | | |

# Hoisting Paths for Some Expressions in the Running Example








# Optimized Version of the Running Example