Introduction to Program Analysis

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July 2017

Part 1

About These Slides

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These slides constitute the lecture notes for CS618 Program Analysis course at IIT Bombay and have been made available as teaching material accompanying the book:

- Uday Khedker, Amitabha Sanyal, and Bageshri Karkare. *Data Flow Analysis: Theory and Practice*. CRC Press (Taylor and Francis Group). 2009.
  (Indian edition published by Ane Books in 2013)

Apart from the above book, some slides are based on the material from the following books

- A. V. Aho, M. Lam, R. Sethi, and J. D. Ullman. *Compilers: Principles, Techniques, and Tools*. Addison-Wesley. 2006.
- M. S. Hecht. *Flow Analysis of Computer Programs*. Elsevier North-Holland Inc. 1977.

Motivating the Need of Program Analysis

- Some representative examples
  - Classical optimizations performed by compilers
  - Optimizing heap memory usage
- Course details, schedule, assessment policies etc.
- Program Model
- Soundness and Precision

Part 2

Classical Optimizations

Examples of Optimising Transformations (ALSU, 2006)

A C program and its optimizations

```
void quicksort(int m, int n)
{  int i, j, v, x;
   if (n <= m) return;
     i = m-1; j = n; v = a[n];                   /* v is the pivot  */
     while(1)                                    /* Move values smaller  */
     {  do i = i + 1; while (a[i] < v);          /* than v to the left of  */
        do j = j - 1; while (a[j] > v);          /* the split point (sp)  */
        if (i >= j) break;                       /* and other values  */
        x = a[i]; a[i] = a[j]; a[j] = x;         /* to the right of sp  */
     }                                           /* of the split point  */
     x = a[i]; a[i] = a[n]; a[n] = x;            /* Move the pivot to sp  */
   quicksort(m,i); quicksort(i+1,n);             /* sort the partitions to  */
}         /* the left of sp and to the right of sp independently  */
```


Intermediate Code

For the boxed source code

1. i = m - 1	12. t5 = a[t4]	23. a[t4] = x
2. j = n	13. if t5 > v goto 10	24. goto 6
3. t1 = 4 ∗ n	14. if i >= j goto 25	25. t2 = 4 ∗ i
4. t6 = a[t1]	15. t2 = 4 ∗ i	26. t3 = a[t2]
5. v = t6	16. t3 = a[t2]	27. x = t3
6. i = i + 1	17. x = t3	28. t2 = 4 ∗ i
7. t2 = 4 ∗ i	18. t2 = 4 ∗ i	29. t1 = 4 ∗ n
8. t3 = a[t2]	19. t4 = 4 ∗ j	30. t6 = a[t1]
9. if t3 < v goto 6	20. t5 = a[t4]	31. a[t2] = t6
10. j = j - 1	21. a[t2] = t5	32. t1 = 4 ∗ n
11. t4 = 4 ∗ j	22. t4 = 4 ∗ j	33. a[t1] = x

Intermediate Code : Observations

- Multiple computations of expressions
- Simple control flow (conditional/unconditional goto)
  Yet undecipherable!
- Array address calculations

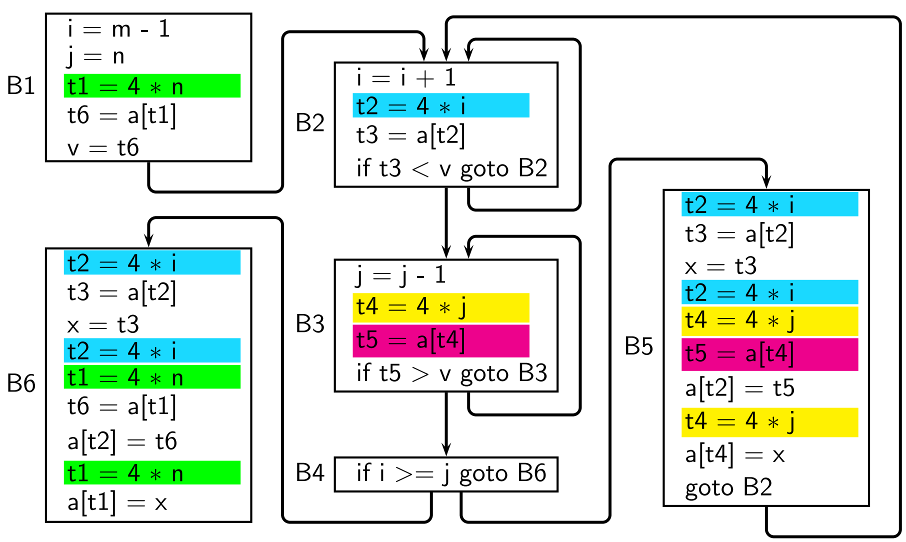
Understanding Control Flow

- Identify maximal sequences of linear control flow
  ⇒ Basic Blocks
- No transfer into or out of basic blocks except the first and last statements
  Control transfer into the block : only at the first statement.
  Control transfer out of the block : only at the last statement.

Intermediate Code with Basic Blocks

```
1.  i = m - 1
2.  j = n
3.  t1 = 4 * n
4.  t6 = a[t1]
5.  v = t6

6.  i = i + 1
7.  t2 = 4 * i
8.  t3 = a[t2]
9.  if t3 < v goto 6

10. j = j - 1
11. t4 = 4 * j
12. t5 = a[t4]
13. if t5 > v goto 10

14. if i >= j goto 25

15. t2 = 4 * i
16. t3 = a[t2]
17. x = t3
18. t2 = 4 * i
19. t4 = 4 * j
20. t5 = a[t4]
21. a[t2] = t5
22. t4 = 4 * j
23. a[t4] = x
24. goto 6

25. t2 = 4 * i
26. t3 = a[t2]
27. x = t3
28. t2 = 4 * i
29. t1 = 4 * n
30. t6 = a[t1]
31. a[t2] = t6
32. t1 = 4 * n
33. a[t1] = x
```

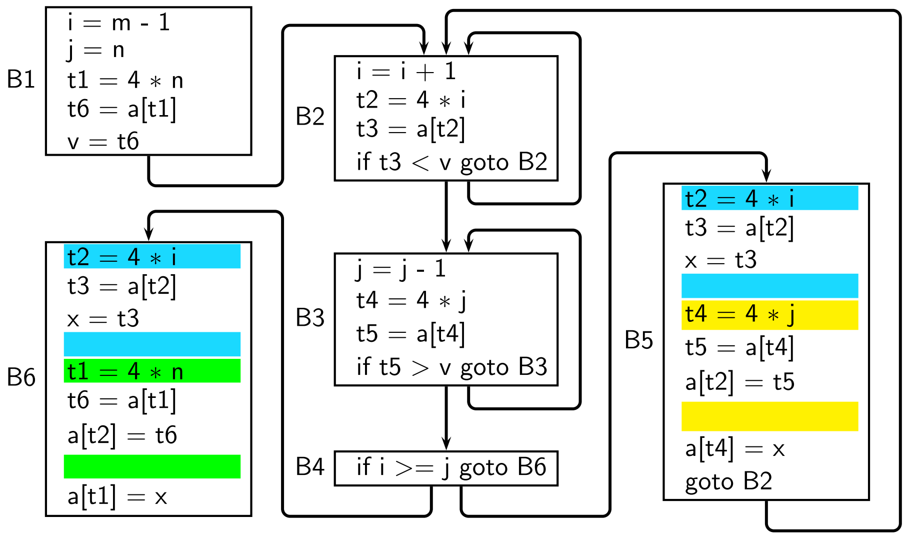

Program Flow Graph



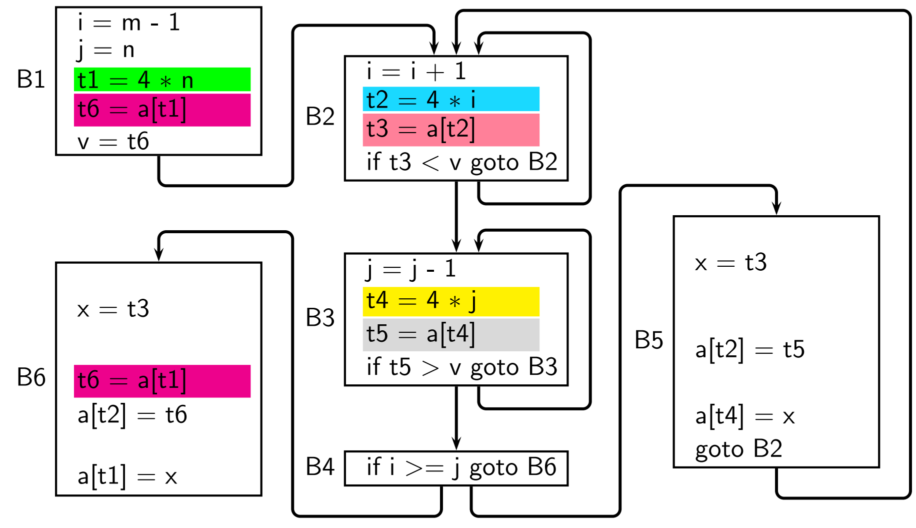
Program Flow Graph : Observations

Nesting Level	Basic Blocks	No. of Statements
0	B1, B6	14
1	B4, B5	11
2	B2, B3	8

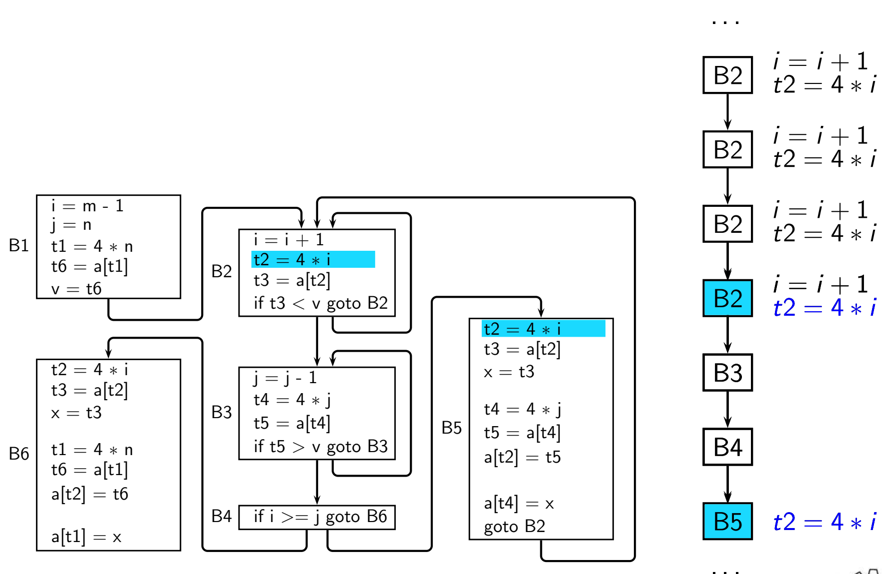
Local Common Subexpression Elimination



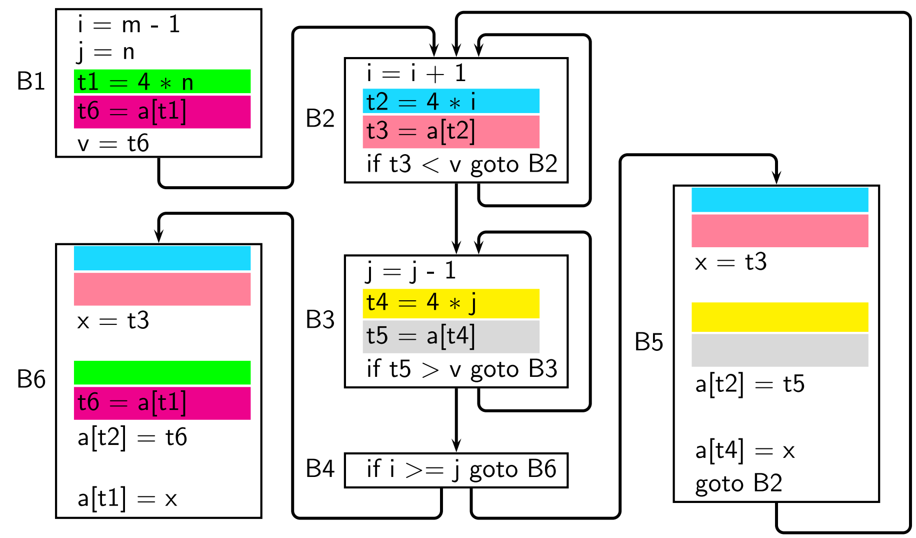
Global Common Subexpression Elimination



Global Common Subexpression Elimination



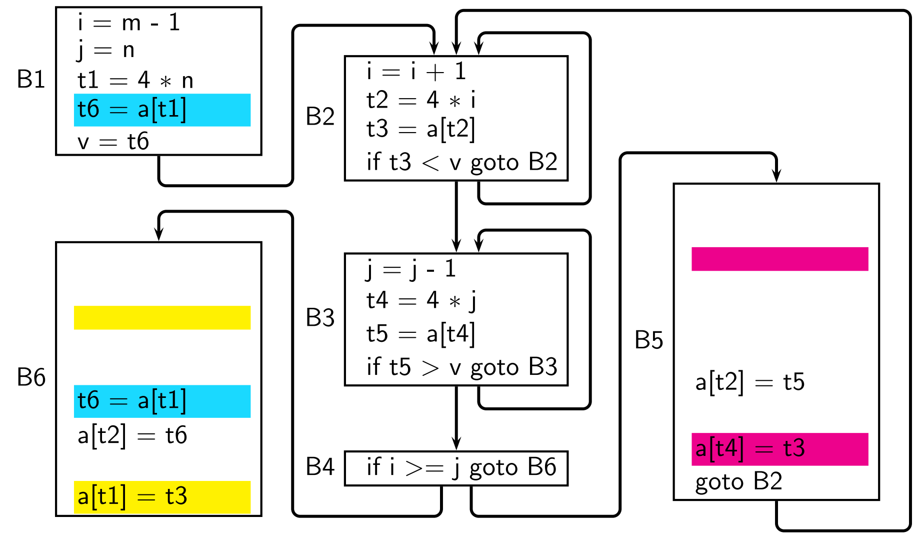
Global Common Subexpression Elimination



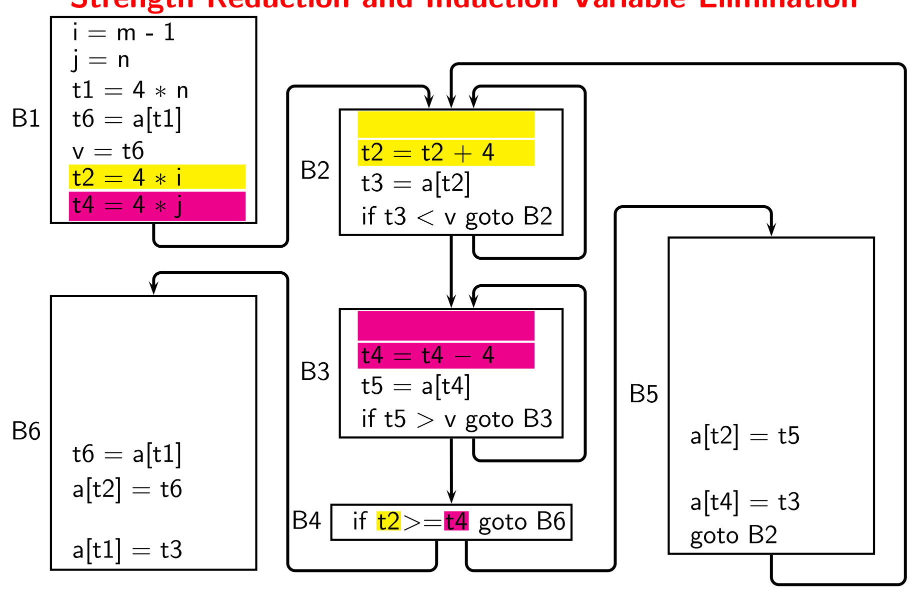
Other Classical Optimizations

- Copy propagation
- Strength Reduction
- Elimination of Induction Variables
- Dead Code Elimination

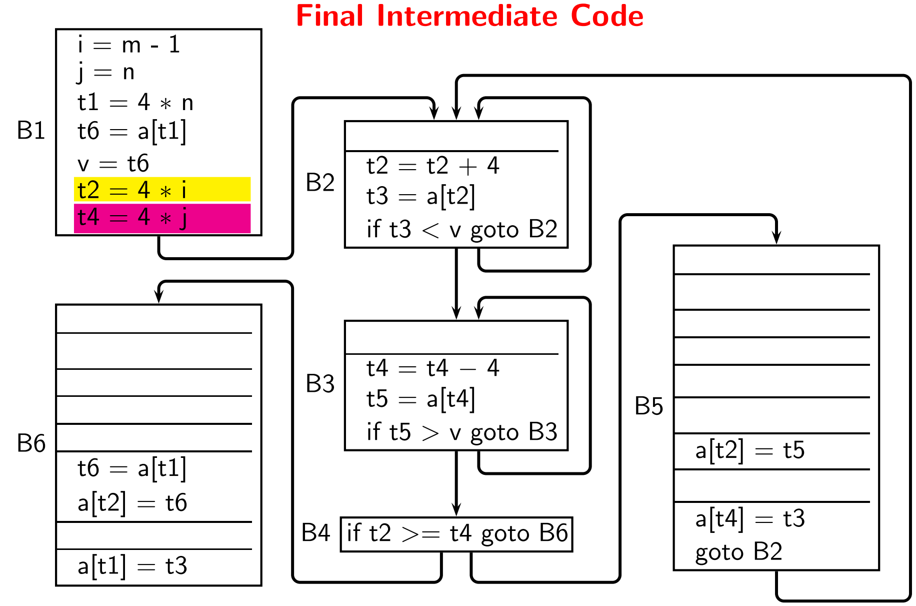
Copy Propagation and Dead Code Elimination



Strength Reduction and Induction Variable Elimination



Final Intermediate Code



Optimized Program Flow Graph

Nesting Level	No. of Statements	
	Original	Optimized
0	14	10
1	11	4
2	8	6

If we assume that a loop is executed 10 times, then the number of computations saved at run time

$$= (14 - 10) + (11 - 4) \times 10 + (8 - 6) \times 10^2 = 4 + 70 + 200 = 274$$

Observations

- Optimizations are transformations based on some information.
- Systematic analysis required for deriving the information.
- We have looked at data flow optimizations.
  Many control flow optimizations can also be performed.

Categories of Optimizing Transformations and Analyses

Code Motion Redundancy Elimination Control flow Optimization	Machine Independent	Flow Analysis (Data + Control)
Loop Transformations	Machine Dependent	Dependence Analysis (Data + Control)
Instruction Scheduling Register Allocation Peephole Optimization	Machine Dependent	Several Independent Techniques
Vectorization Parallelization	Machine Dependent	Dependence Analysis (Data + Control)

What is Program Analysis?

Discovering information about a given program

- Representing the dynamic behaviour of the program
- Most often obtained without executing the program
  - Static analysis Vs. Dynamic Analysis
  - Example of loop tiling for parallelization
- Must represent all execution instances of the program

Why is it Useful?

- Code optimization
  - Improving time, space, energy, or power efficiency
  - Compilation for special architecture (eg. multi-core)
- Verification and validation

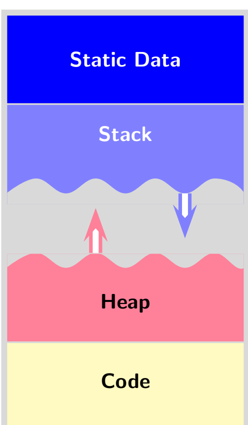
  Giving guarantees such as: The program will
  - never divide a number by zero
  - never dereference a NULL pointer
  - close all opened files, all opened socket connections
  - not allow buffer overflow security violation
- Software engineering
  - Maintenance, bug fixes, enhancements, migration
  - Example: Y2K problem
- Reverse engineering

  To understand the program

Part 3

Optimizing Heap Memory Usage

Standard Memory Architecture of Programs



Heap allocation provides the flexibility of

- *Variable Sizes.* Data structures can grow or shrink as desired at runtime.

  (Not bound to the declarations in program.)
- *Variable Lifetimes.* Data structures can be created and destroyed as desired at runtime.

  (Not bound to the activations of procedures.)

Managing Heap Memory

Decision 1: When to Allocate?

- Explicit. Specified in the programs. (eg. Imperative/OO languages)
- Implicit. Decided by the language processors. (eg. Declarative Languages)

Decision 2: When to Deallocate?

- Explicit. Manual Memory Management (eg. C/C++)
- Implicit. Automatic Memory Management aka Garbage Collection (eg. Java/Declarative languages)

State of Art in Manual Deallocation

- Memory leaks
  10% to 20% of last development effort goes in plugging leaks
- Tool assisted manual plugging
  Purify, Electric Fence, RootCause, GlowCode, yakTest, Leak Tracer, BDW Garbage Collector, mtrace, memwatch, dmalloc etc.
- All leak detectors
  - are dynamic (and hence specific to execution instances)
  - generate massive reports to be perused by programmers
  - usually do not locate last use but only allocation escaping a call
    $\Rightarrow$ At which program point should a leak be “plugged”?

Garbage Collection $\equiv$ Automatic Deallocation

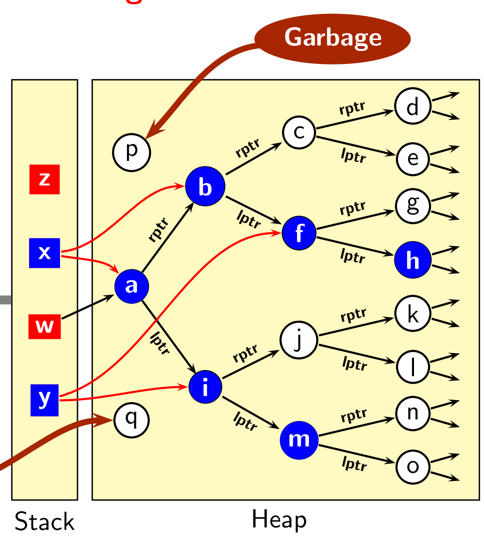
- Retain active data structure.
  Deallocate inactive data structure.
- What is an Active Data Structure?

  If an object does not have an access path, (i.e. it is unreachable)
  then its memory can be reclaimed.

What if an object has an access path, but is not accessed after the given program point?

What is Garbage?

```
w = x          // x points to m_a
if (x.data < max)
      x = x.rptr
y = x.lptr

z = New  class_of_z
y = y.lptr
z.sum = x.data + y.data
```



Garbage

All white nodes are unused and should be considered garbage

Is Reachable Same as Live?

From www.memorymanagement.org/glossary

live (also known as alive, active) : Memory(2) or an object is live if the program will read from it in future. *The term is often used more broadly to mean reachable.*

It is not possible, in general, for garbage collectors to determine exactly which objects are still live. Instead, they use some approximation to detect objects that are provably dead, *such as those that are not reachable.*

Similar terms: reachable. Opposites: dead. See also: undead.

Is Reachable Same as Live?

- Not really. Most of us know that.

  Even with the state of art of garbage collection, 24% to 76% unused memory remains unclaimed

- The state of art compilers, virtual machines, garbage collectors cannot distinguish between the two

Reachability and Liveness

Comparison between different sets of objects:

Live ? Reachable ? Allocated

The objects that are not live must be reclaimed.

Reachability and Liveness

Comparison between different sets of objects:

$$\text{Live} \subseteq \text{Reachable} \subseteq \text{Allocated}$$

The objects that are not live must be reclaimed.

Reachability and Liveness

Comparison between different sets of objects:

$$\text{Live} \subseteq \text{Reachable} \subseteq \text{Allocated}$$

The objects that are not live must be reclaimed.

$\neg$ Live ? $\neg$ Reachable ? $\neg$ Allocated

Reachability and Liveness

Comparison between different sets of objects:

$$\text{Live} \quad \subseteq \quad \text{Reachable} \quad \subseteq \quad \text{Allocated}$$

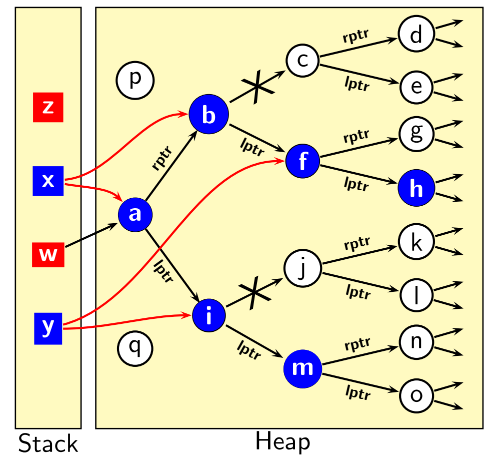
The objects that are not live must be reclaimed.

$$\neg\,\text{Live} \quad \supseteq \quad \neg\,\text{Reachable} \quad \supseteq \quad \neg\,\text{Allocated}$$

Garbage collectors collect these

Cedar Mesa Folk Wisdom

Make the unused memory unreachable by setting references to NULL. (GC FAQ: `http://www.iecc.com/gclist/GC-harder.html`)



Cedar Mesa Folk Wisdom

- Most promising, simplest to understand, yet the hardest to implement.
- Which references should be set to NULL?
  - Most approaches rely on feedback from profiling.
  - No systematic and clean solution.

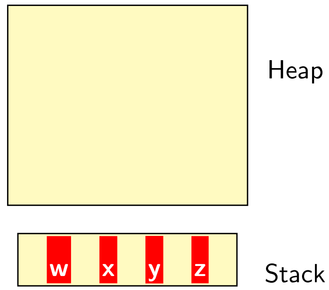
Distinguishing Between Reachable and Live

The state of art

- Eliminating objects reachable from root variables which are not live.
- Implemented in current Sun JVMs.
- Uses liveness data flow analysis of root variables (stack data).
- What about liveness of heap data?

Liveness of Stack Data: An Informal Introduction

```
w = x        // x points to m_a
while (x.data < max)
      x = x.rptr
y = x.lptr
z = New class_of_z
y = y.lptr
z.sum = x.data + y.data
```



if changed to while

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Liveness of Stack Data: An Informal Introduction

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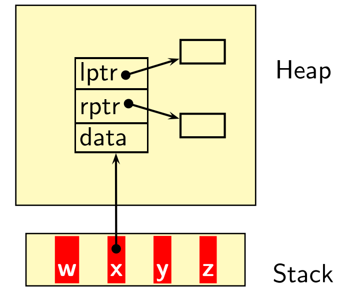

What is the meaning of the use of data?

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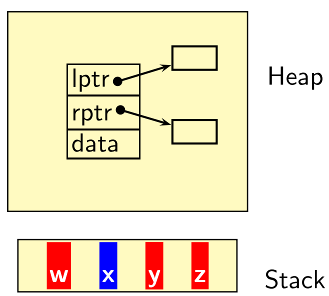
Liveness of Stack Data: An Informal Introduction

Accessing the location and reading its contents

```
w = x        // x points to m_a
while (x.data < max)
      x = x.rptr
y = x.lptr
z = New class_of_z
y = y.lptr
z.sum = x.data + y.data
```



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Liveness of Stack Data: An Informal Introduction

Accessing the location and reading its contents

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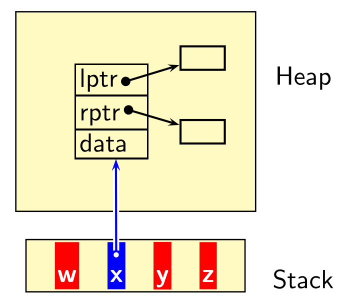
```



Reading x (Stack data)

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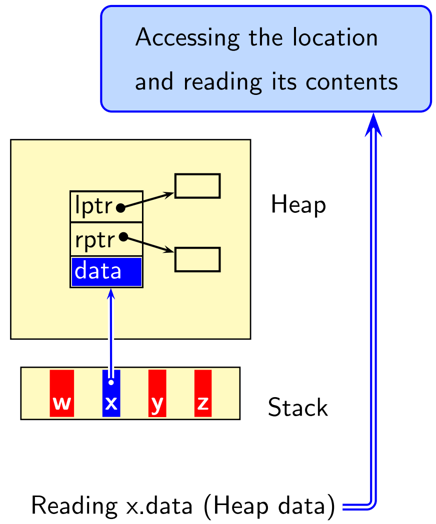
Liveness of Stack Data: An Informal Introduction

```
w = x          // x points to m_a
while (x.data < max)
      x = x.rptr
y = x.lptr
z = New class_of_z
y = y.lptr
z.sum = x.data + y.data
```



Liveness of Stack Data: An Informal Introduction

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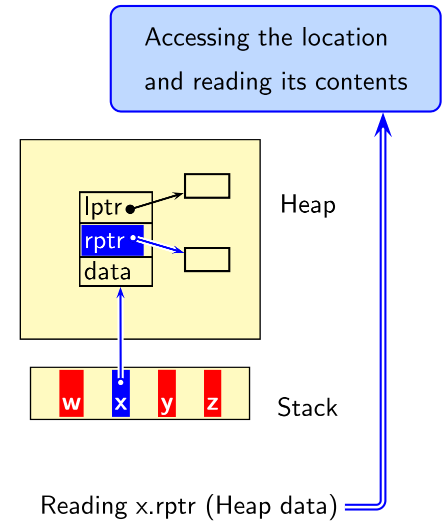

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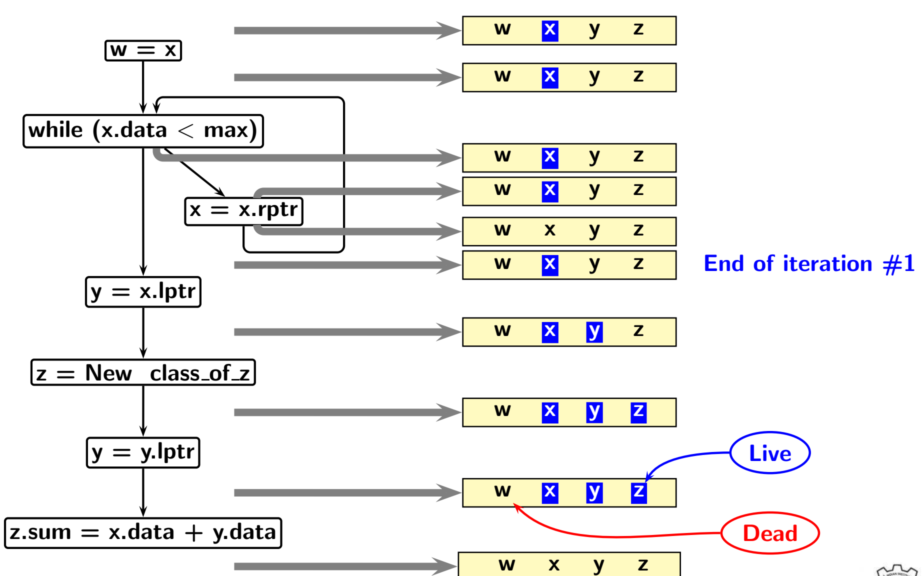

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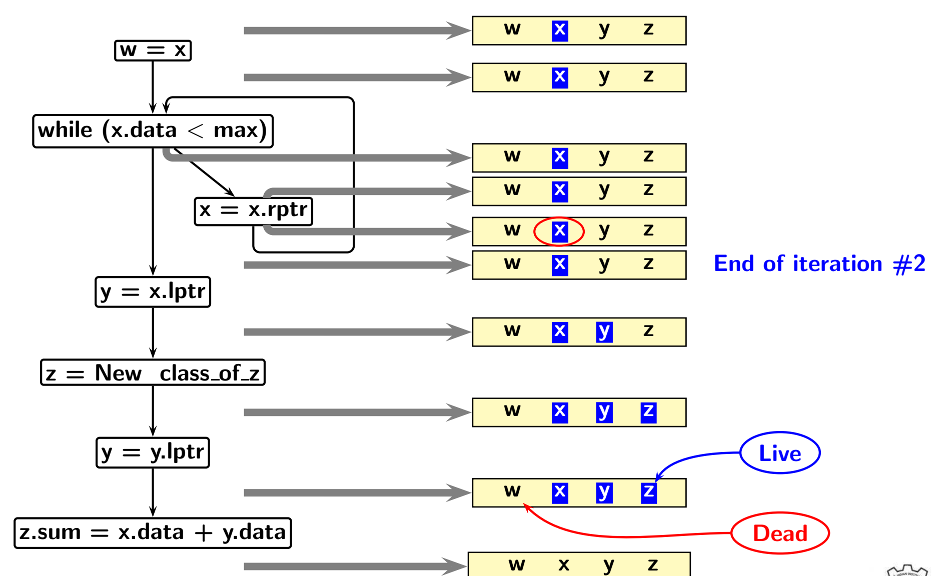


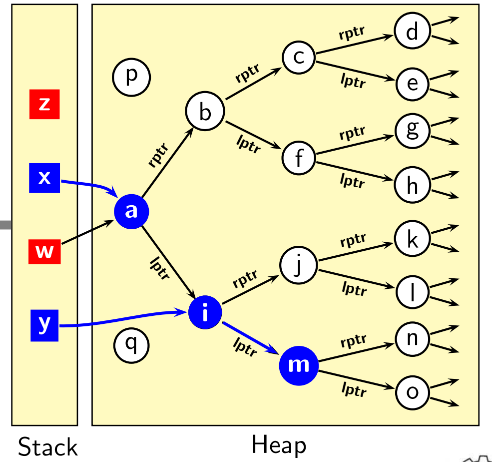
Applying Cedar Mesa Folk Wisdom to Heap Data

Liveness Analysis of Heap Data

If the while loop is not executed even once.

```
w = x          // x points to m_a
while (x.data < max)
      x = x.rptr
y = x.lptr

z = New class_of_z
y = y.lptr
z.sum = x.data + y.data
```



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Applying Cedar Mesa Folk Wisdom to Heap Data

Liveness Analysis of Heap Data

If the while loop is executed once.

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w = x          // x points to m_a
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z = New class_of_z
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```

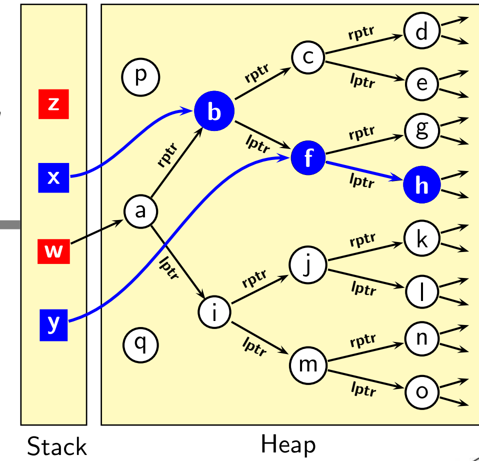

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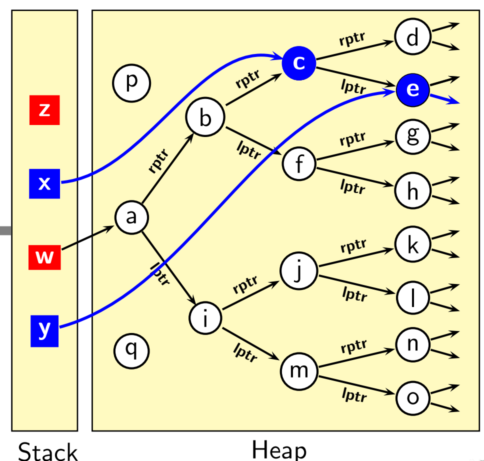
Applying Cedar Mesa Folk Wisdom to Heap Data

Liveness Analysis of Heap Data

If the while loop is executed twice.

```
w = x          // x points to m_a
while (x.data < max)
      x = x.rptr
y = x.lptr

z = New class_of_z
y = y.lptr
z.sum = x.data + y.data
```



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The Moral of the Story

- Mappings between access expressions and l-values keep changing
- This is a *rule* for heap data

  For stack and static data, it is an *exception*!
- Static analysis of programs has made significant progress for stack and static data.

  What about heap data?
  - Given two access expressions at a program point, do they have the same l-value?
  - Given the same access expression at two program points, does it have the same l-value?

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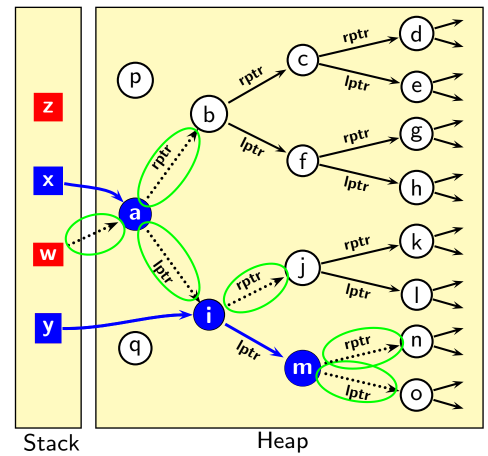
Our Solution

```
     y = z = null
1  w = x
     w = null
2  while (x.data < max)
   {     x.lptr = null
3        x = x.rptr      }
     x.rptr = x.lptr.rptr = null
     x.lptr.lptr.lptr = null
     x.lptr.lptr.rptr = null
4  y = x.lptr
     x.lptr = y.rptr = null
     y.lptr.lptr = y.lptr.rptr = null
5  z = New class_of_z
     z.lptr = z.rptr = null
6  y = y.lptr
     y.lptr = y.rptr = null
7  z.sum = x.data + y.data
     x = y = z = null
```


Our Solution

```
   y = z = null
1  w = x
   w = null
2  while (x.data < max)
   {     x.lptr = null
3        x = x.rptr      }
   x.rptr = x.lptr.rptr = null
   x.lptr.lptr.lptr = null
   x.lptr.lptr.rptr = null
4  y = x.lptr
   x.lptr = y.rptr = null
   y.lptr.lptr = y.lptr.rptr = null
5  z = New class_of_z
   z.lptr = z.rptr = null
6  y = y.lptr
   y.lptr = y.rptr = null
7  z.sum = x.data + y.data
   x = y = z = null
```

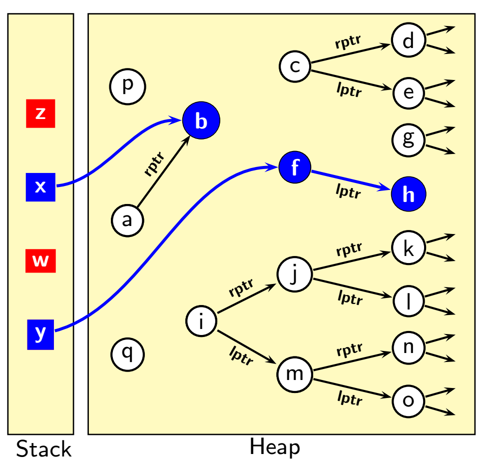
While loop is not executed even once



Our Solution

```
   y = z = null
1  w = x
   w = null
2  while (x.data < max)
   {     x.lptr = null
3        x = x.rptr      }
   x.rptr = x.lptr.rptr = null
   x.lptr.lptr.lptr = null
   x.lptr.lptr.rptr = null
4  y = x.lptr
   x.lptr = y.rptr = null
   y.lptr.lptr = y.lptr.rptr = null
5  z = New class_of_z
   z.lptr = z.rptr = null
6  y = y.lptr
   y.lptr = y.rptr = null
7  z.sum = x.data + y.data
   x = y = z = null
```

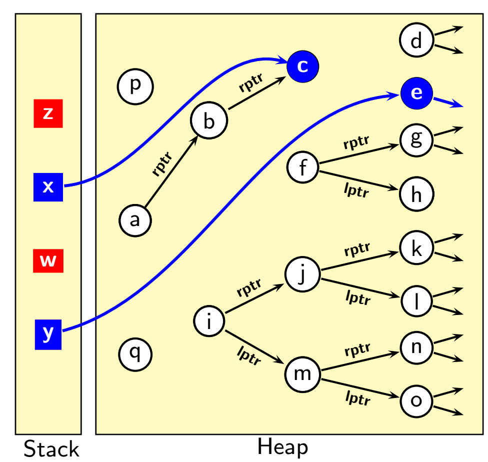
While loop is executed once



Our Solution

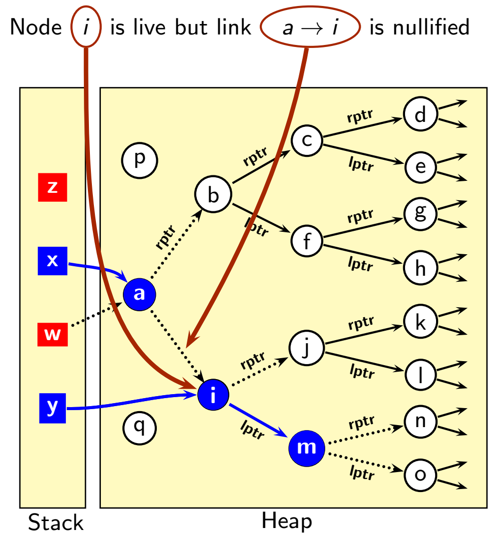
```
   y = z = null
1  w = x
   w = null
2  while (x.data < max)
   {     x.lptr = null
3        x = x.rptr      }
   x.rptr = x.lptr.rptr = null
   x.lptr.lptr.lptr = null
   x.lptr.lptr.rptr = null
4  y = x.lptr
   x.lptr = y.rptr = null
   y.lptr.lptr = y.lptr.rptr = null
5  z = New class_of_z
   z.lptr = z.rptr = null
6  y = y.lptr
   y.lptr = y.rptr = null
7  z.sum = x.data + y.data
   x = y = z = null
```

While loop is executed twice



Some Observations

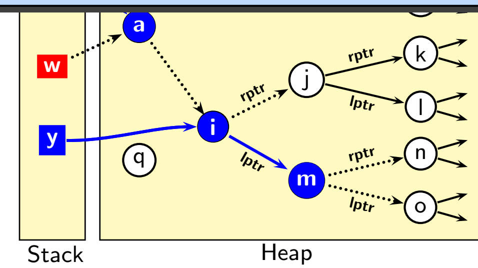
```
    y = z = null
1   w = x
    w = null
2   while (x.data < max)
    {     x.lptr = null
3         x = x.rptr       }
    x.rptr = x.lptr.rptr = null
    x.lptr.lptr.lptr = null
    x.lptr.lptr.rptr = null
4   y = x.lptr
    x.lptr = y.rptr = null
    y.lptr.lptr = y.lptr.rptr = null
5   z = New class_of_z
    z.lptr = z.rptr = null
6   y = y.lptr
    y.lptr = y.rptr = null
7   z.sum = x.data + y.data
    x = y = z = null
```



Some Observations

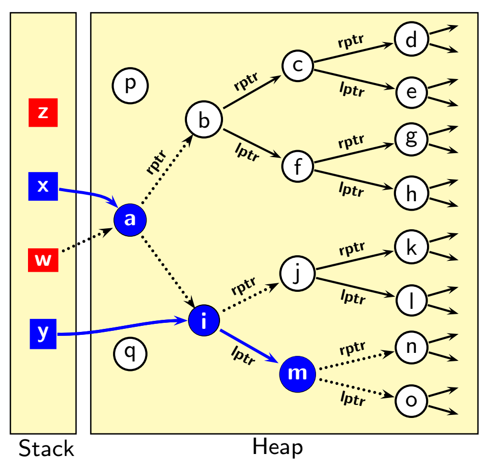

```
    y = z = null
1   w = x
    w = null
2   while (x.data < max)
    {     x.lptr = null
3         x = x.rptr       }
    x.rptr = x.lptr.rptr = null
    x.lptr.lptr.lptr = null
    x.lptr.lptr.rptr = null
4   y = x.lptr
    x.lptr = y.rptr = null
    y.lptr.lptr = y.lptr.rptr = null
5   z = New class_of_z
    z.lptr = z.rptr = null
6   y = y.lptr
    y.lptr = y.rptr = null
7   z.sum = x.data + y.data
    x = y = z = null
```


- The memory address that x holds when the execution reaches a given program point is not an invariant of program execution
- Whether we dereference lptr out of x or rptr out of x at a given program point is an invariant of program execution
- *A static analysis can discover only* some *invariants*



Some Observations


```
    y = z = null
1   w = x
    w = null
2   while (x.data < max)
    {     x.lptr = null
3         x = x.rptr       }
    x.rptr = x.lptr.rptr = null
    x.lptr.lptr.lptr = null
    x.lptr.lptr.rptr = null
4   y = x.lptr
    x.lptr = y.rptr = null
    y.lptr.lptr = y.lptr.rptr = null
5   z = New class_of_z
    z.lptr = z.rptr = null
6   y = y.lptr
    y.lptr = y.rptr = null
7   z.sum = x.data + y.data
    x = y = z = null
```

Some Observations


```
    y = z = null
1   w = x
    w = null
2   while (x.data < max)
    {     x.lptr = null
3         x = x.rptr       }
    x.rptr = x.lptr.rptr = null
    x.lptr.lptr.lptr = null
    x.lptr.lptr.rptr = null
4   y = x.lptr
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5   z = New class_of_z
    z.lptr = z.rptr = null
6   y = y.lptr
    y.lptr = y.rptr = null
7   z.sum = x.data + y.data
    x = y = z = null
```
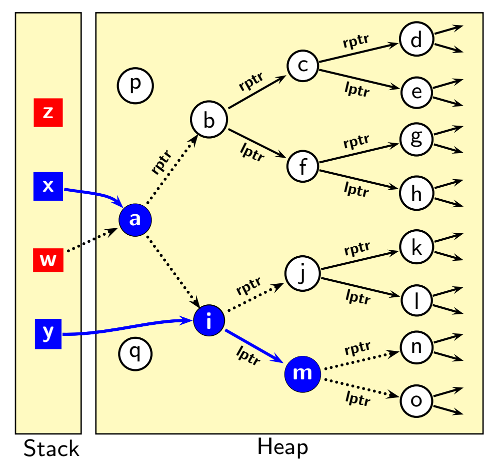
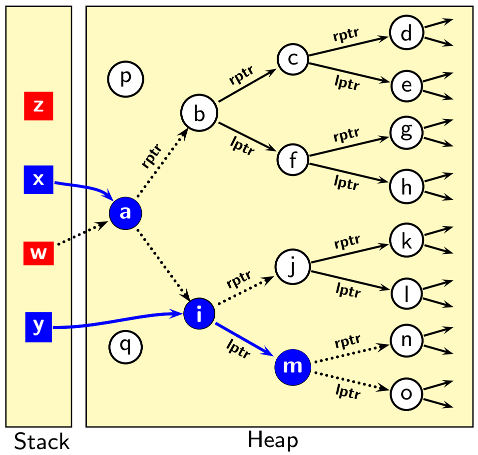




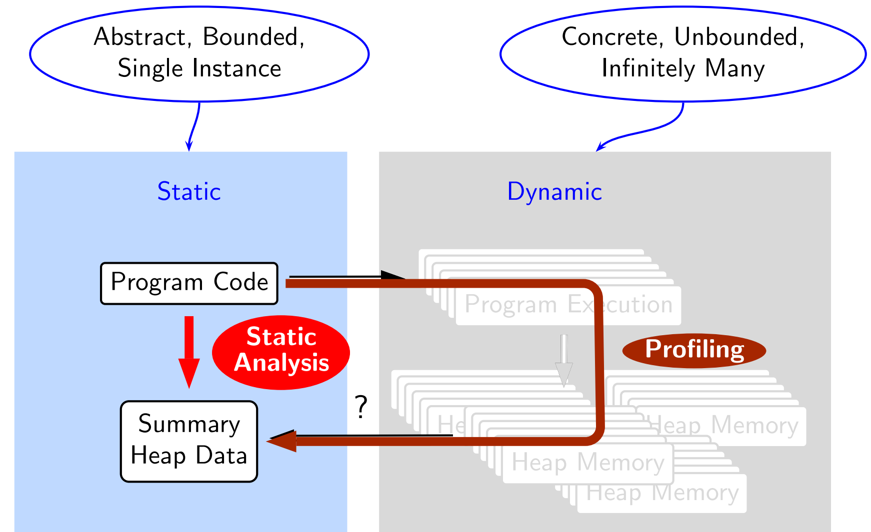
Some Observations

```
   y = z = null
1  w = x
   w = null
2  while (x.data < max)
   {      x.lptr = null
3         x = x.rptr       }
   x.rptr = x.lptr.rptr = null
   x.lptr.lptr.lptr = null
   x.lptr.lptr.rptr = null
4  y = x.lptr
   x.lptr = y.rptr = null
   y.lptr.lptr = y.lptr.rptr = null
5  z = New class_of_z
   z.lptr = z.rptr = null
6  y = y.lptr
   y.lptr = y.rptr = null
7  z.sum = x.data + y.data
   x = y = z = null
```

New access expressions are created.
Can they cause exceptions?



BTW, What is Static Analysis of Heap?

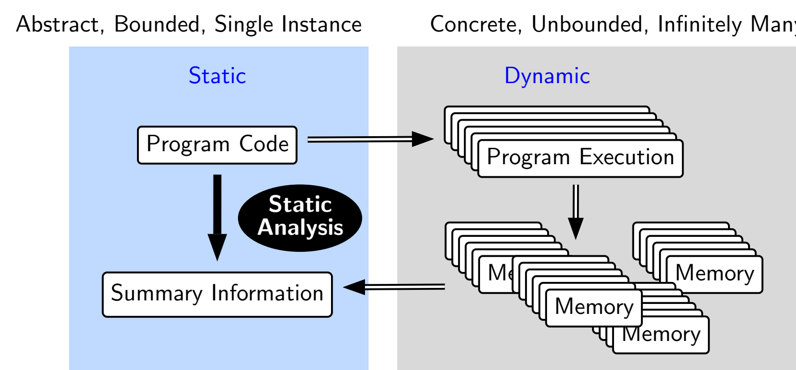


Part 4

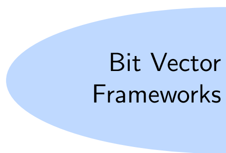
Course Details

The Main Theme of the Course

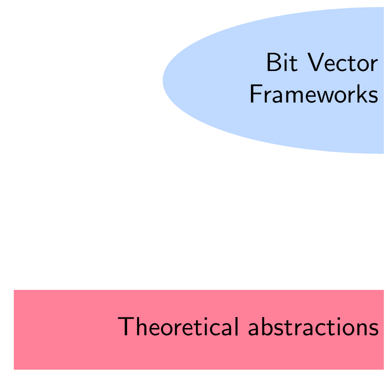
Constructing *suitable abstractions* for
sound & precise modelling of
runtime behaviour of programs
efficiently



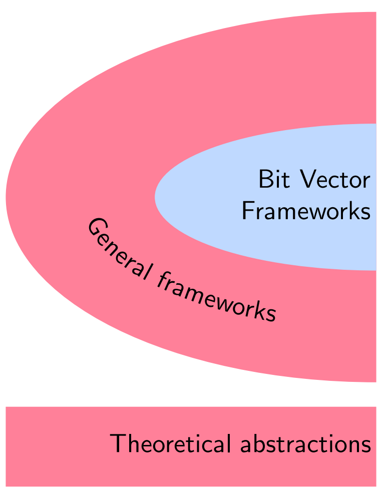
Sequence of Generalizations in the Course Modules



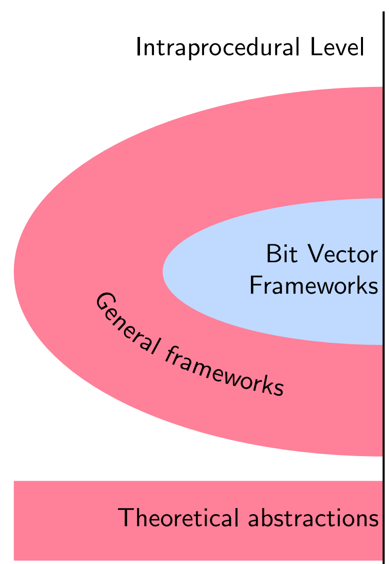
Sequence of Generalizations in the Course Modules



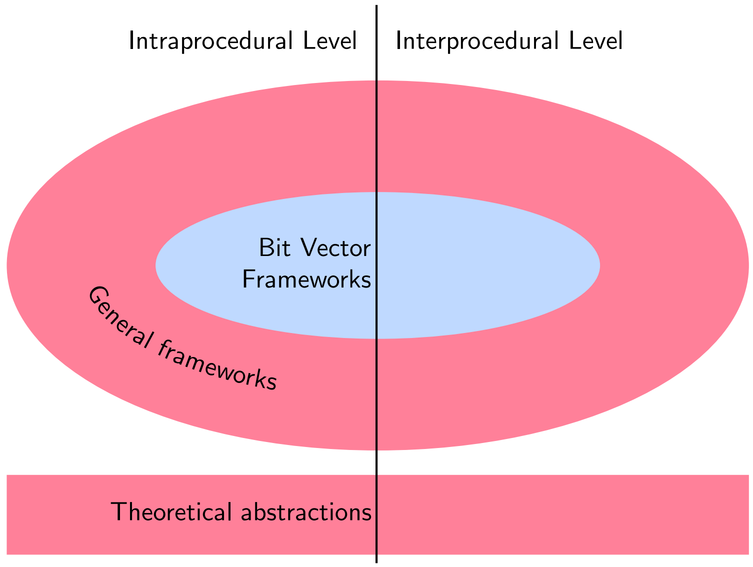
Sequence of Generalizations in the Course Modules



Sequence of Generalizations in the Course Modules



Sequence of Generalizations in the Course Modules



Course Pedagogy

- Interleaved lectures and tutorials
- Plenty of problem solving
- Practice problems will be provided,
  - Ready-made solutions will not be provided
  - Your solutions will be checked
- Detailed course plan can be found at the course page:
  `http://www.cse.iitb.ac.in/~uday/courses/cs618-17/`
- Moodle will be used extensively for announcements and discussions

Assessment Scheme

- Tentative plan

Mid Semester Examination	30%
End Semester Examination	40%
Two Quizzes	10%
Project	20%
Total	100%

- Can be fine tuned based on the class feedback

Course Strength and Selection Criteria

- Unavailability of TAs forces restricting the strength
  Less than 30 is preferable, 40 is tolerable
- Course primarily aimed at M.Tech. 1 students
  Follow up course and MTPs
- If the number is large, selection will be based on a test
  - Separate selection for M.Tech.1 and other students
  - Preference to M.Tech.1 students
  - May allow a reasonable number of audits
    - Attending all lectures is sufficient
    - No need to appear in examinations or do projects
  - Need to finalize the logistics of the test

Questions ??

Part 5

Program Model

Program Representation

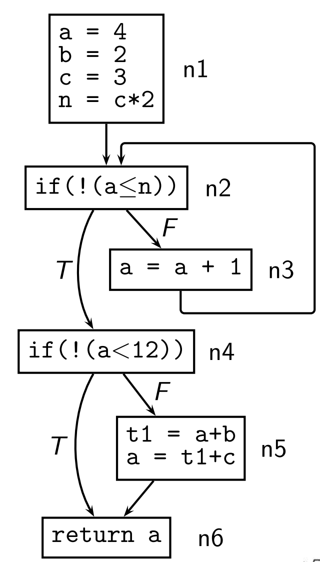
- Three address code statements
  - Result, operator, operand1, operand2
  - Assignments, expressions, conditional jumps
  - Initially only scalars
    Pointers, structures, arrays modelled later
- Control flow graph representation
  - Nodes represent maximal groups of statements
    devoid of any control transfer except fall through
  - Edges represent control transfers across basic blocks
  - A unique *Start* node and a unique *End* node
    Every node reachable from *Start*, and *End* reachable from every node
- Initially only intraprocedural programs

  Function calls brought in later

An Example Program

```
int main()
{ int a, b, c, n;

  a = 4;
  b = 2;
  c = 3;
  n = c*2;
  while (a <= n)
  {
    a = a+1;
  }
  if (a < 12)
    a = a+b+c;
   return a;
}
```

```
 1. a = 4
 2. b = 2
 3. c = 3
 4. n = c*2
 5. if (!(a≤n))
         goto 8
 6. a = a + 1
 7. goto 5
 8. if (!(a<12))
         goto 11
 9. t1 = a+b
10. a = t1+c
11. return a
```



Part 6

Soundness and Precision

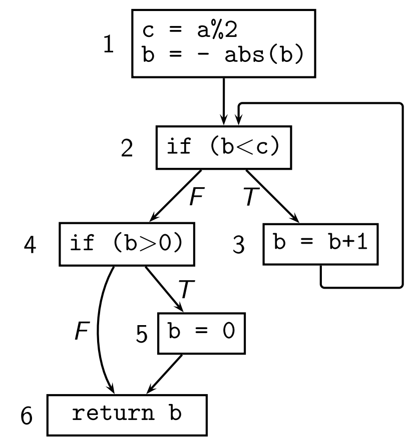
Soundness and Precision of Static Analysis

Example Program

Absolute

```
int a;
int f(int b)
{  int c;
   c = a%2;
   b = - abs(b);
   while (b < c)
      b = b+1;
   if (b > 0)
      b = 0;
   return b;
}
```

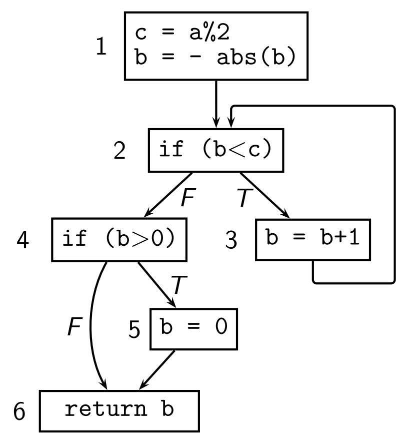
Control Flow Graph



Execution Traces for Concrete Semantics (1)

- States
  - A *data* state: Variables → Values
  - A *program* state: (Program Point, A data state)
- Execution traces (or traces, for short)
  - Valid sequences of program states starting with a given initial state

Execution Traces for Concrete Semantics (2)



Trace 1 (a b c)	*Trace 2* (a b c)
$Entry_1, (5, 2, 7)$	$Entry_1, (-5, -2, 8)$
$Entry_2, (5, -2, 1)$	$Entry_2, (-5, -2, -1)$
$Entry_3, (5, -2, 1)$	$Entry_3, (-5, -2, -1)$
$Entry_2, (5, -1, 1)$	$Entry_2, (-5, -1, -1)$
$Entry_3, (5, -1, 1)$	$Entry_4, (-5, -1, -1)$
$Entry_2, (5, 0, 1)$	$Entry_6, (-5, -1, -1)$
$Entry_3, (5, 0, 1)$	
$Entry_2, (5, 1, 1)$	
$Entry_4, (5, 1, 1)$	
$Entry_5, (5, 1, 1)$	
$Entry_6, (5, 0, 1)$	

Execution Traces for Concrete Semantics (2)

- A separate trace for each combination of inputs
  - The number of traces is potentially infinite
- Program points may repeat in the traces
  - Traces may be very long
  - Non-terminating traces: Infinitely long

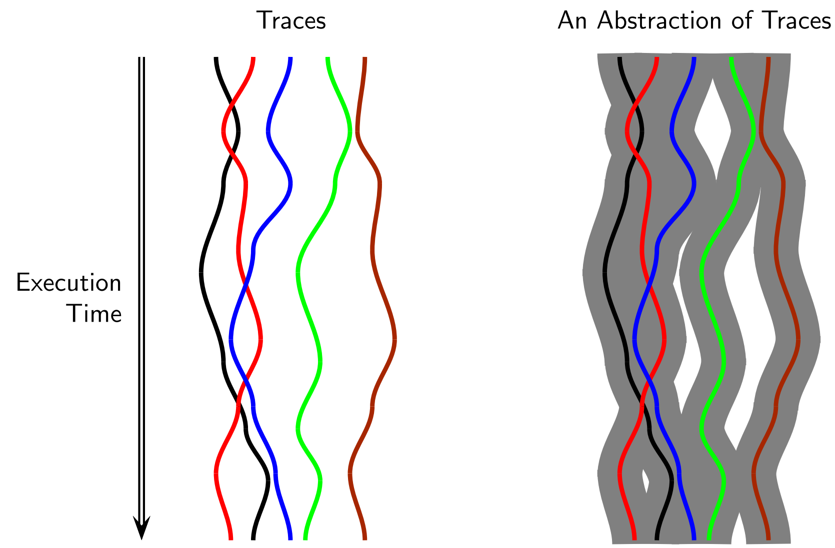
4 if

F

6 return b

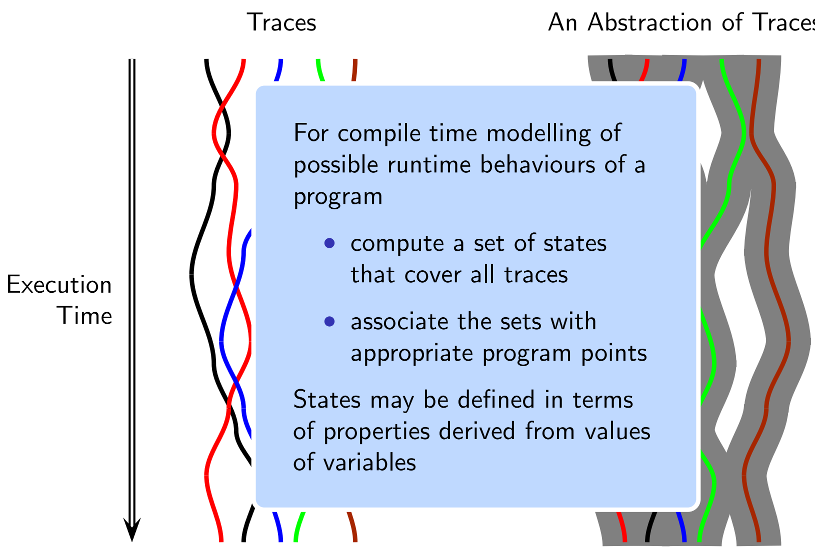
$Entry_5, (5, 1, 1)$
$Entry_6, (5, 0, 1)$

Static Analysis Computes Abstractions of Traces (1)



CS 618 Intro to PA: Soundness and Precision 53/62

Static Analysis Computes Abstractions of Traces (1)



For compile time modelling of possible runtime behaviours of a program

- compute a set of states that cover all traces
- associate the sets with appropriate program points

States may be defined in terms of properties derived from values of variables

Static Analysis Computes Abstractions of Traces (2)

A possible static abstraction using sets

Trace 1

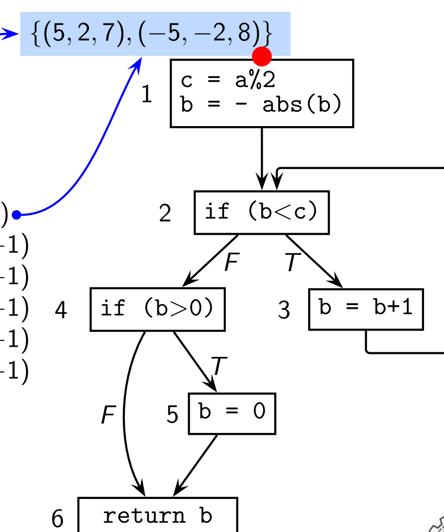
a b c

$Entry_1, (5, 2, 7)$
$Entry_2, (5, -2, 1)$
$Entry_3, (5, -2, 1)$
$Entry_2, (5, -1, 1)$
$Entry_3, (5, -1, 1)$
$Entry_2, (5, 0, 1)$
$Entry_3, (5, 0, 1)$
$Entry_2, (5, 1, 1)$
$Entry_4, (5, 1, 1)$
$Entry_5, (5, 1, 1)$
$Entry_6, (5, 0, 1)$

Trace 2

a b c

$Entry_1, (-5, -2, 8)$
$Entry_2, (-5, -2, -1)$
$Entry_3, (-5, -2, -1)$
$Entry_2, (-5, -1, -1)$
$Entry_4, (-5, -1, -1)$
$Entry_6, (-5, -1, -1)$



Static Analysis Computes Abstractions of Traces (2)

A possible static abstraction using sets

Trace 1

$a\ b\ c$

$Entry_1, (5, 2, 7)$
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$Entry_3, (5, -2, 1)$
$Entry_2, (5, -1, 1)$
$Entry_3, (5, -1, 1)$
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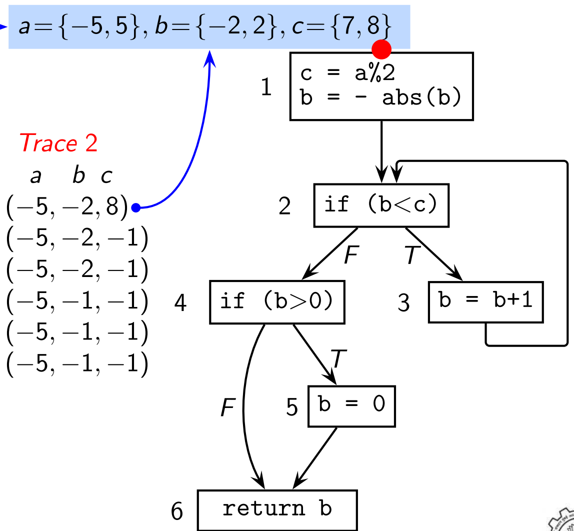
$Entry_3, (5, 0, 1)$
$Entry_2, (5, 1, 1)$
$Entry_4, (5, 1, 1)$
$Entry_5, (5, 1, 1)$
$Entry_6, (5, 0, 1)$

Trace 2

$a\ b\ c$

$Entry_1, (-5, -2, 8)$
$Entry_2, (-5, -2, -1)$
$Entry_3, (-5, -2, -1)$
$Entry_2, (-5, -1, -1)$
$Entry_4, (-5, -1, -1)$
$Entry_6, (-5, -1, -1)$



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Trace 2

$a\ b\ c$

$Entry_1, (-5, -2, 8)$
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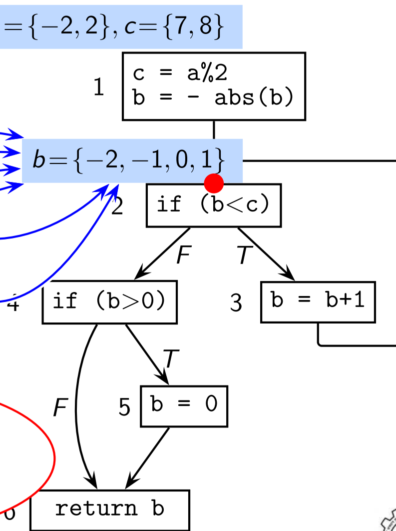
$Entry_3, (-5, -2, -1)$
$Entry_2, (-5, -1, -1)$
$Entry_4, (-5, -1, -1)$
$Entry_6, (-5, -1, -1)$

We only show the values of b

Combine the values across all occurrences of a program point



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Static Analysis Computes Abstractions of Traces (2)

A possible static abstraction using sets

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$a\ b\ c$

$Entry_1, (5, 2, 7)$
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$Entry_3, (5, -1, 1)$
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$Entry_3, (5, 0, 1)$
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Trace 2

$a\ b\ c$

$Entry_1, (-5, -2, 8)$
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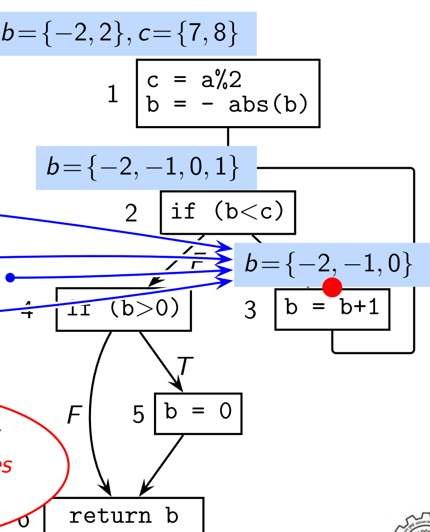
Combine the values across all occurrences of a program point



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Static Analysis Computes Abstractions of Traces (2)

A possible static abstraction using sets

Trace 1

$a\ b\ c$

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Trace 2

$a\ b\ c$

$Entry_1, (-5, -2, 8)$
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$Entry_3, (-5, -2, -1)$
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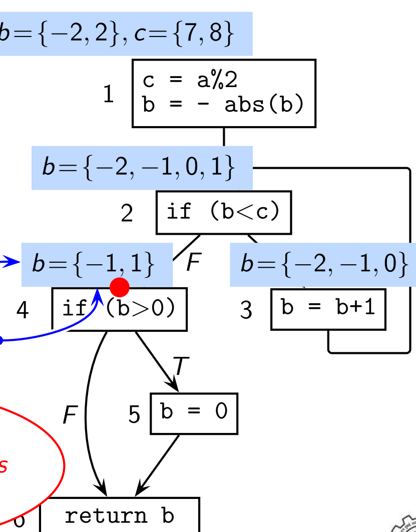
We only show the values of b

Combine the values across all occurrences of a program point



Static Analysis Computes Abstractions of Traces (2)

A possible static abstraction using sets

$a=\{-5,5\}, b=\{-2,2\}, c=\{7,8\}$

We only show the values of b

Trace 1

$a \; b \; c$

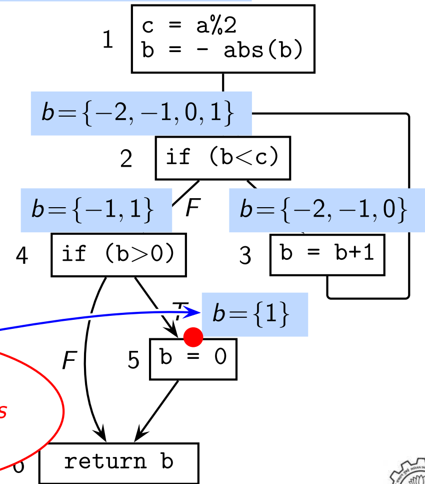
$Entry_1, (5,2,7)$
$Entry_2, (5,-2,1)$
$Entry_3, (5,-2,1)$
$Entry_2, (5,-1,1)$
$Entry_3, (5,-1,1)$
$Entry_2, (5,0,1)$
$Entry_3, (5,0,1)$
$Entry_2, (5,1,1)$
$Entry_4, (5,1,1)$
$Entry_5, (5,1,1)$
$Entry_6, (5,0,1)$

Trace 2

$a \; b \; c$

$Entry_1, (-5,-2,8)$
$Entry_2, (-5,-2,-1)$
$Entry_3, (-5,-2,-1)$
$Entry_2, (-5,-1,-1)$
$Entry_4, (-5,-1,-1)$
$Entry_6, (-5,-1,-1)$

Combine the values across all occurrences of a program point



Static Analysis Computes Abstractions of Traces (2)

A possible static abstraction using sets

$a=\{-5,5\}, b=\{-2,2\}, c=\{7,8\}$

We only show the values of b

Trace 1

$a \; b \; c$

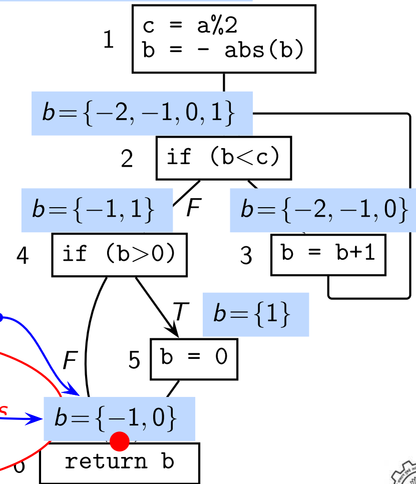
$Entry_1, (5,2,7)$
$Entry_2, (5,-2,1)$
$Entry_3, (5,-2,1)$
$Entry_2, (5,-1,1)$
$Entry_3, (5,-1,1)$
$Entry_2, (5,0,1)$
$Entry_3, (5,0,1)$
$Entry_2, (5,1,1)$
$Entry_4, (5,1,1)$
$Entry_5, (5,1,1)$
$Entry_6, (5,0,1)$

Trace 2

$a \; b \; c$

$Entry_1, (-5,-2,8)$
$Entry_2, (-5,-2,-1)$
$Entry_3, (-5,-2,-1)$
$Entry_2, (-5,-1,-1)$
$Entry_4, (-5,-1,-1)$
$Entry_6, (-5,-1,-1)$

Combine the values across all occurrences of a program point



Computing Static Abstraction for Liveness of Variables

At a program point p
$a \mapsto 1 \Rightarrow a$ is live at p
$a \mapsto 0 \Rightarrow a$ is not live at p

Trace 1

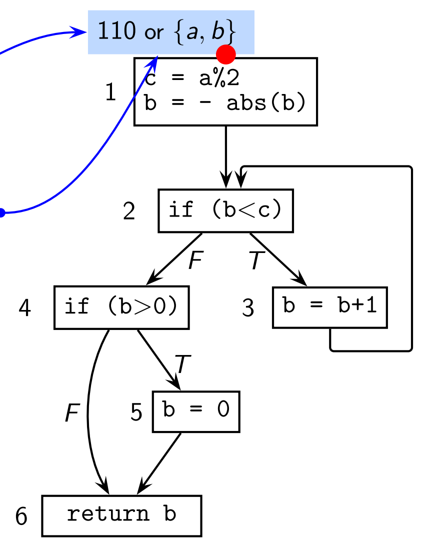
$a \; b \; c$

$Entry_1, (1,1,0)$
$Entry_2, (0,1,1)$
$Entry_3, (0,1,1)$
$Entry_2, (0,1,1)$
$Entry_3, (0,1,1)$
$Entry_2, (0,1,1)$
$Entry_3, (0,1,1)$
$Entry_2, (0,1,1)$
$Entry_4, (0,1,0)$
$Entry_5, (0,0,0)$
$Entry_6, (0,1,0)$

Trace 2

$a \; b \; c$

$Entry_1, (1,1,0)$
$Entry_2, (0,1,1)$
$Entry_3, (0,0,1)$
$Entry_2, (0,1,1)$
$Entry_4, (0,1,0)$
$Entry_6, (0,1,0)$



Computing Static Abstraction for Liveness of Variables

At a program point p
$a \mapsto 1 \Rightarrow a$ is live at p
$a \mapsto 0 \Rightarrow a$ is not live at p

Trace 1

$a \; b \; c$

$Entry_1, (1,1,0)$
$Entry_2, (0,1,1)$
$Entry_3, (0,1,1)$
$Entry_2, (0,1,1)$
$Entry_3, (0,1,1)$
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$Entry_3, (0,1,1)$
$Entry_2, (0,1,1)$
$Entry_4, (0,1,0)$
$Entry_5, (0,0,0)$
$Entry_6, (0,1,0)$

Trace 2

$a \; b \; c$

$Entry_1, (1,1,0)$
$Entry_2, (0,1,1)$
$Entry_3, (0,0,1)$
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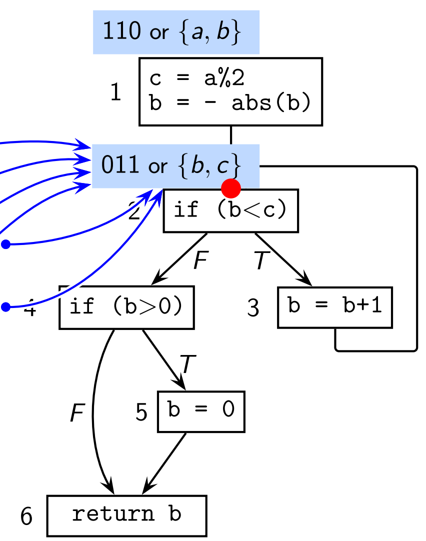
$Entry_4, (0,1,0)$
$Entry_6, (0,1,0)$



Computing Static Abstraction for Liveness of Variables

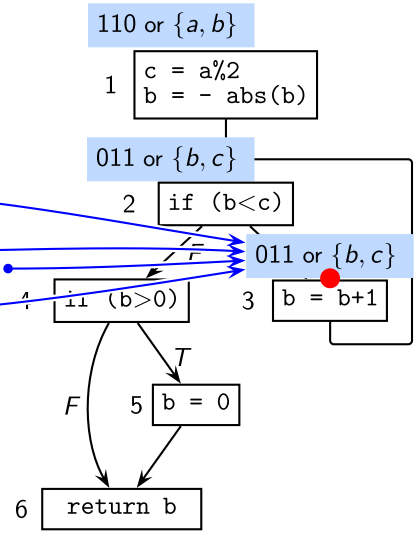
At a program point p
$a \mapsto 1 \Rightarrow a$ is live at p
$a \mapsto 0 \Rightarrow a$ is not live at p

Trace 1

	a b c
$Entry_1$,	$(1,1,0)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,1,1)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,1,1)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,1,1)$
$Entry_2$,	$(0,1,1)$
$Entry_4$,	$(0,1,0)$
$Entry_5$,	$(0,0,0)$
$Entry_6$,	$(0,1,0)$

Trace 2

	a b c
$Entry_1$,	$(1,1,0)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,0,1)$
$Entry_2$,	$(0,1,1)$
$Entry_4$,	$(0,1,0)$
$Entry_6$,	$(0,1,0)$



Computing Static Abstraction for Liveness of Variables

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	a b c
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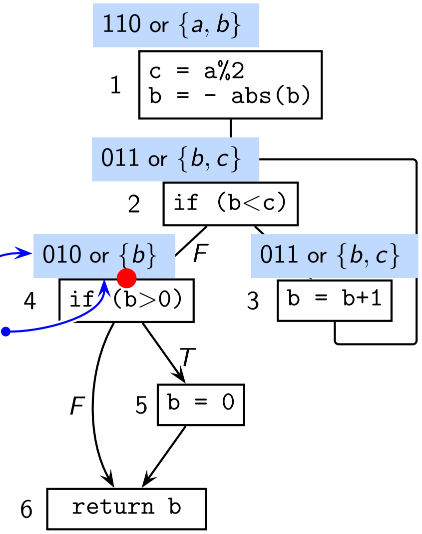
$Entry_3$,	$(0,1,1)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,1,1)$
$Entry_2$,	$(0,1,1)$
$Entry_4$,	$(0,1,0)$
$Entry_5$,	$(0,0,0)$
$Entry_6$,	$(0,1,0)$

Trace 2

	a b c
$Entry_1$,	$(1,1,0)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,0,1)$
$Entry_2$,	$(0,1,1)$
$Entry_4$,	$(0,1,0)$
$Entry_6$,	$(0,1,0)$



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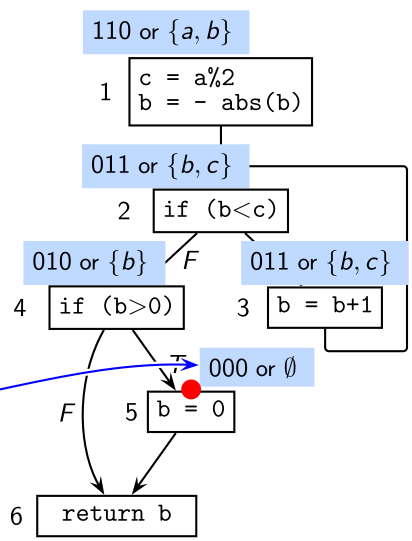
$Entry_2$,	$(0,1,1)$
$Entry_4$,	$(0,1,0)$
$Entry_5$,	$(0,0,0)$
$Entry_6$,	$(0,1,0)$

Trace 2

	a b c
$Entry_1$,	$(1,1,0)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,0,1)$
$Entry_2$,	$(0,1,1)$
$Entry_4$,	$(0,1,0)$
$Entry_6$,	$(0,1,0)$



Computing Static Abstraction for Liveness of Variables

At a program point p
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Trace 1

	a b c
$Entry_1$,	$(1,1,0)$
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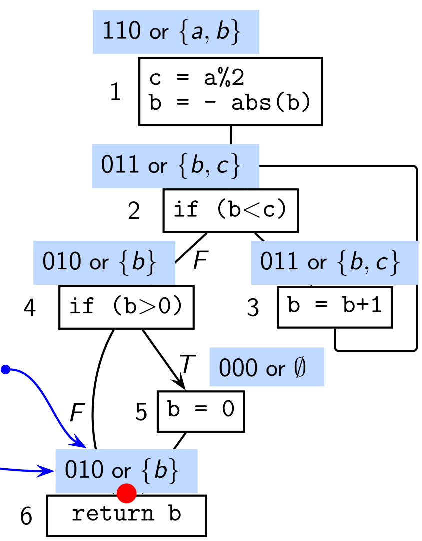
$Entry_3$,	$(0,1,1)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,1,1)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,1,1)$
$Entry_2$,	$(0,1,1)$
$Entry_4$,	$(0,1,0)$
$Entry_5$,	$(0,0,0)$
$Entry_6$,	$(0,1,0)$

Trace 2

	a b c
$Entry_1$,	$(1,1,0)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,0,1)$
$Entry_2$,	$(0,1,1)$
$Entry_4$,	$(0,1,0)$
$Entry_6$,	$(0,1,0)$



Computing Static Abstraction for Liveness of Variables

At a program point p
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$a \mapsto 0 \Rightarrow a$ is not live at p

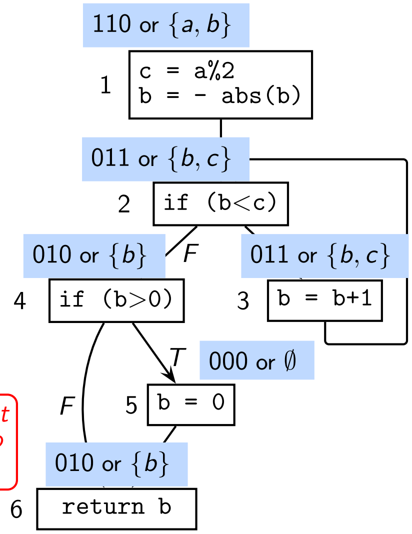
Trace 1

	a b c
$Entry_1$,	$(1,1,0)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,1,1)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,1,1)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,1,1)$
$Entry_2$,	$(0,1,1)$
$Entry_4$,	$(0,1,0)$
$Entry_5$,	$(0,0,0)$
$Entry_6$,	$(0,1,0)$

Trace 2

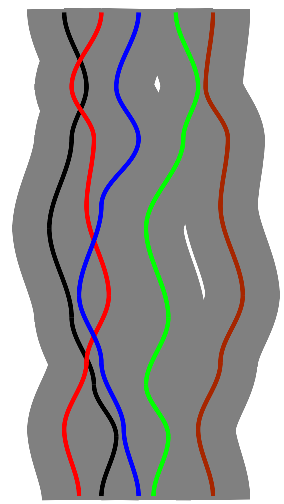
	a b c
$Entry_1$,	$(1,1,0)$
$Entry_2$,	$(0,1,1)$
$Entry_3$,	$(0,0,1)$
$Entry_2$,	$(0,1,1)$
$Entry_4$,	$(0,1,0)$
$Entry_6$,	$(0,1,0)$

Trace 2 does not add anything to the abstraction



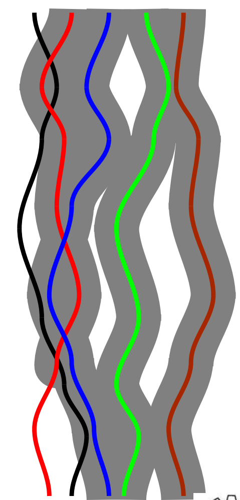
Soundness of Abstractions (1)

Sound



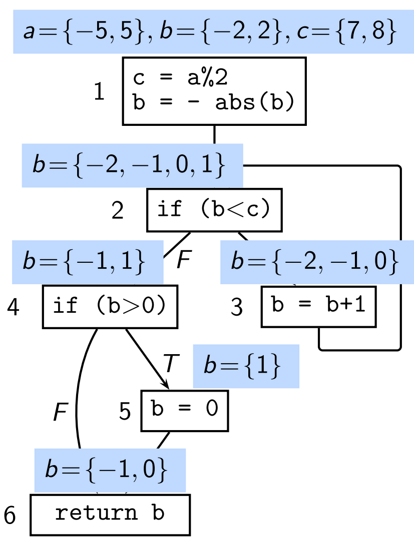
- An over-approximation of traces is sound
- Missing any state in any trace causes unsoundness

Unsound



Soundness of Abstractions (2)

An unsound abstraction



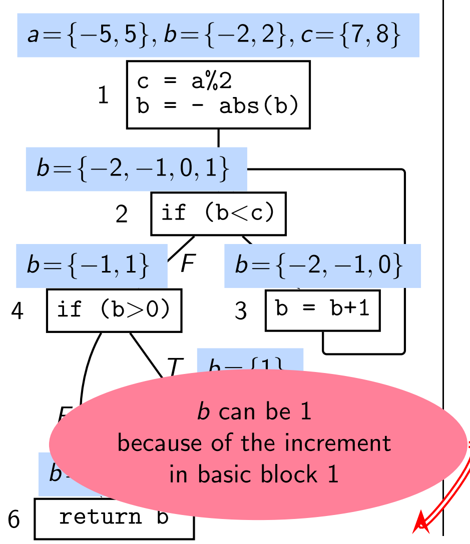
All variables can have arbitrary values at the start.

b can have many more values at the entry of

- blocks 2 and 3 (e.g. -3, -8, ...)
- block 4 (e.g. 0)

Soundness of Abstractions (2)

An unsound abstraction



A sound abstraction using intervals

- Overapproximated range of values denoted by

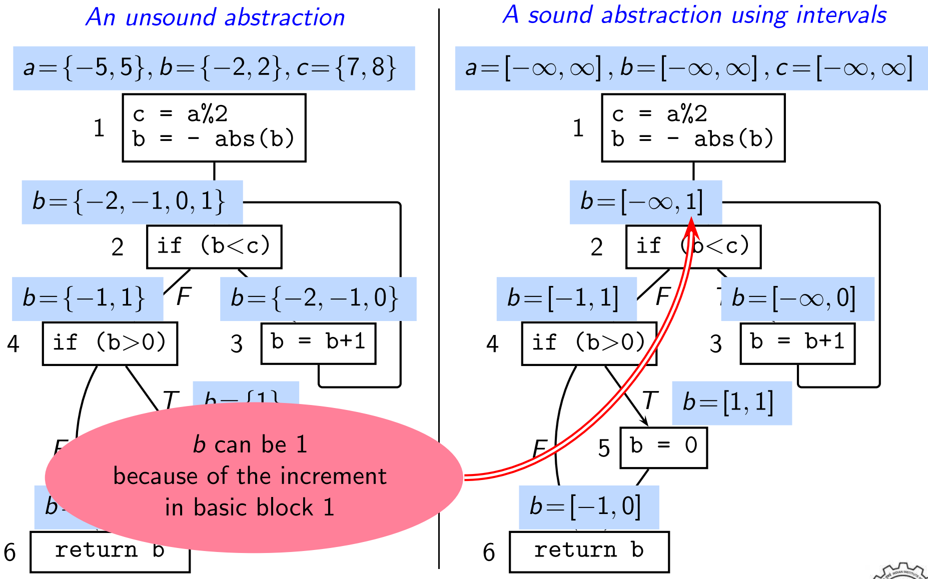
$$[low\_limit, high\_limit]$$

- Inclusive limits with

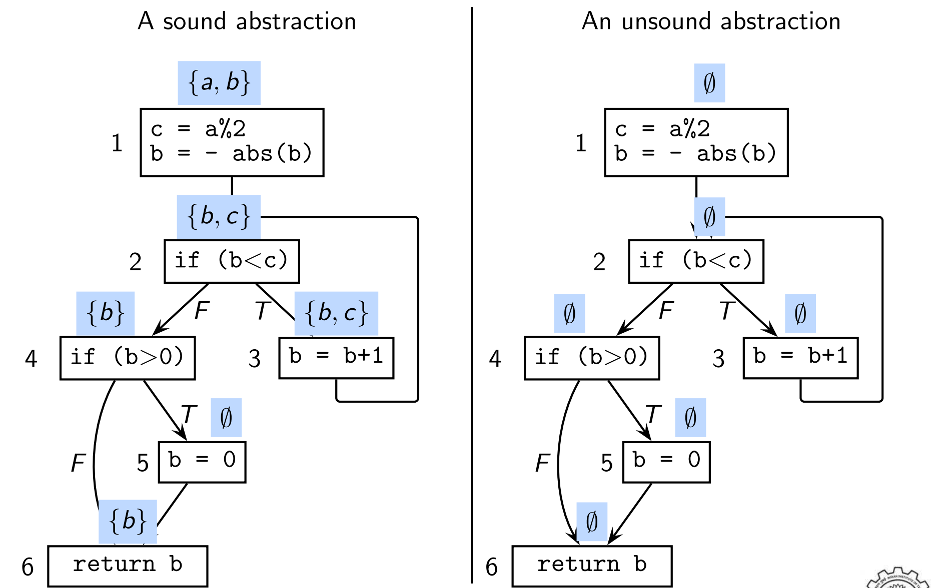
$$low\_limit \leq high\_limit$$

- One continuous range per variable with no "holes"

Soundness of Abstractions (2)

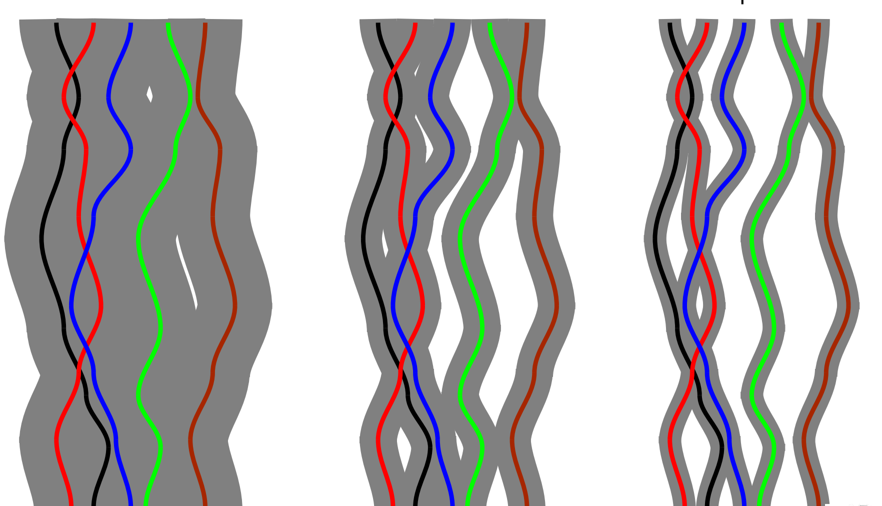


Soundness of Abstractions for Liveness Analysis



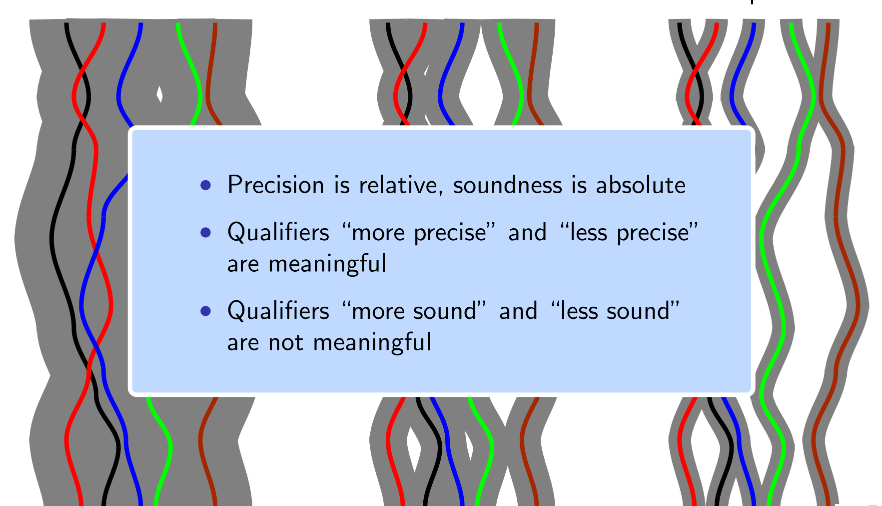
Precision of Sound Abstractions(1)

Sound but imprecise | Sound and more precise | Sound and even more precise



Precision of Sound Abstractions(1)

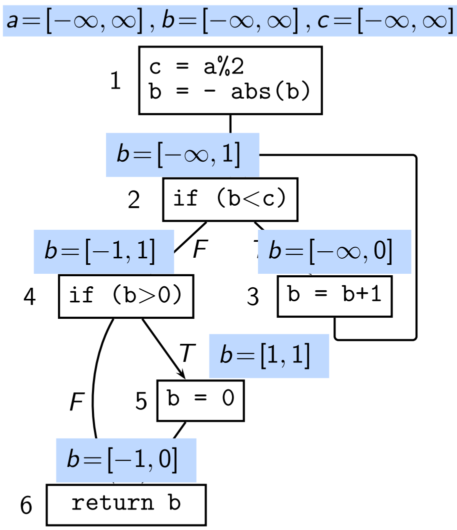
Sound but imprecise | Sound and more precise | Sound and even more precise



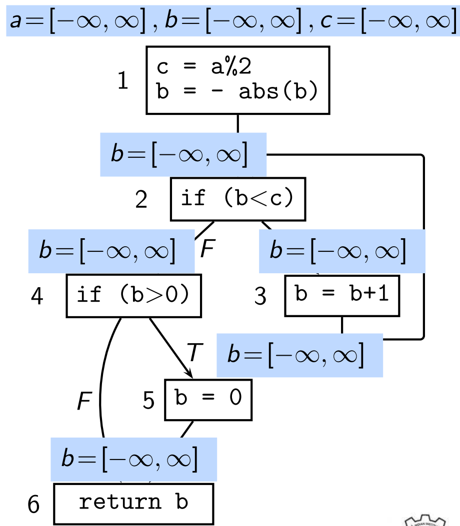
- Precision is relative, soundness is absolute
- Qualifiers "more precise" and "less precise" are meaningful
- Qualifiers "more sound" and "less sound" are not meaningful

Precision of Sound Abstractions(2)

A precise abstraction using intervals

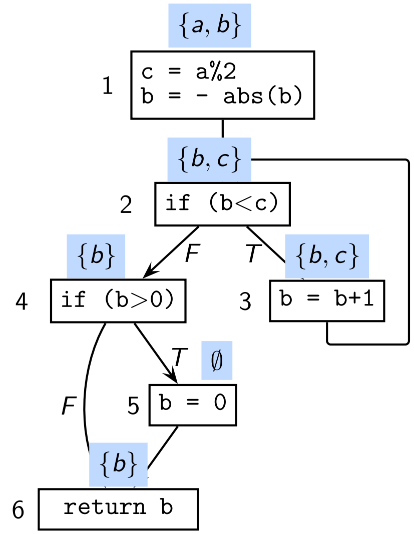


An imprecise abstraction using intervals

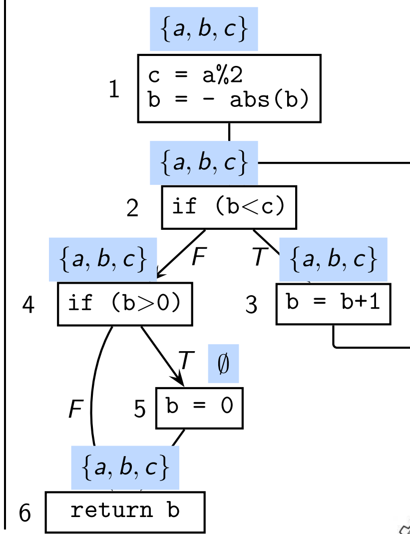


Precision of Abstractions for Liveness Analysis

A precise abstraction



An imprecise abstraction



Limitations of Static Analysis

- In general, the computation of *exact* static abstraction is *undecidable*
  - Possible reasons
    - Values of variables not known
    - Branch outcomes not known
    - Infinitely many paths in the presence of loops or recursion
    - Infinitely many values
  - We have to settle for some imprecision
  - How are data states compared to distinguish between a sound and unsound (or a precise or an imprecise result)?
    - We have introduced the concepts intuitively
    - Will define them formally in a later module
- *Goodness of a static analysis lies in minimizing imprecision without compromising on soundness*

  Additional expectations: Efficiency and scalability