

Assignment 3

Introduction to Complexity Theory (CS 721)

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1. An oracle separating NP from coNP.

- (a) Construct an oracle C such that $\mathbf{NP}^C \neq \mathbf{coNP}^C$.
- (b) For the same oracle C , what can you conclude about $\mathbf{P}^C$ versus $\mathbf{NP}^C$? Justify.

2. Two faces of NL.

- (a) Prove that the nondeterministic logspace machine definition of **NL** and the read-once certificate / logspace verifier definition describe the same class.
- (b) Show that $2\text{SAT} \in \mathbf{NL}$.

3. TQBF is PSPACE-complete under logspace reductions. Show that every $L \in \mathbf{PSPACE}$ is reducible to TQBF by a *logspace* reduction.

4. CYCLE is NL-complete. Define

$$\text{CYCLE} = \{ \langle G \rangle \mid G \text{ is a directed graph containing a directed cycle} \}.$$

Show that CYCLE is **NL**-complete under logspace reductions. (Prove both membership in **NL** and **NL**-hardness.)

5. Antakshari is PSPACE-complete. Here is a formalism of the game *antakshari* (also called word-building, geography, etc.). Let Σ be a finite alphabet and S a finite set of strings in Σ^* . For a string $s = s_1 s_2 \cdots s_n$ (each $s_i \in \Sigma$), let $\text{last}(s) = s_n$ and $\text{first}(s) = s_1$.

Two players P_0, P_1 play the game, starting with P_0 and alternating turns. On the first turn P_0 chooses any $s \in S$. On every subsequent turn the other player must choose a string $s' \in S$ that has not been chosen before and satisfies $\text{last}(s) = \text{first}(s')$, where s is the string chosen on the previous turn. The first player unable to choose a next string loses.

Define

$$\text{ANTAKSHARI} = \{ (\Sigma, S) \mid S \text{ is a finite set of strings in } \Sigma^* \\ \text{for which the starting player } P_0 \text{ has a winning strategy} \}.$$

Show that ANTAKSHARI is **PSPACE**-complete.