FO definable Transformations of Infinite strings

Vrunda Dave, S. Krishna, Ashutosh Trivedi

Outline

- Introduction
  - Three formalisms for transductions
  - Related work
- Aperiodic transformations for Infinite strings
  - Aperiodic two way transducer
  - Aperiodic streaming string transducer
- Equivalence results and Proof ideas
  - $SST_{sf} \subset FOT = 2WST_{sf} \subset SST_{sf}$
- Conclusion

Introduction

- Three formalisms for transformations:

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  - Logic

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  - Two way machines

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- Three formalisms for transformations:
  - Logic
  - Two way machines
  - One way machines with finite registers

Logic Transducer

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[Courcelle'94]

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input

Logic Transducer

[Courcelle'94]

input a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8

Logic Transducer

[Courcelle'94]

input a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 $\overset{\Sigma}{\phi_{dom}}$ ✔

Logic Transducer

[Courcelle'94]

input $a_1 \to a_2 \to a_3 \to a_4 \to a_5 \to a_6 \to a_7 \to a_8$

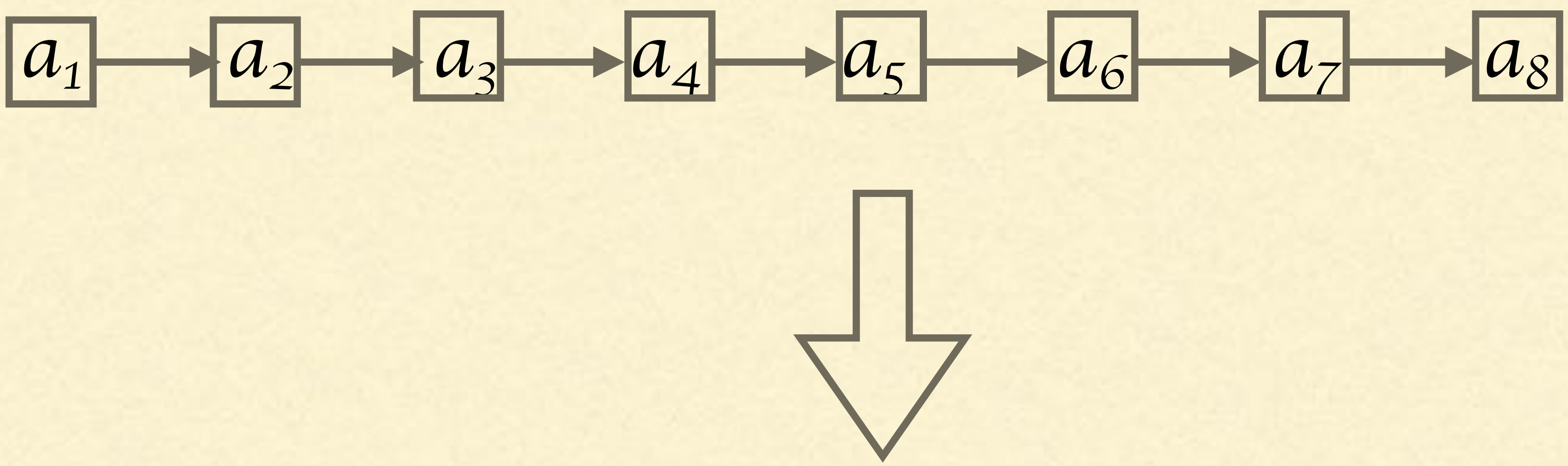
Σ

ϕ_{dom} ✔

Logic Transducer

[Courcelle'94]

input



Σ

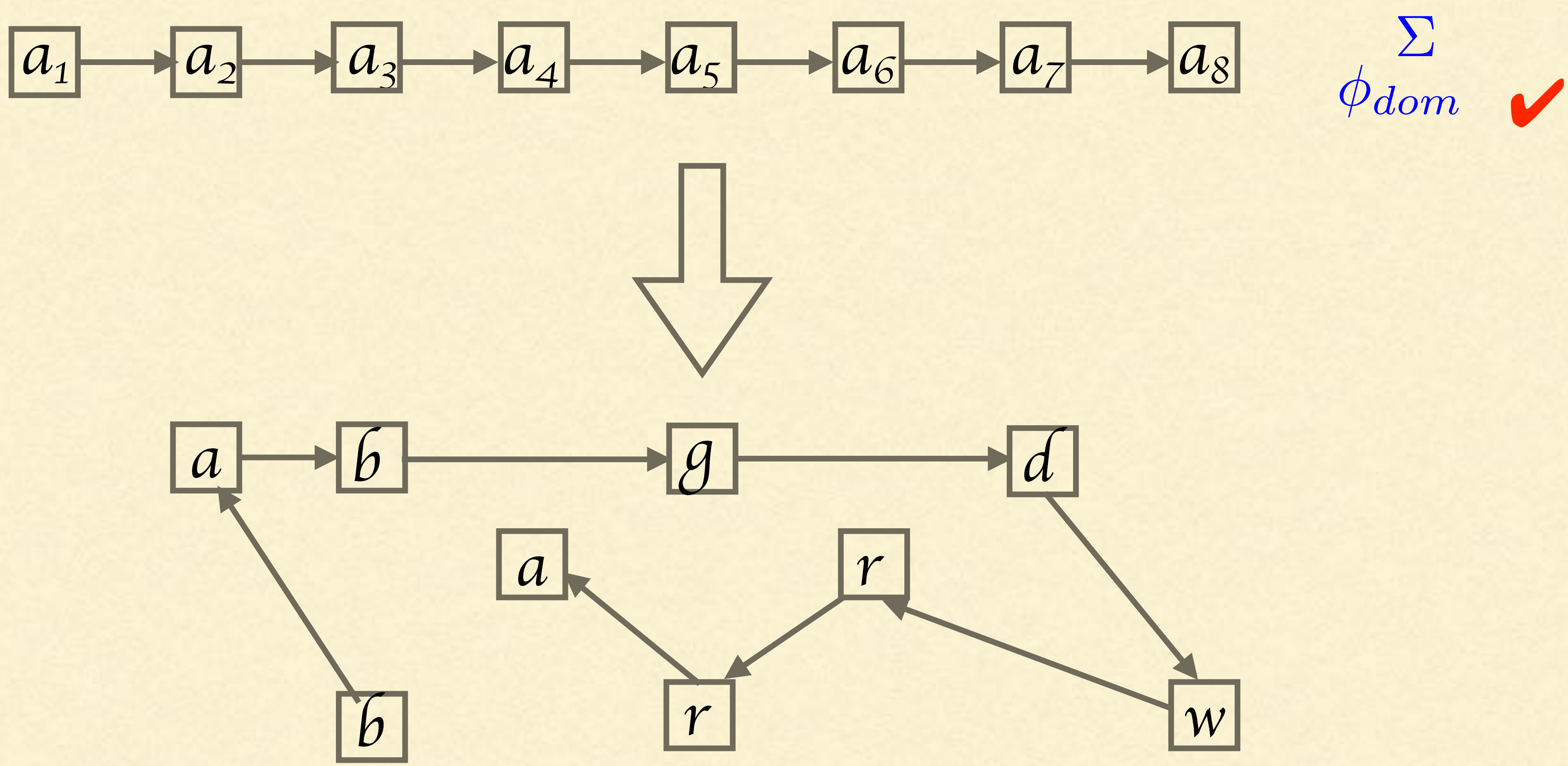
ϕ_{dom} ✔

output

Logic Transducer

[Courcelle'94]

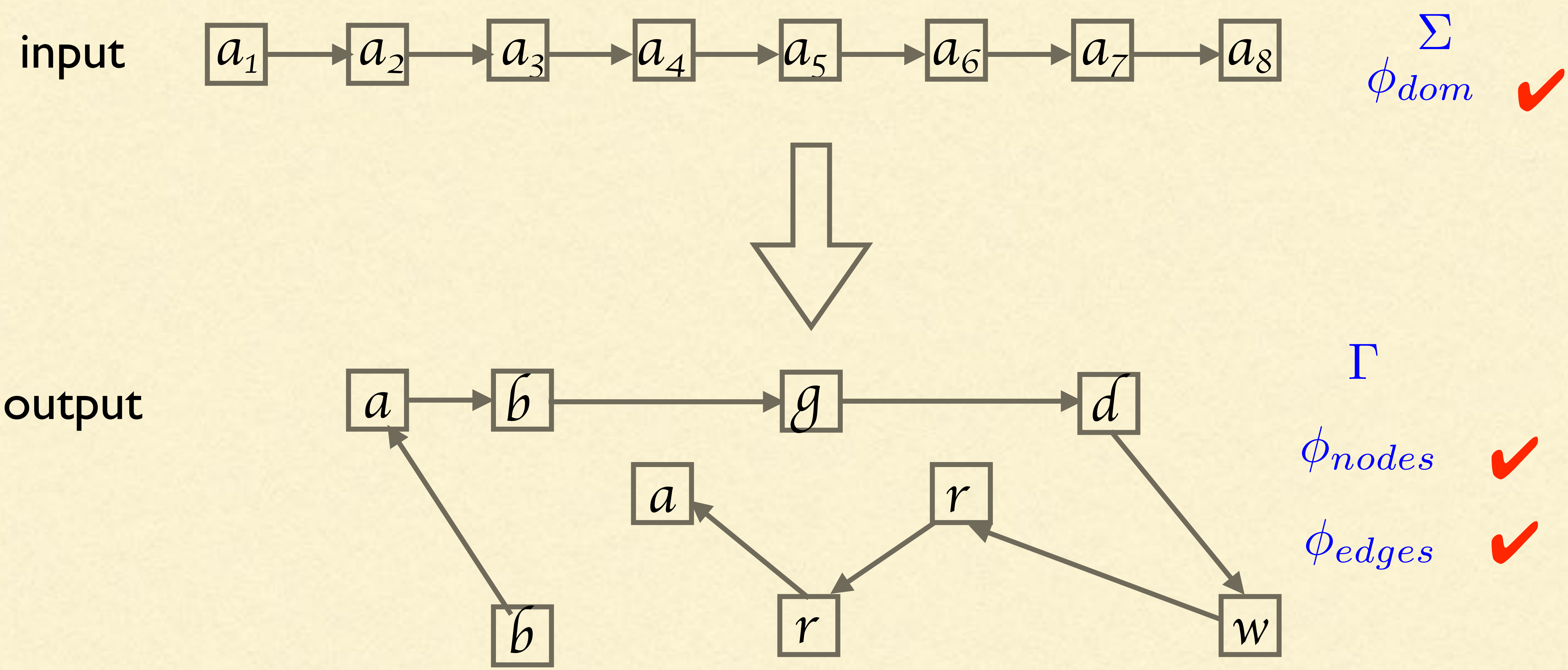
input



output

Logic Transducer

[Courcelle'94]



$$f(u_1\#u_2\#\ldots\#u_n\#v) = u_1^R u_1\#u_2^R u_2\#\ldots\#u_n^R u_n\#v$$

where $u_i \in \{a,b\}^*$ and $v \in \{a,b\}^\omega$

FO/ MSO transducer

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where $u_i \in \{a,b\}^*$ and $v \in \{a,b\}^\omega$

FO/ MSO transducer

input:	a	b	b	$\#$	b	a	$\#$	$(a,b)^\omega$

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where $u_i \in \{a,b\}^*$ and $v \in \{a,b\}^\omega$

FO/ MSO transducer

input: $\boxed{a} \to \boxed{b} \to \boxed{b} \to \boxed{\#} \to \boxed{b} \to \boxed{a} \to \boxed{\#} \to \boxed{(a,b)^\omega}$

$$f(u_1\#u_2\#\ldots\#u_n\#v) = u_1^R u_1\#u_2^R u_2\#\ldots\#u_n^R u_n\#v$$

where $u_i \in \{a,b\}^*$ and $v \in \{a,b\}^\omega$

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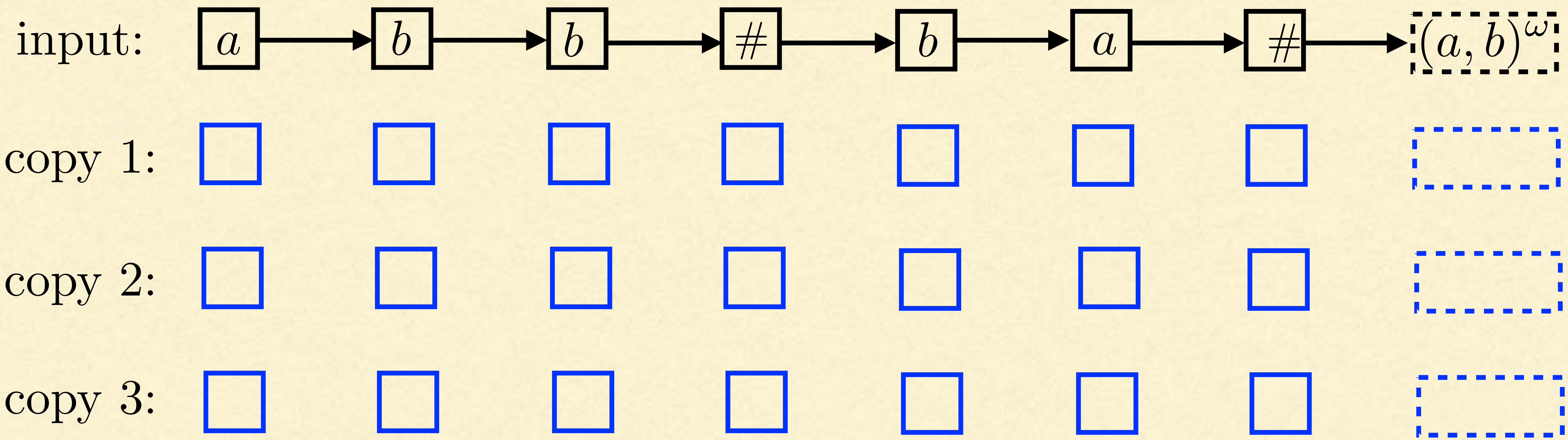

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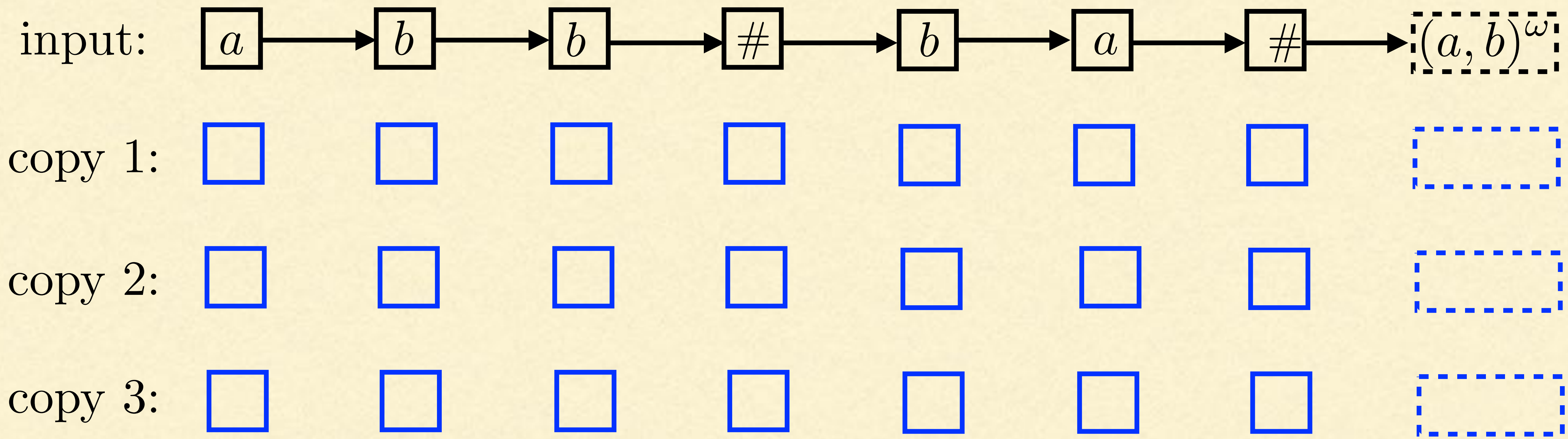

$$\phi_\gamma^1(x) = \phi_\gamma^2(x) = L_\gamma(x) \wedge \neg L_\#(x) \wedge \text{reach}_\#(x)$$

$$\phi_\gamma^3(x) = L_\#(x) \vee \neg\text{reach}_\#(x)$$

$$\text{reach}_\#(x) = \exists y(x \prec y \wedge L_\#(y))$$

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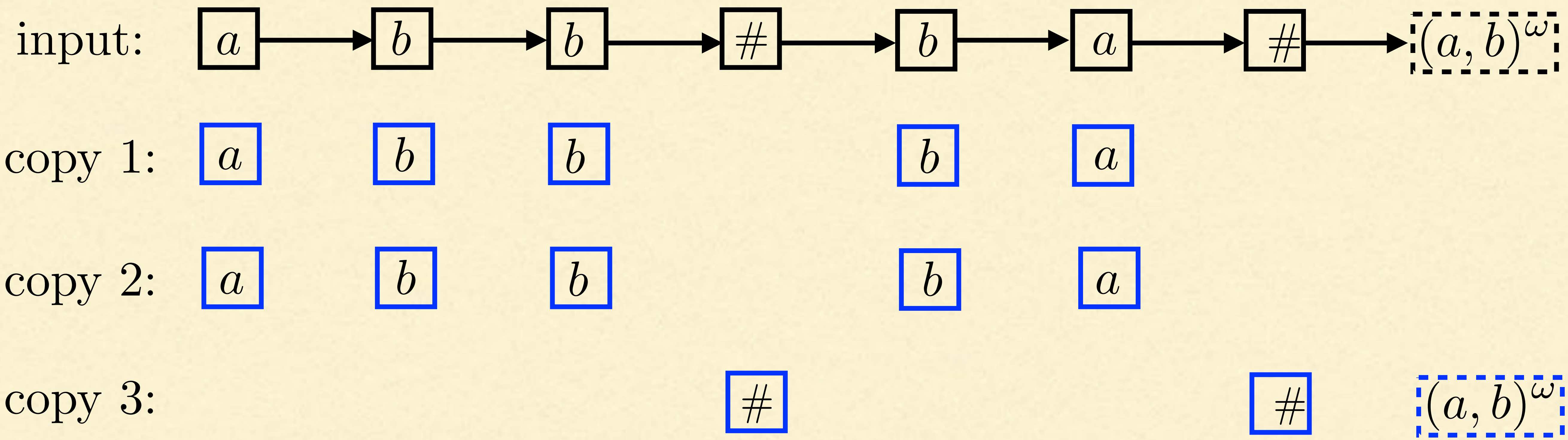

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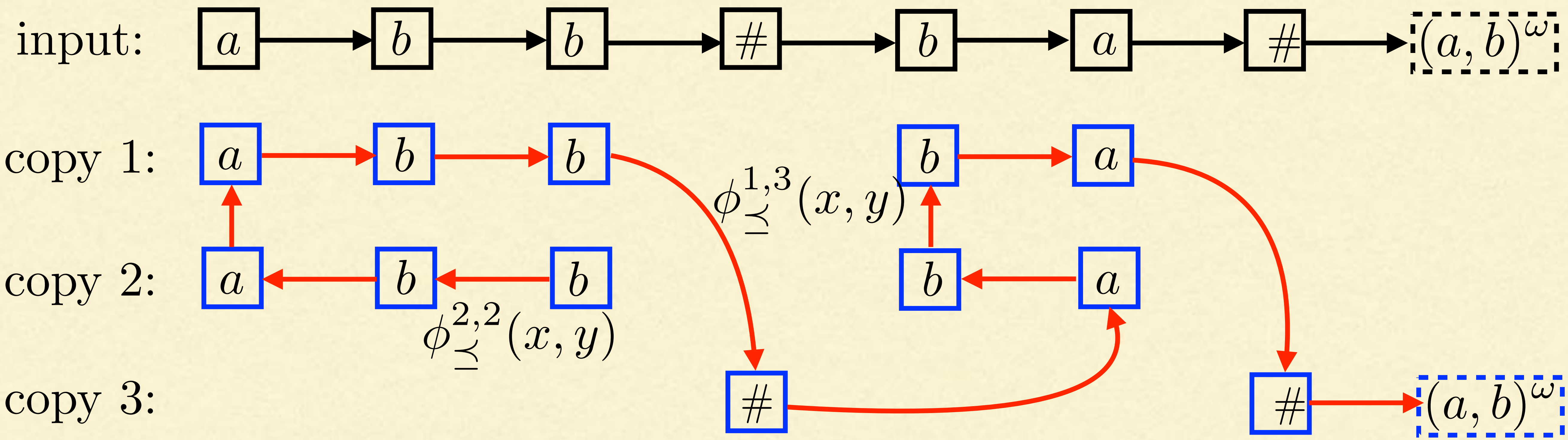
where $u_i \in \{a,b\}^*$ and $v \in \{a,b\}^\omega$

FO/ MSO transducer



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Two way machine

Two way machine

[Rabin, Scott'59]

[Ehrich, Yau'71]

Two way machine

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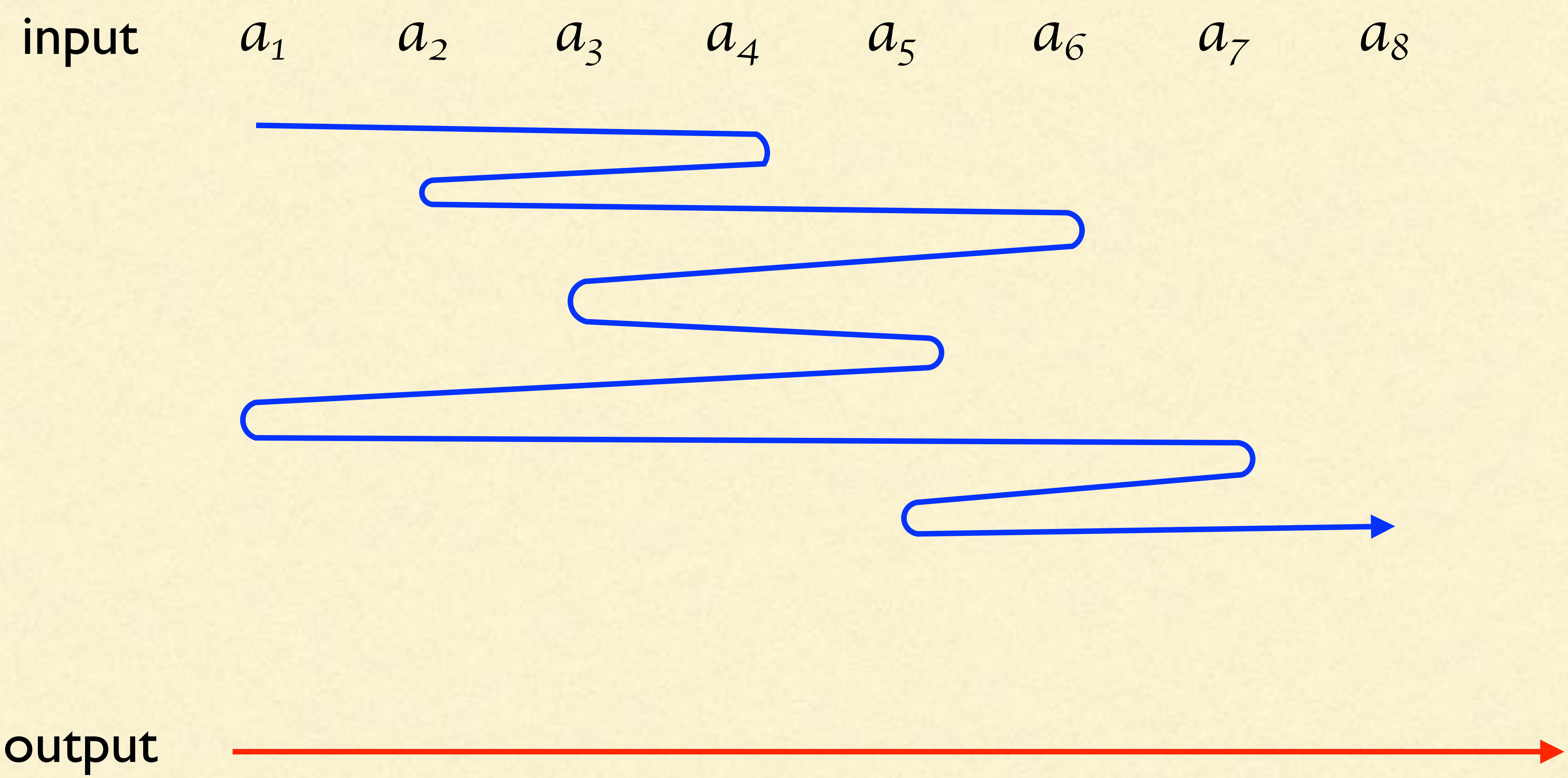

Look-around automaton

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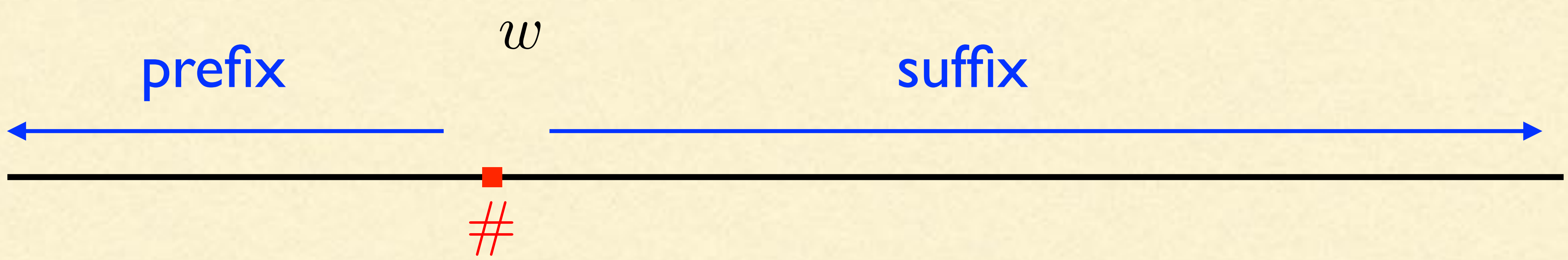

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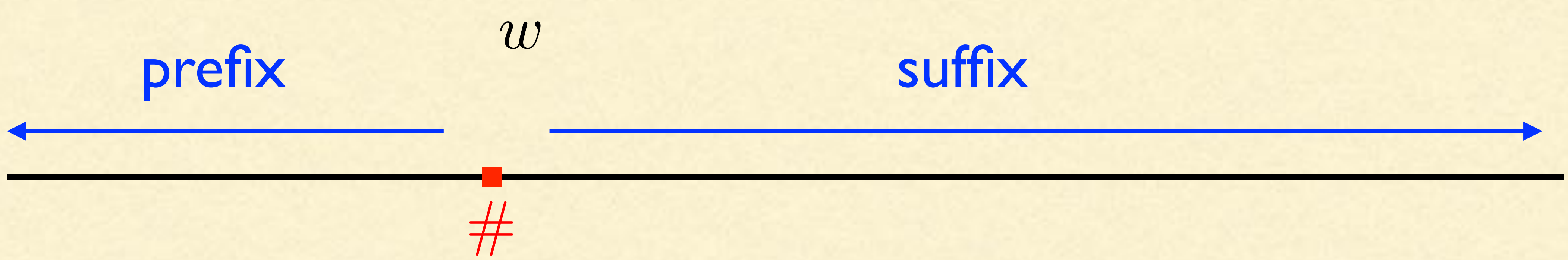

property: suffix does not contain #

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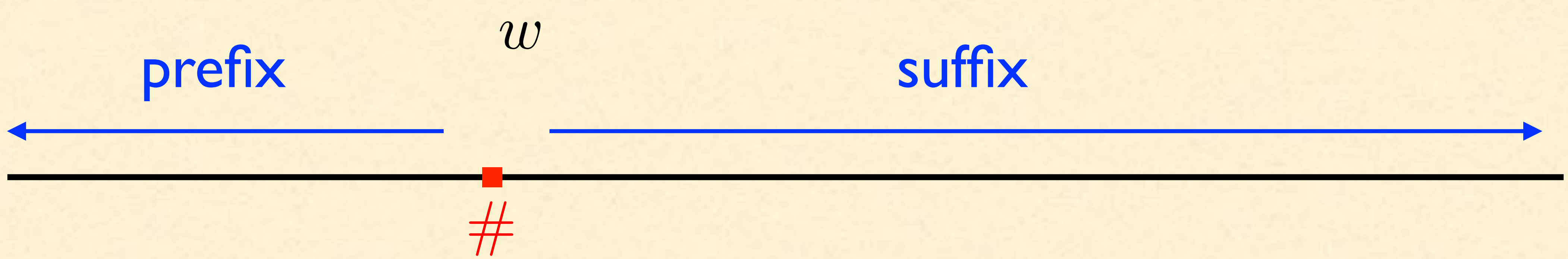

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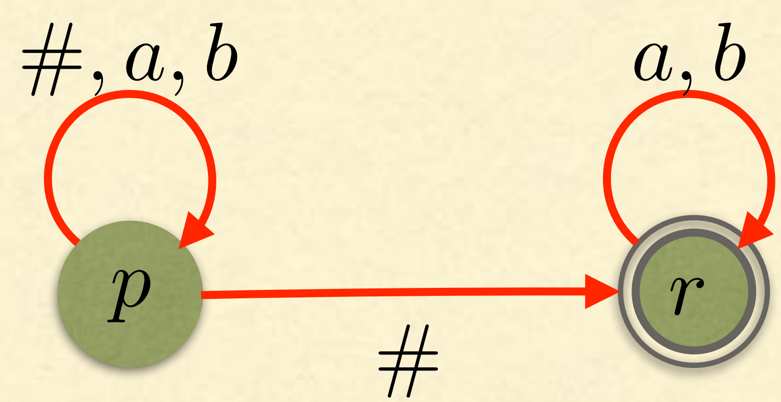

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Two way Transducer (2WST)

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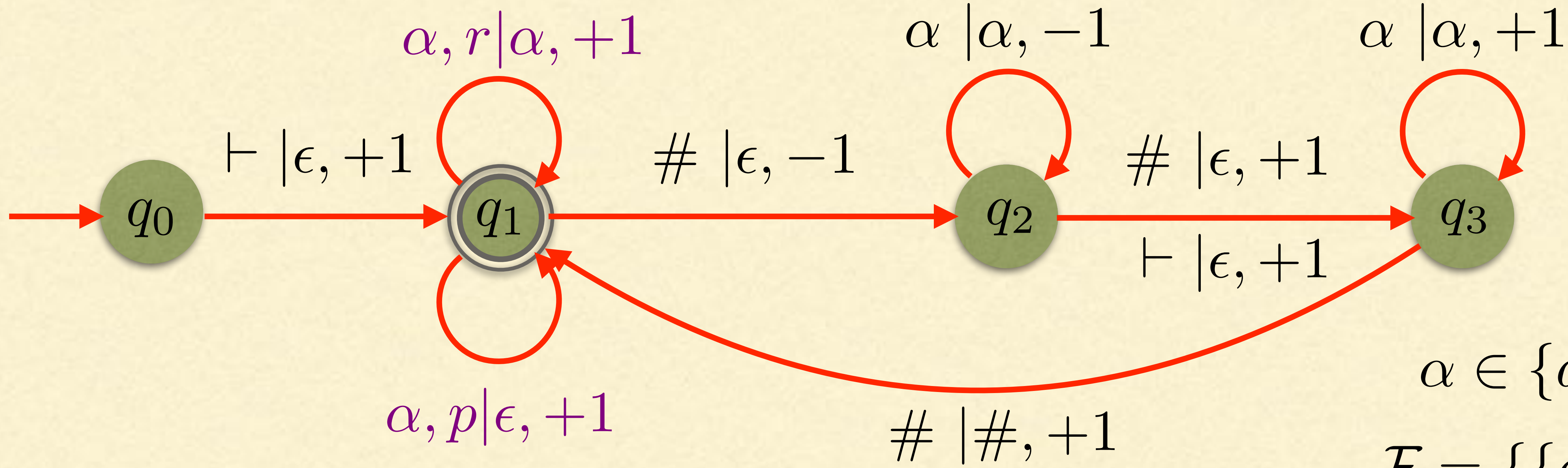

$\alpha \in \{a,b\}$

$\mathcal{F} = \{\{q_1\}\}$

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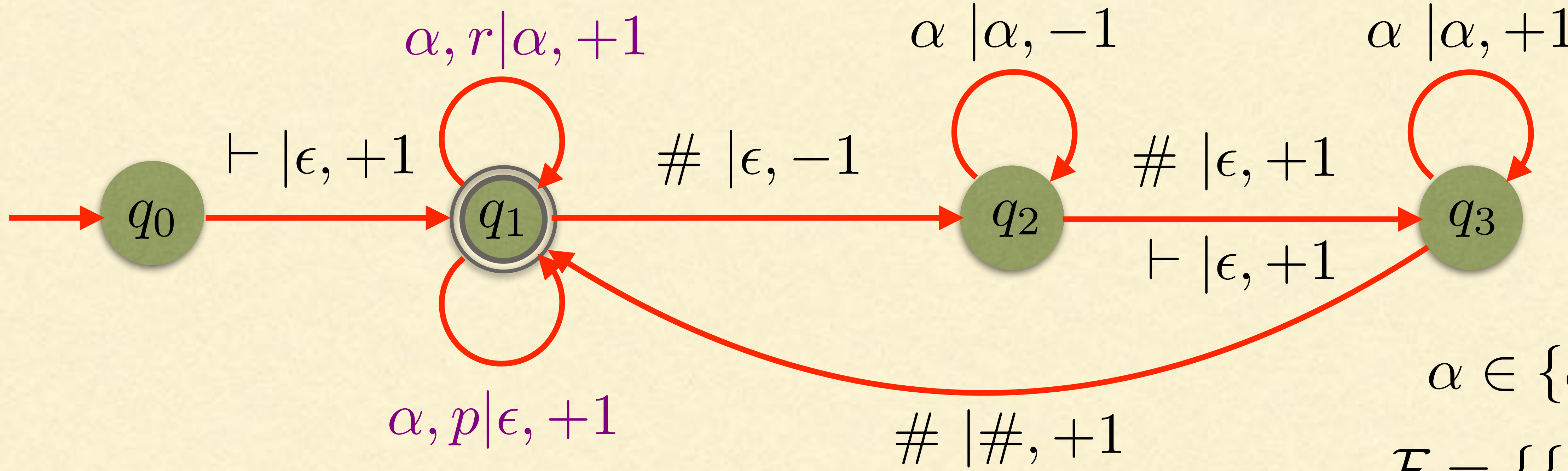

$\alpha \in \{a,b\}$

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$\vdash$ a b b # (a + b)$^\omega$

$\uparrow$

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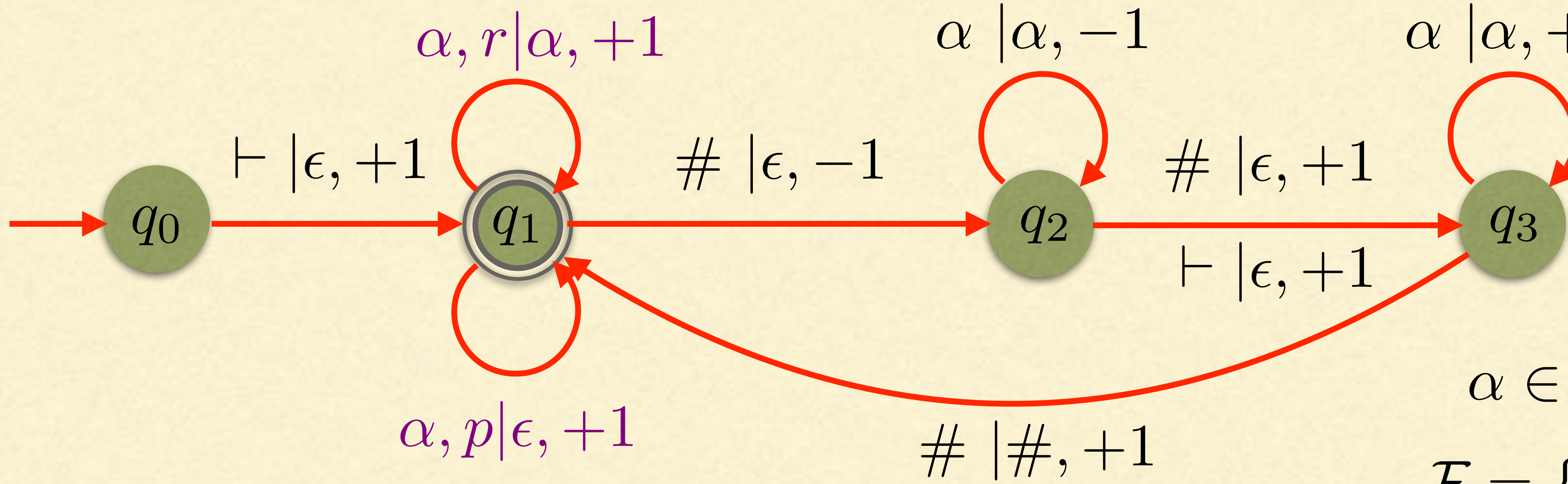

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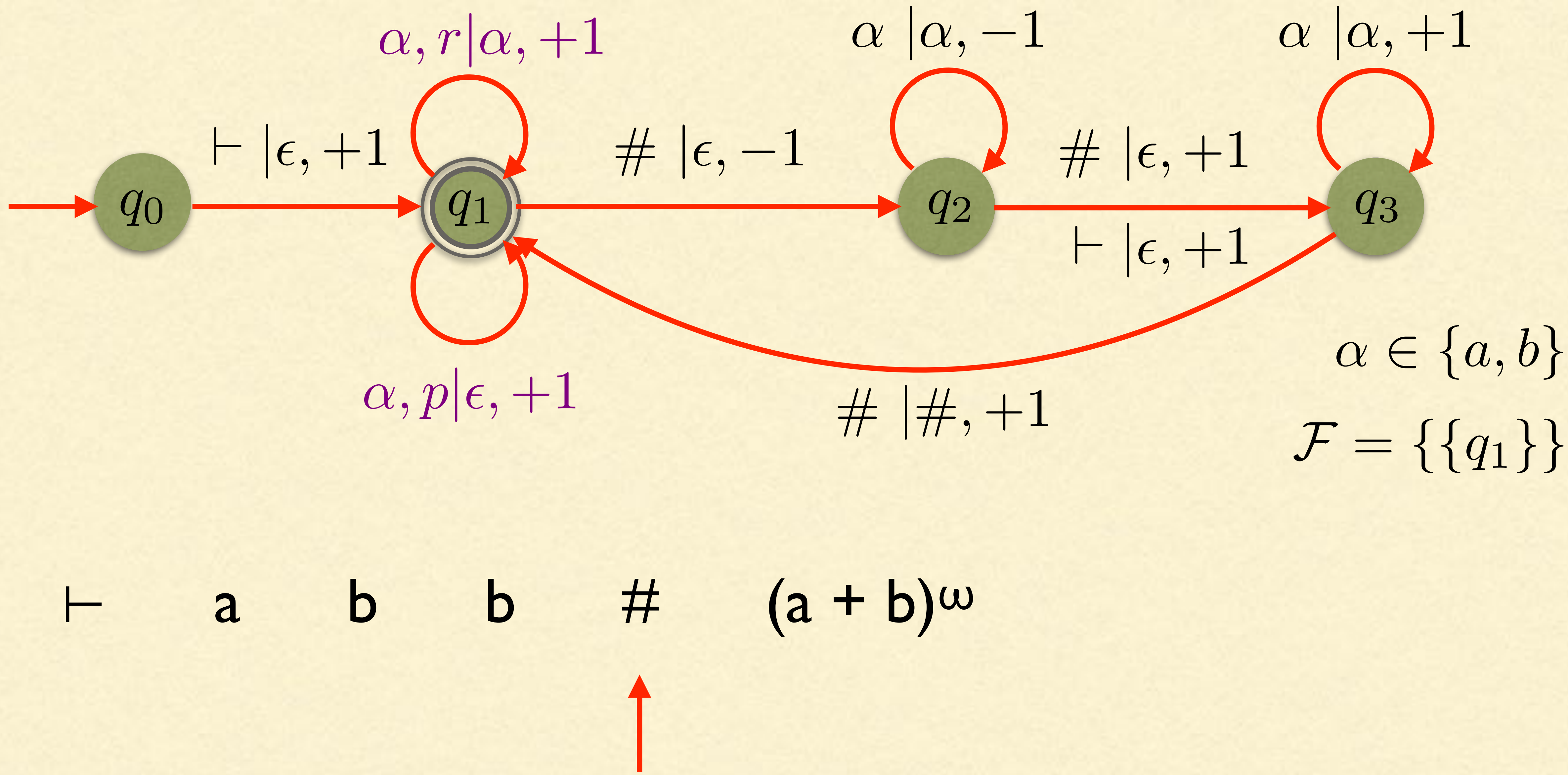

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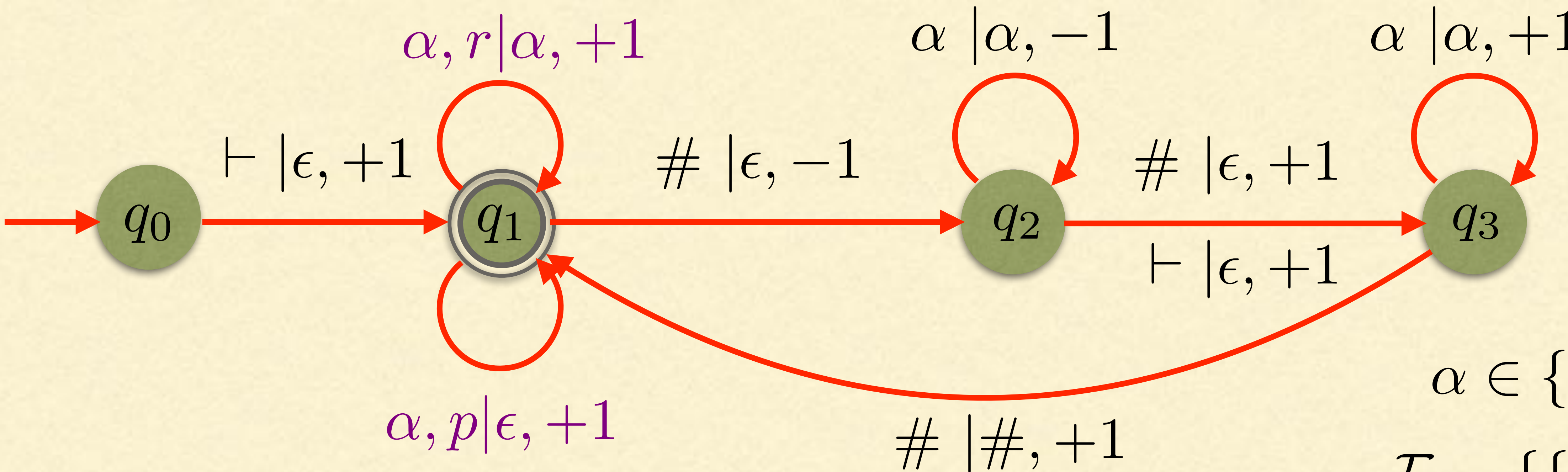

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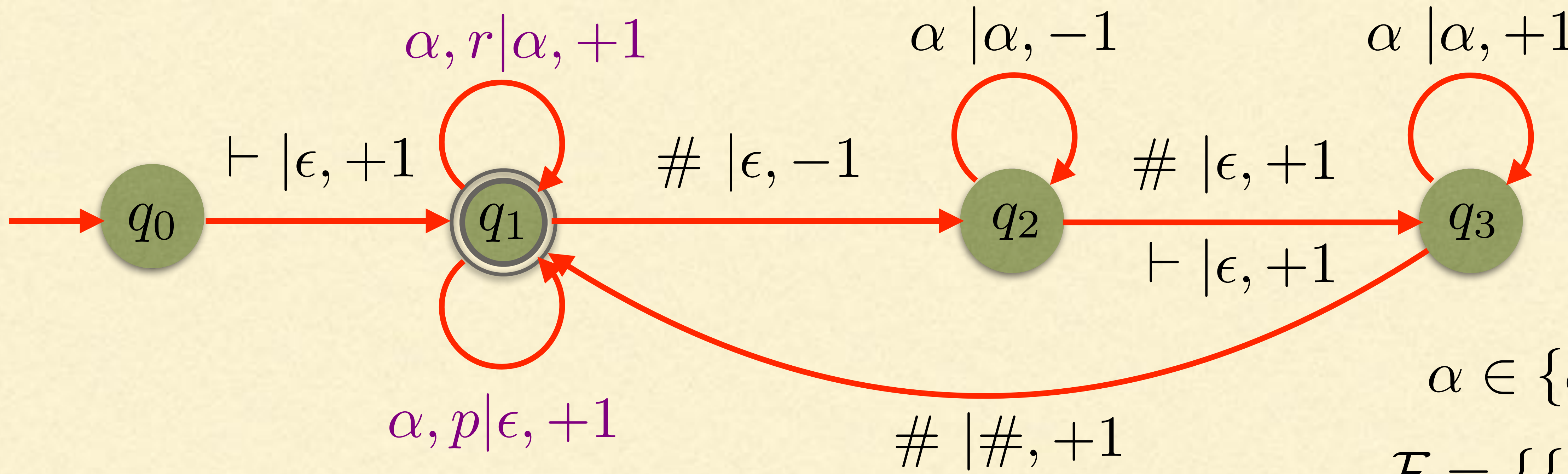

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b b a

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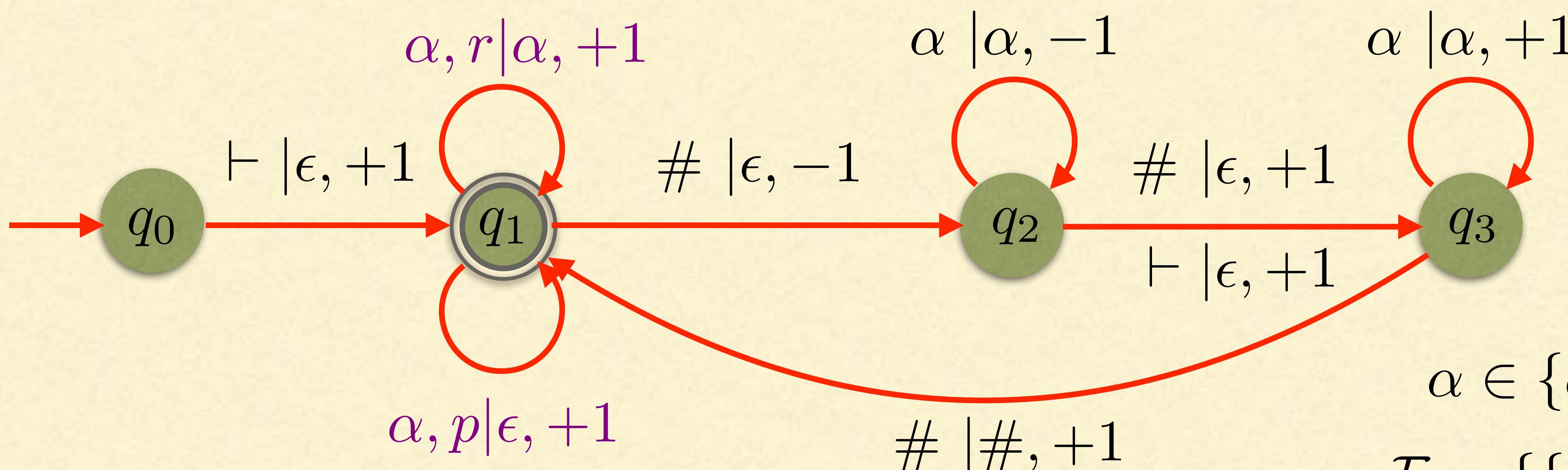

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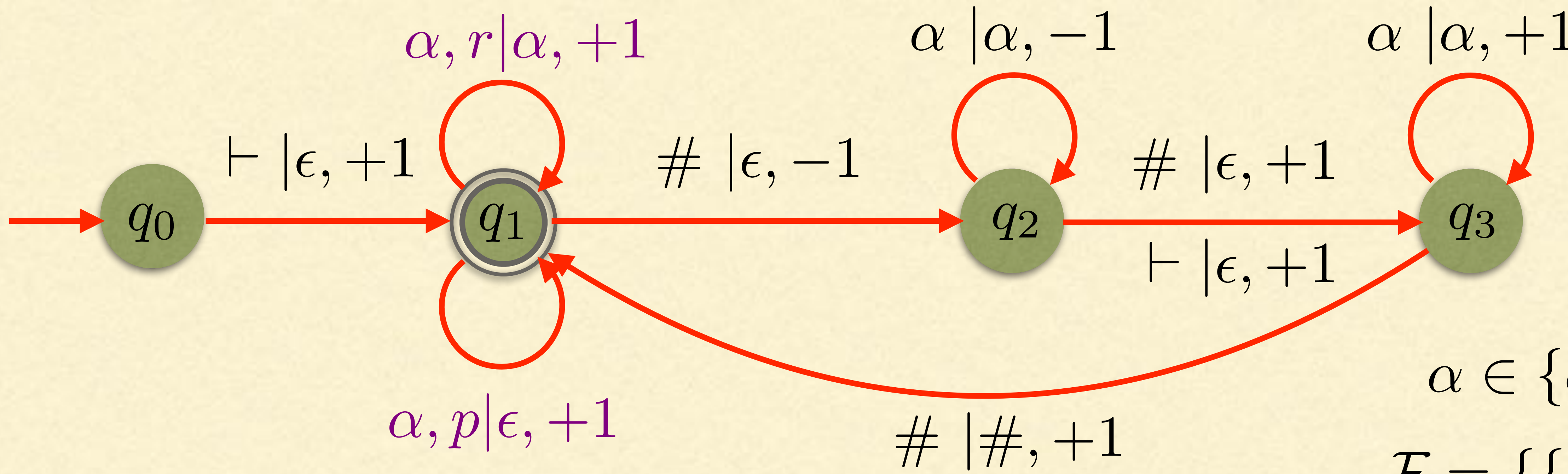

$\alpha \in \{a,b\}$

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⊢ a b b # (a + b)$^\omega$

b b a a b b

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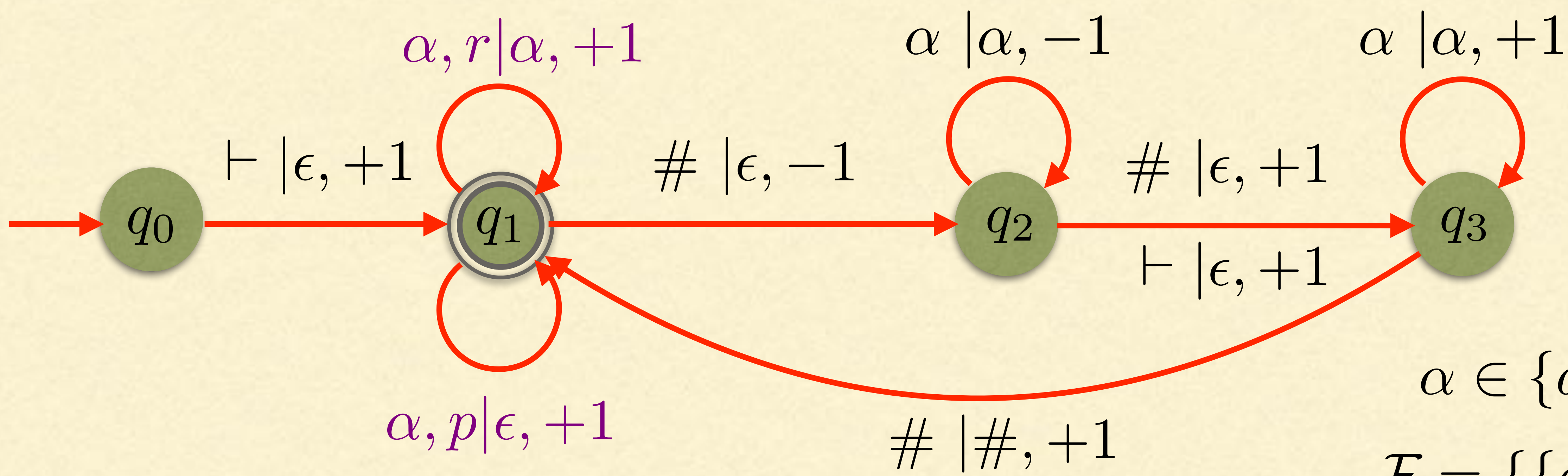

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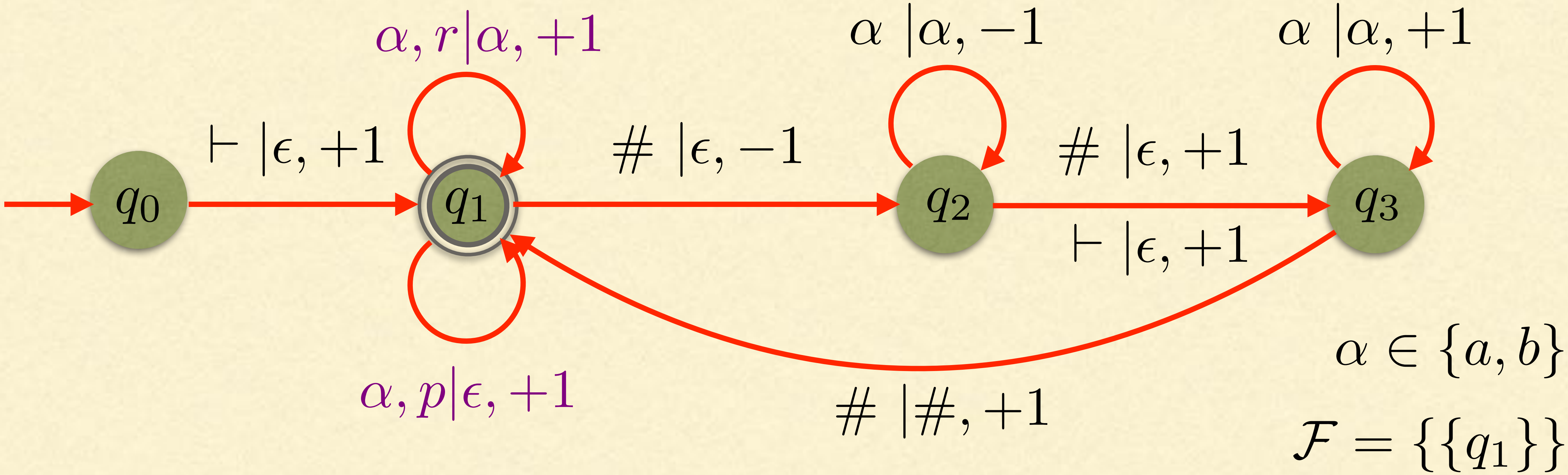

⊢ a b b # $(a + b)^\omega$

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One way machine with finite registers

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[Alur, Černý'10]

One way machine with finite registers

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input : a_1 a_2 a_3 a_4

One way machine with finite registers

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input : a_1 a_2 a_3 a_4

registers : $x, y, z \ldots$

One way machine with finite registers

[Alur, Černý'10]

input : a_1 a_2 a_3 a_4

a_1	a_2	a_3	a_4
$x = ax$	$x = \varepsilon$	$x = ax$	$x = \varepsilon$
$y = bycz$	$y = cz$	$y = bycz$	$y = cz$
$z = c$	$z = by$	$z = c$	$z = by$

registers : $x, y, z\ldots$

One way machine with finite registers

[Alur, Černý'10]

input :

a_1	a_2	a_3	a_4
$x = ax$	$x = \varepsilon$	$x = ax$	$x = \varepsilon$
$y = bycz$	$y = cz$	$y = bycz$	$y = cz$
$z = c$	$z = by$	$z = c$	$z = by$

registers : $x, y, z \ldots$

output : $x\ a\ y\ z$

One way machine with finite registers

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registers : $x, y, z \ldots$

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copyless update :

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registers : $x, y, z \ldots$

output : $x\ a\ y\ z$

copyless update :

$$x = ax \qquad y = bycz \qquad z = c$$

$$x = ax \qquad y = bycz \qquad z = cx$$

One way machine with finite registers

[Alur, Černý'10]

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registers : $x, y, z\ldots$

output : $x\ a\ y\ z$

copyless update :

$$x = ax \quad y = bycz \quad z = c \qquad ✔$$

$$x = ax \quad y = bycz \quad z = cx \qquad ✘$$

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Streaming String Transducer (SST)

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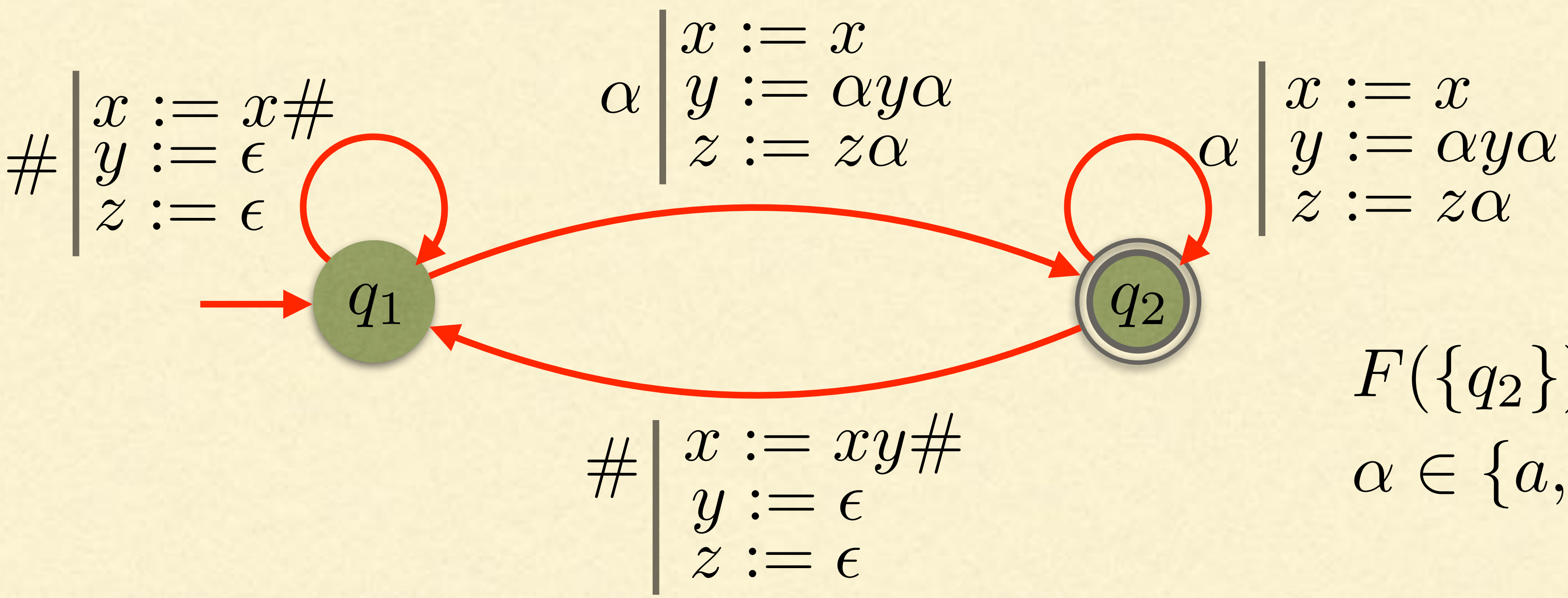

$F(\{q_2\}) = xz$

$\alpha \in \{a, b\}$

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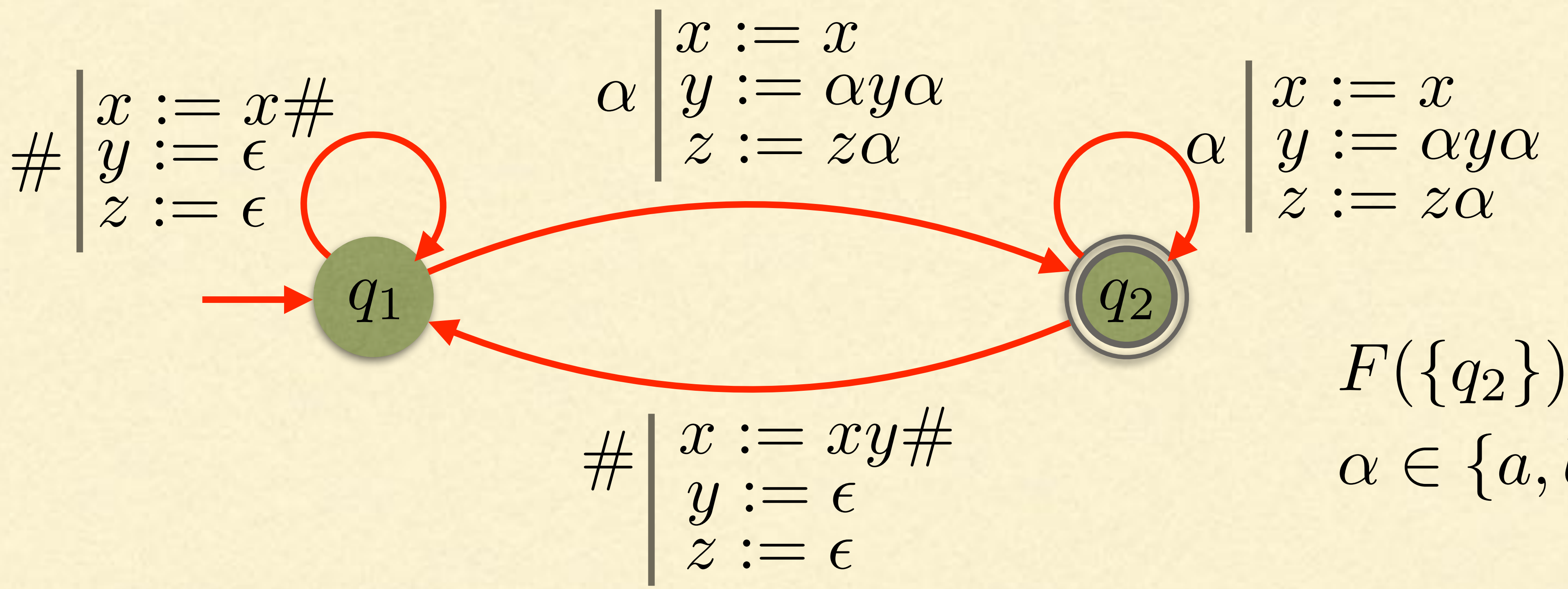

$F(\{q_2\}) = xz$

$\alpha \in \{a,b\}$

	a	b	b	#	(a + b)$^\omega$
x:	ε				
y:	ε				
z:	ε				

$$f(u_1\#u_2\#\ldots\#u_n\#v) = u_1^R u_1\#u_2^R u_2\#\ldots\#u_n^R u_n\#v$$

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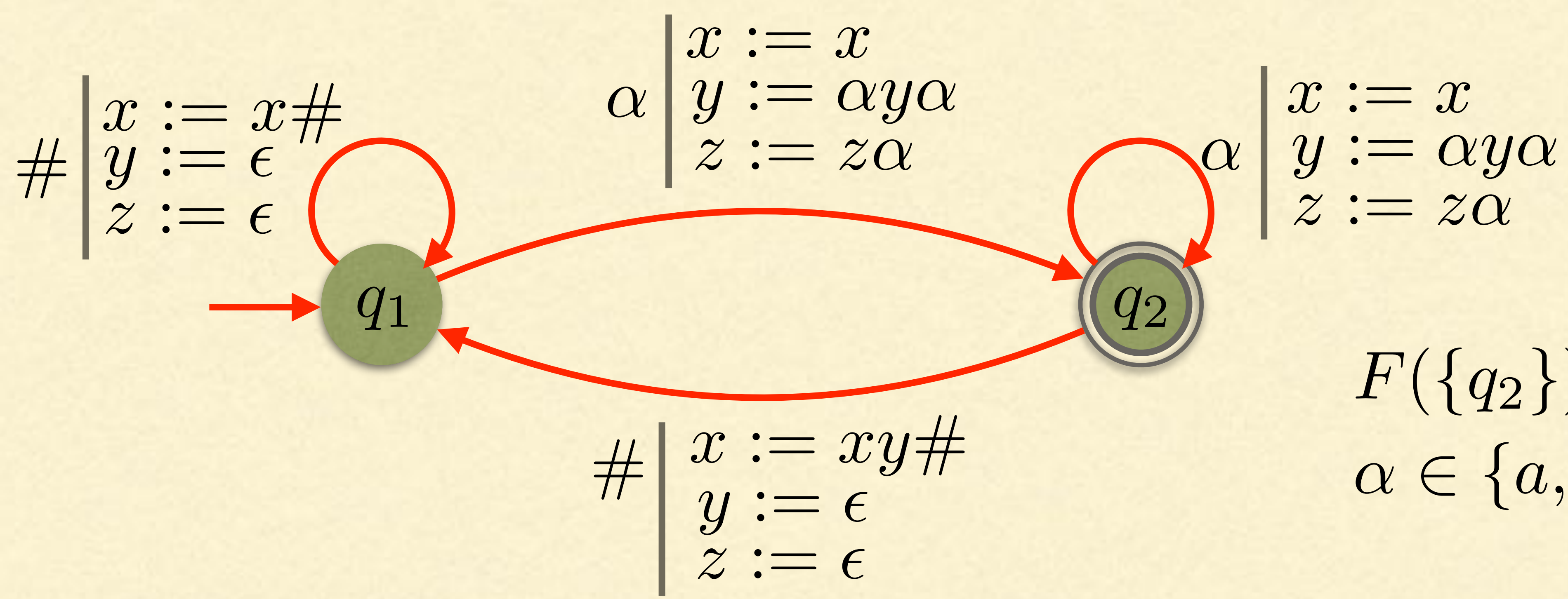

$F(\{q_2\}) = xz$

$\alpha \in \{a,b\}$

		a	b	b	#	$(a+b)^\omega$
x:	ε	ε				
y:	ε	aa				
z:	ε	a				

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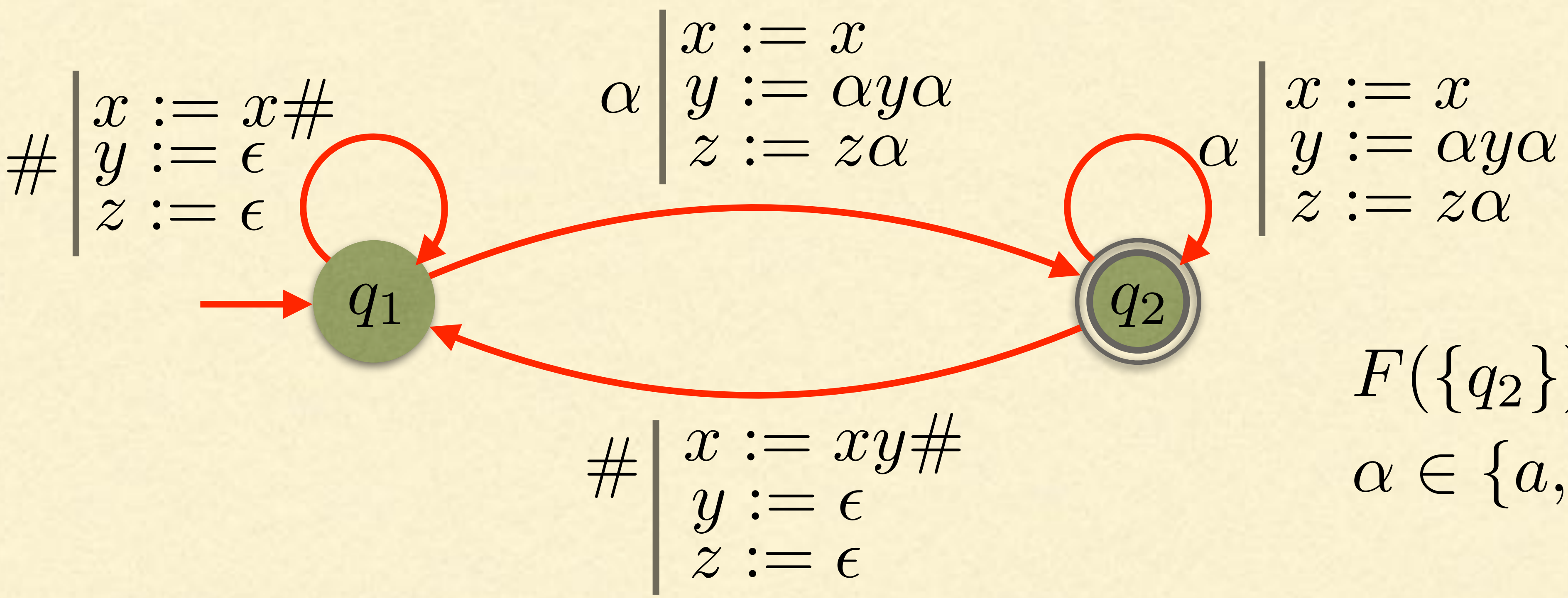


$F(\{q_2\}) = xz$

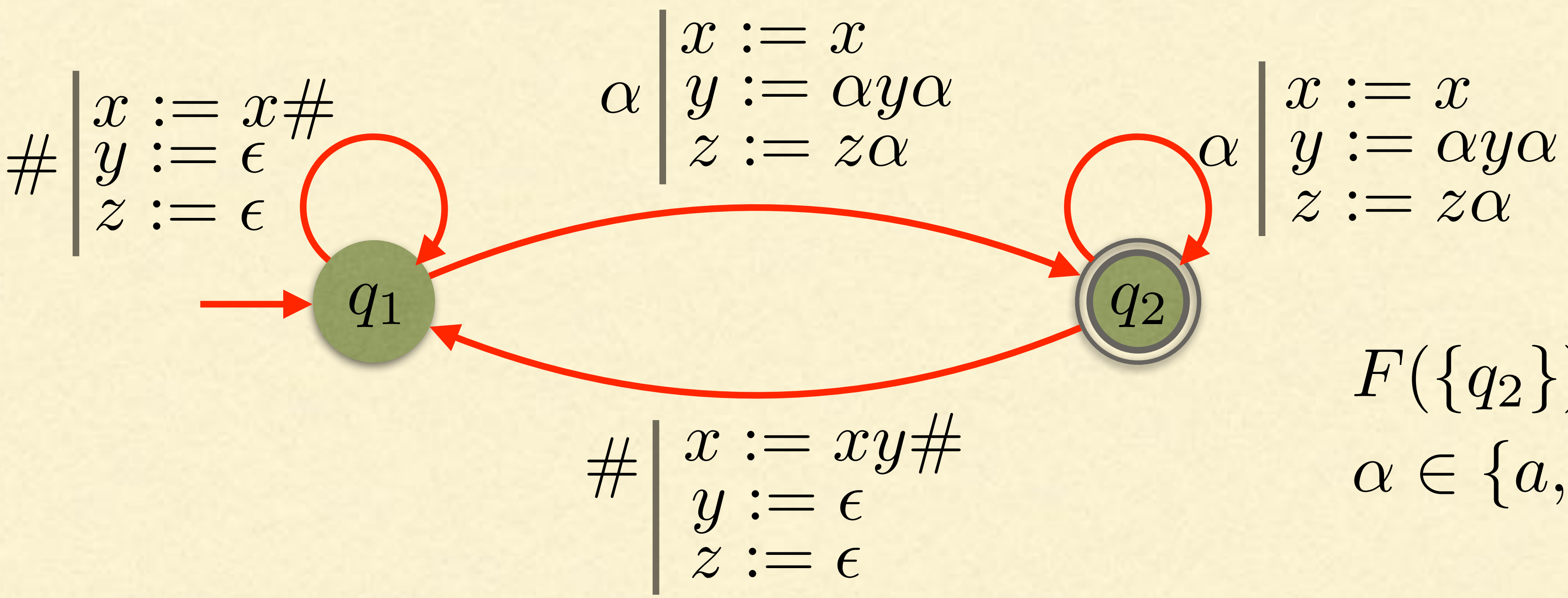
$\alpha \in \{a,b\}$

		a	b	b	#	$(a + b)^\omega$
x:	ε	ε	ε	ε		
y:	ε	aa	baab	bbaabb		
z:	ε	a	ab	abb		

$$f(u_1\#u_2\#\ldots\#u_n\#v) = u_1^R u_1\#u_2^R u_2\#\ldots\#u_n^R u_n\#v$$

where $u_i \in \{a,b\}^*$ and $v \in \{a,b\}^\omega$

Streaming String Transducer (SST)



$F(\{q_2\}) = xz$

$\alpha \in \{a,b\}$

		a	b	b	#	$(a+b)^\omega$
x:	ε	ε	ε	ε	bbaabb#	
y:	ε	aa	baab	bbaabb	ε	
z:	ε	a	ab	abb	ε	

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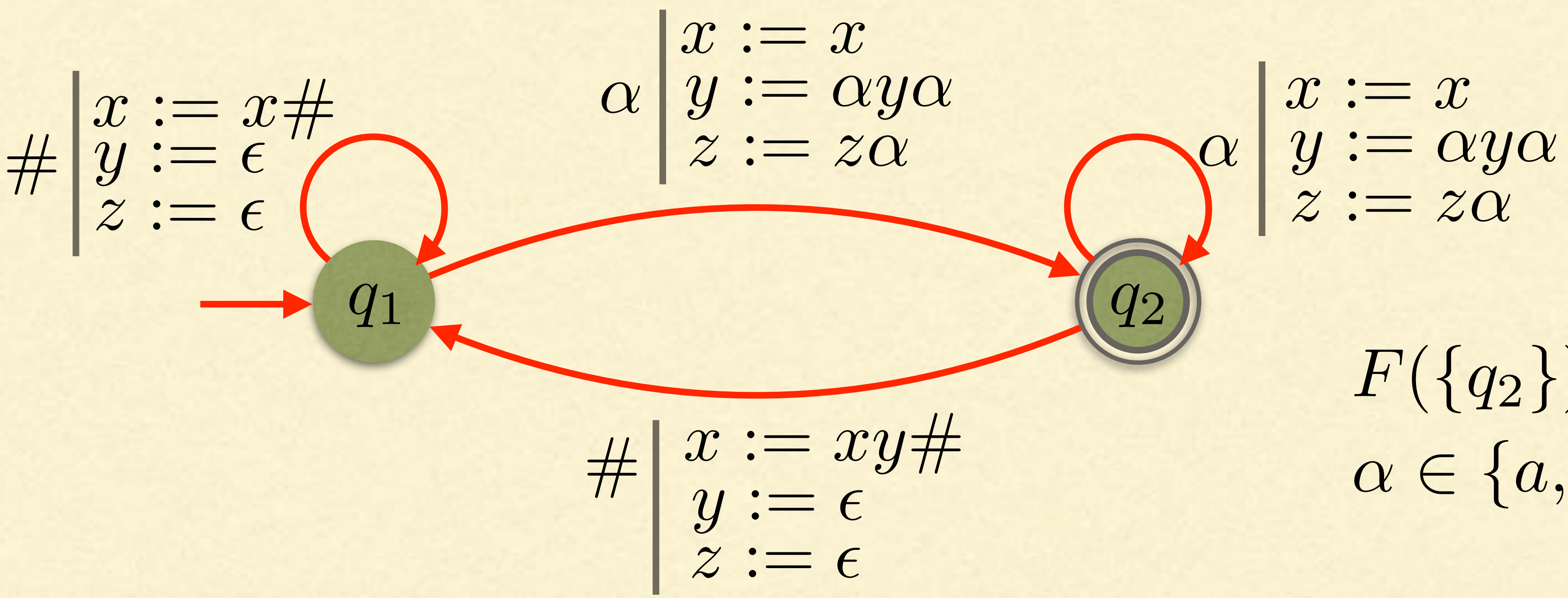

$F(\{q_2\}) = xz$

$\alpha \in \{a,b\}$

		a	b	b	#	(a + b)$^\omega$
x:	ε	ε	ε	ε	bbaabb#	bbaabb#
y:	ε	aa	baab	bbaabb	ε	
z:	ε	a	ab	abb	ε	(a + b)$^\omega$

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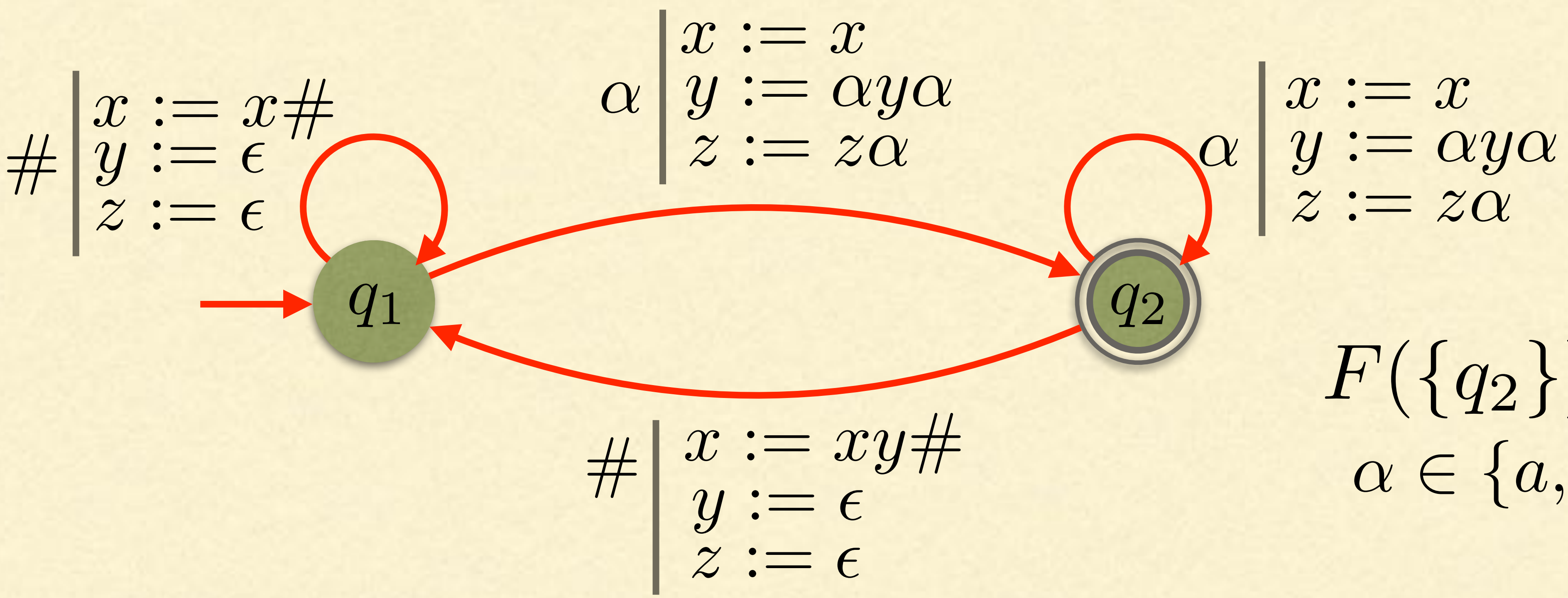
Streaming String Transducer (SST)



$F(\{q_2\}) = xz$

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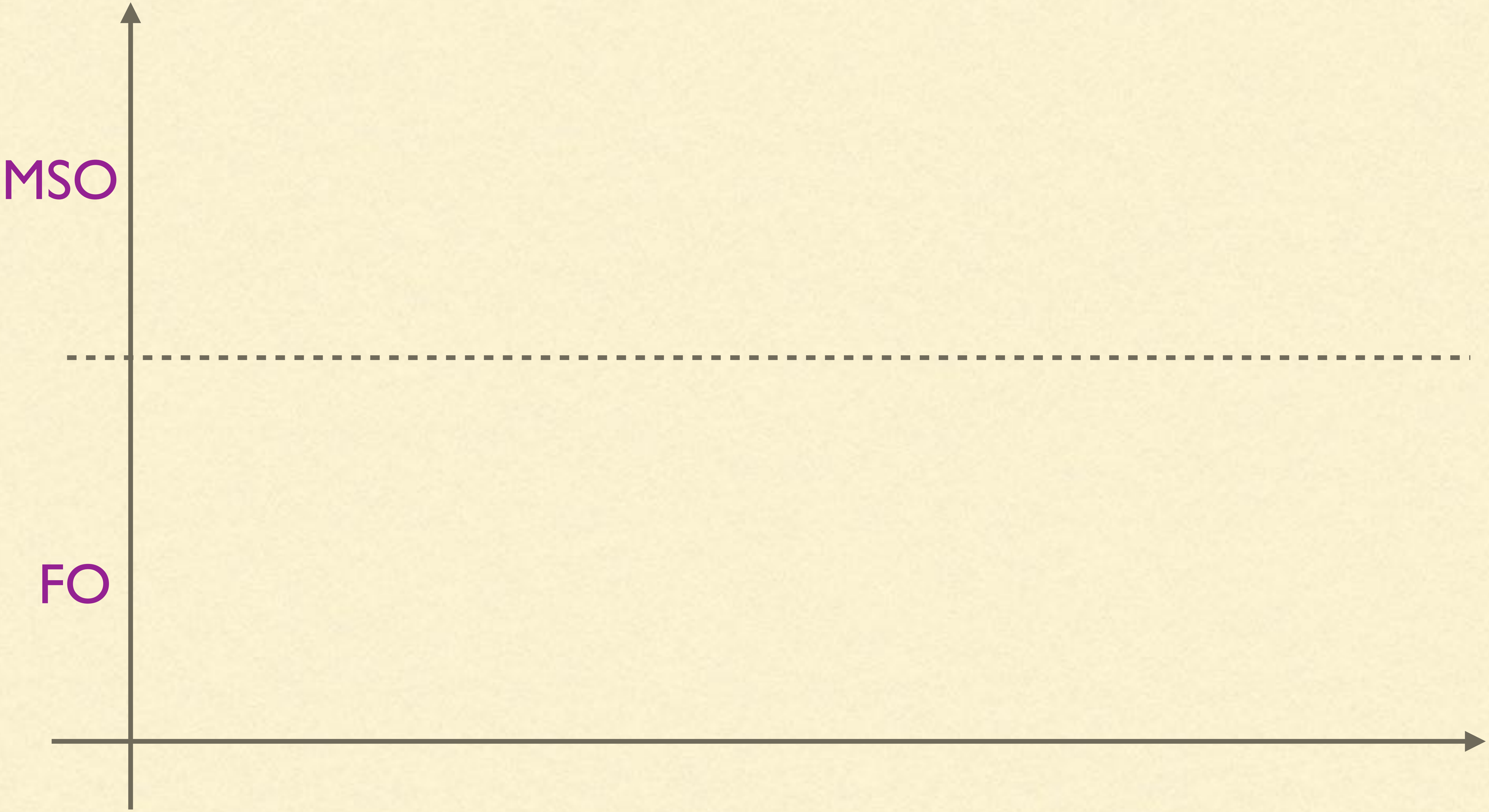
		a	b	b	#	$(a + b)^\omega$
x:	ε	ε	ε	ε	bbaabb#	bbaabb#
y:	ε	aa	baab	bbaabb	ε	
z:	ε	a	ab	abb	ε	$(a + b)^\omega$

Related Work

Related Work

Related Work



Related Work

Related Work

MSOT = 2WST

[Engelfriet, Hoogeboom'01]

MSO

MSOT = SST

[Alur, Černý'10]

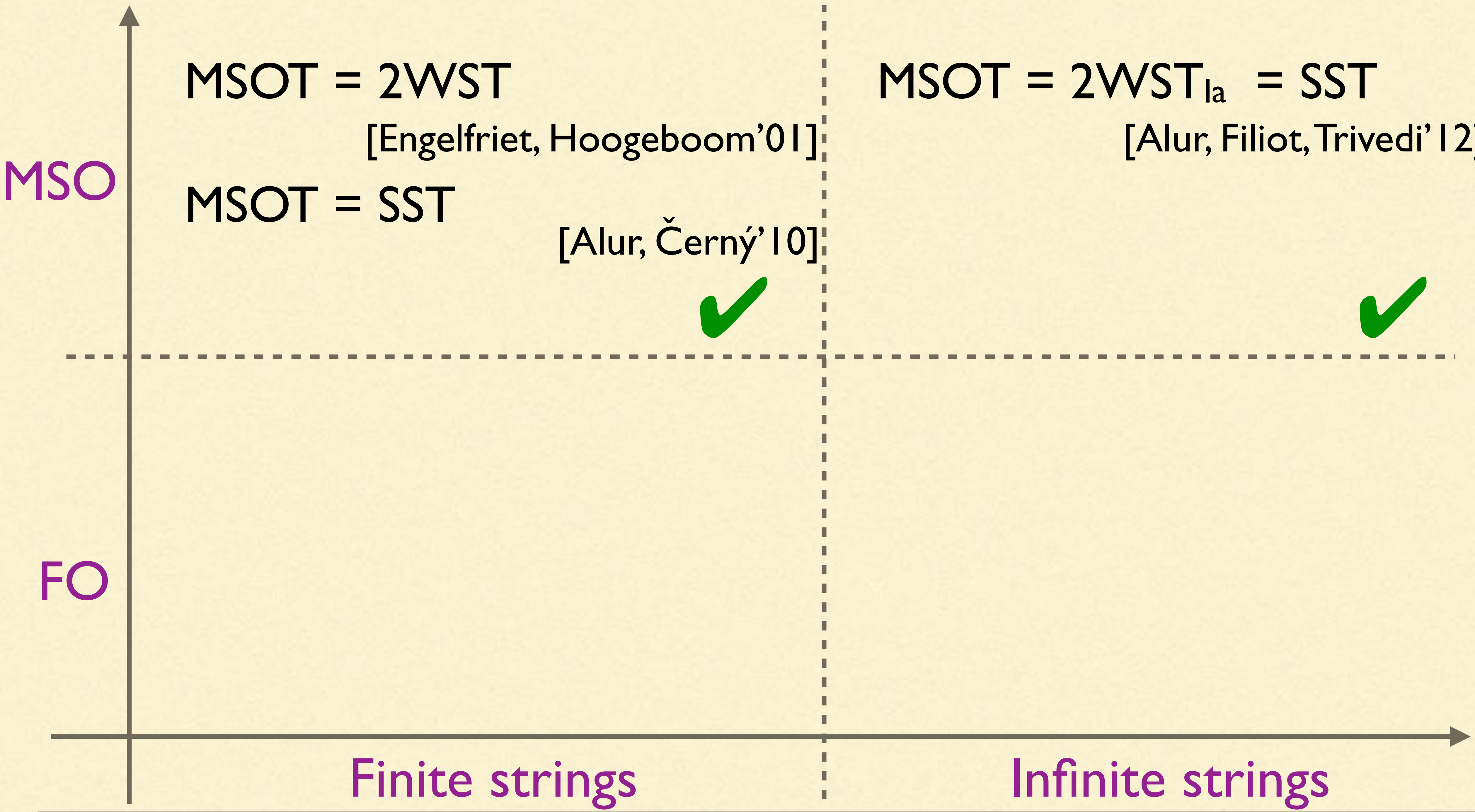
✔

FO

Finite strings

Infinite strings

Related Work



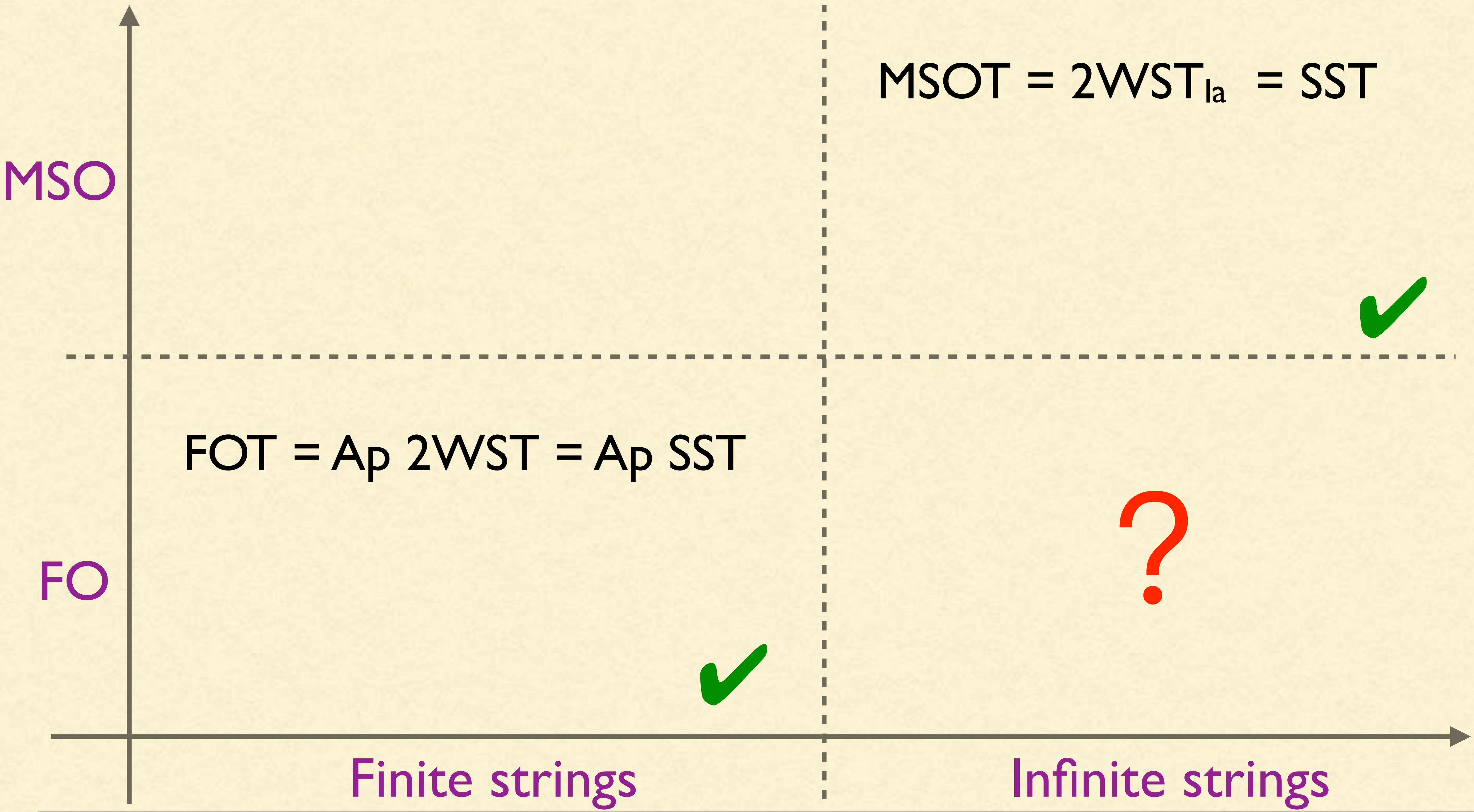
Related Work

	Finite strings	Infinite strings
MSO	MSOT = 2WST [Engelfriet, Hoogeboom'01] MSOT = SST [Alur, Černý'10] ✔	MSOT = $2WST_{la}$ = SST [Alur, Filiot, Trivedi'12] ✔
FO	FOT = Ap 2WST [Carton, Dartois'15] FOT = Ap SST [Filiot, Krishna, Trivedi'14] ✔	

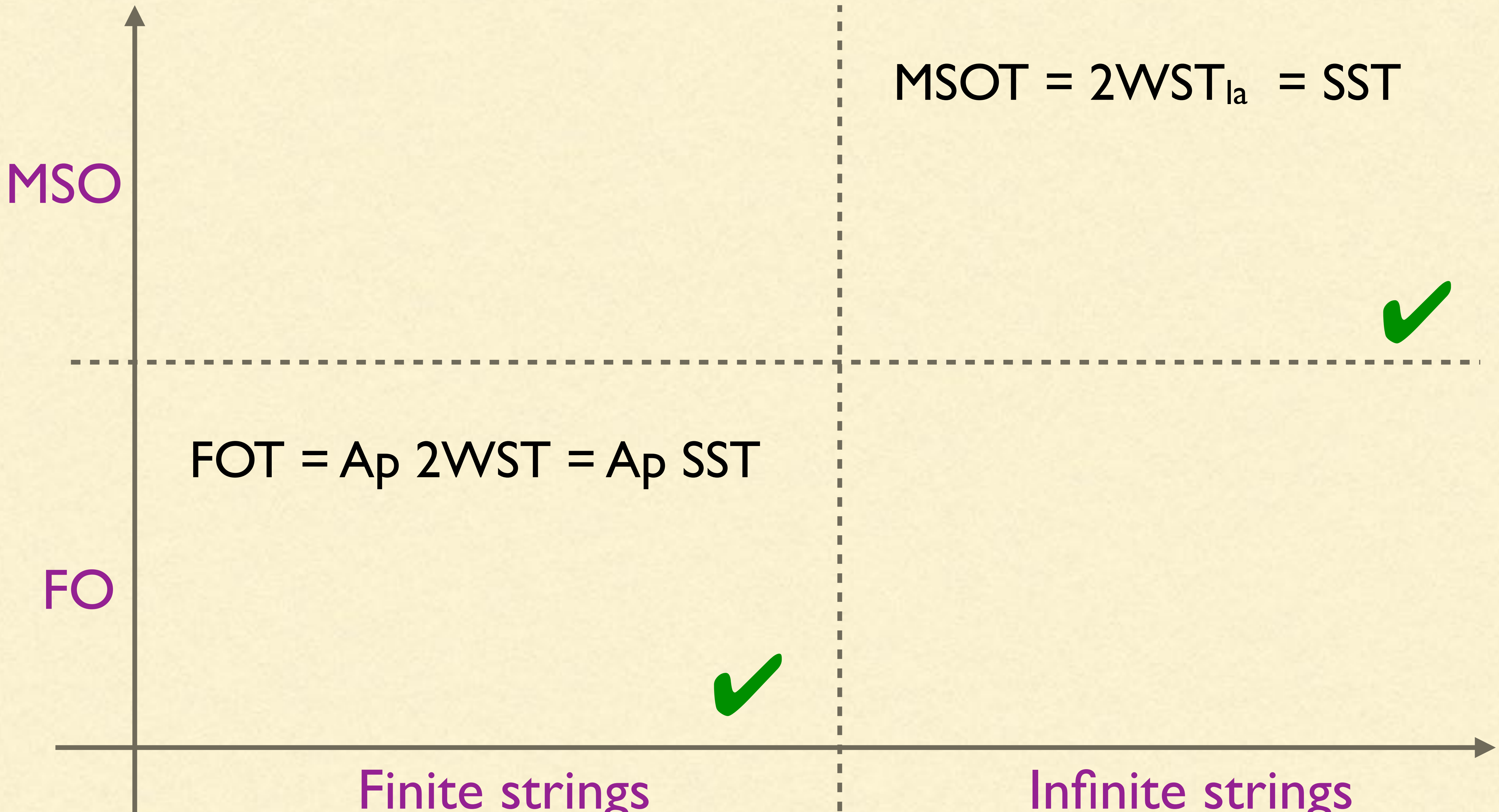
Related Work

	Finite strings	Infinite strings
MSO	MSOT = 2WST [Engelfriet, Hoogeboom'01] MSOT = SST [Alur, Černý'10] ✔	MSOT = $2WST_{la}$ = SST [Alur, Filiot, Trivedi'12] ✔
FO	FOT = Ap 2WST [Carton, Dartois'15] FOT = Ap SST [Filiot, Krishna, Trivedi'14] ✔	?

Related Work



Related Work



Related Work

$\mathrm{MSOT} = \mathrm{2WST}_{la} = \mathrm{SST}$ ✔

MSO

$\mathrm{FOT} = \mathrm{Ap}\ \mathrm{2WST} = \mathrm{Ap}\ \mathrm{SST}$ ✔

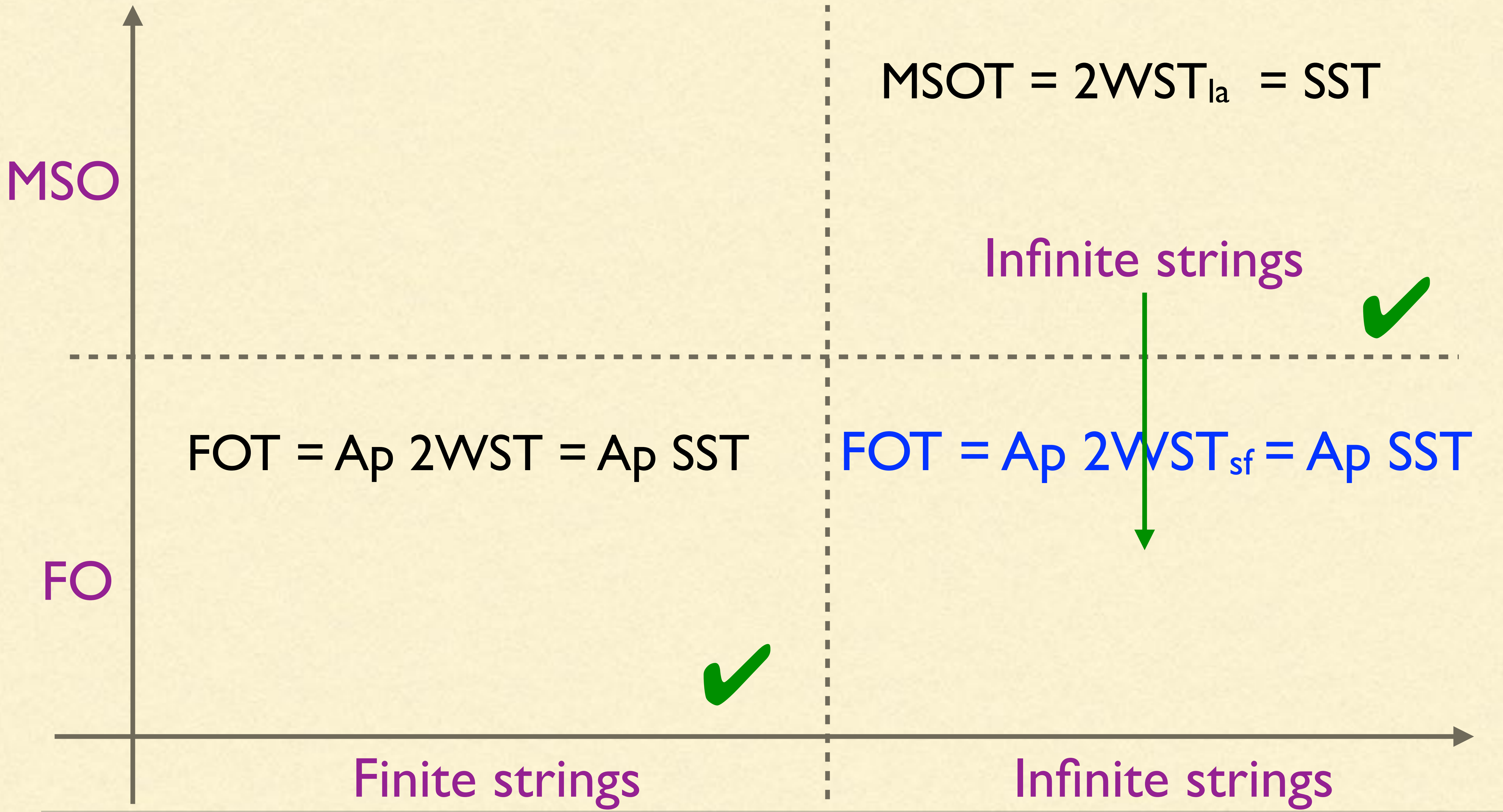
$\mathrm{FOT} = \mathrm{Ap}\ \mathrm{2WST}_{sf} = \mathrm{Ap}\ \mathrm{SST}$

FO

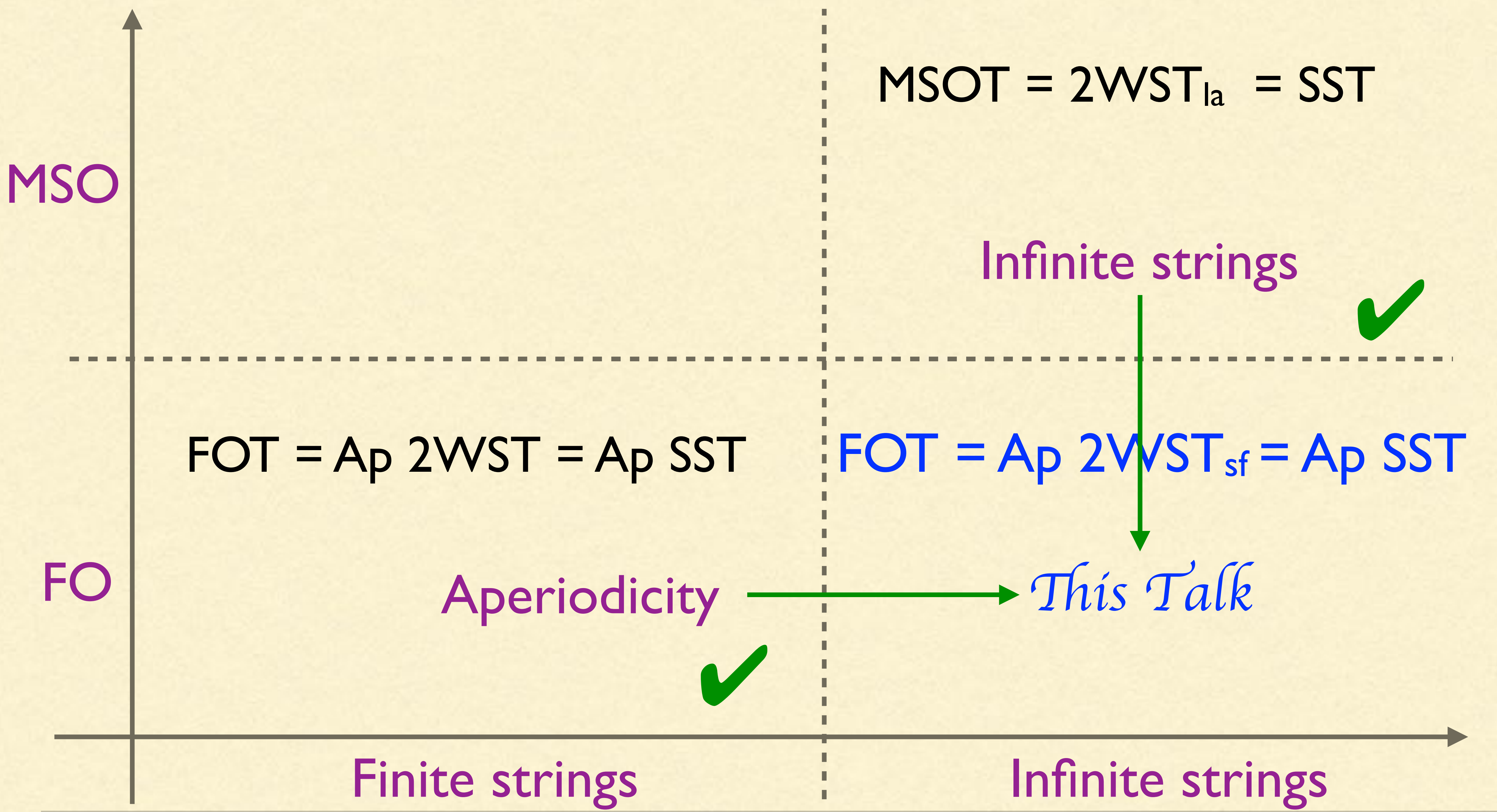
Finite strings

Infinite strings

Related Work



Related Work



Outline

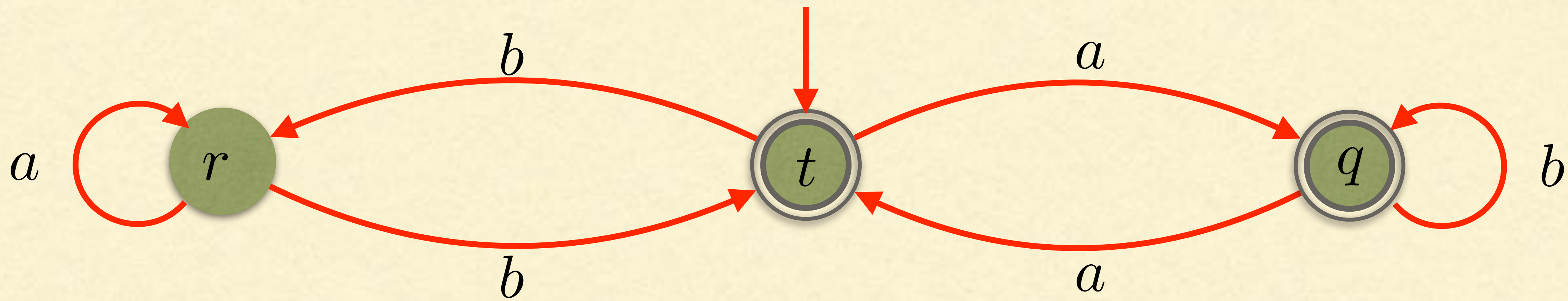
- Introduction
  - Three formalisms for transductions
  - Related work
- Aperiodic transformations for Infinite strings
  - Aperiodic two way transducer
  - Aperiodic streaming string transducer
- Equivalence results and Proof ideas
  - $SST_{sf} \subset FOT = 2WST_{sf} \subset SST_{sf}$
- Conclusion

Transition monoid of a Finite state automaton

[Schutzenberger'65]

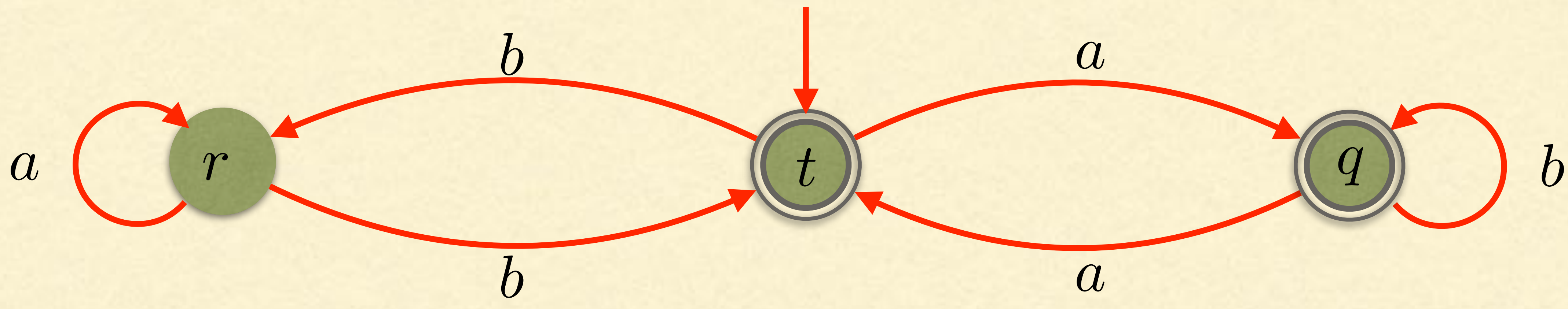
Transition monoid of a Finite state automaton

[Schutzenberger'65]



Transition monoid of a Finite state automaton

[Schutzenberger'65]

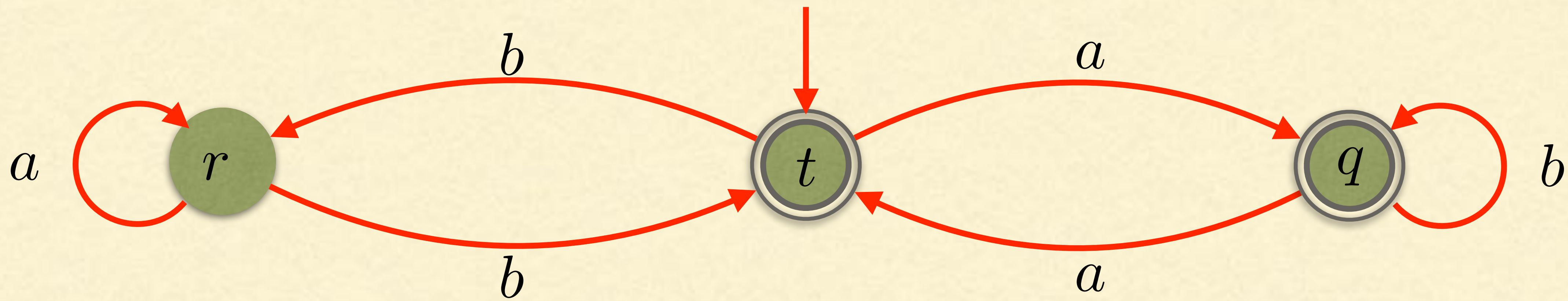


$\mathcal{M}_{\mathcal{A}} = (M_{\mathcal{A}}, \times, 1)$

$M_{\mathcal{A}} \subseteq \{0,1\}^{|Q| \times |Q|}$

Transition monoid of a Finite state automaton

[Schutzenberger'65]

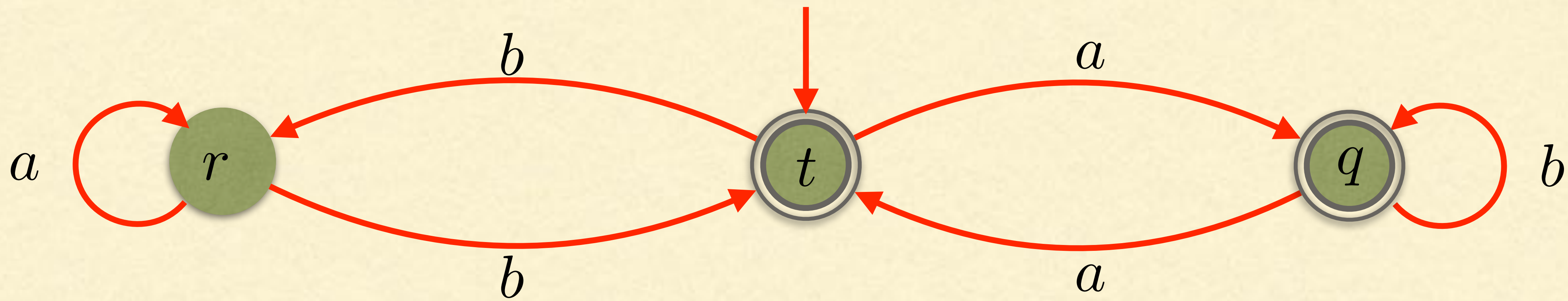


$$\mathcal{M}_{\mathcal{A}} = (M_{\mathcal{A}}, \times, 1) \qquad M_{\mathcal{A}} \subseteq \{0,1\}^{|Q|\times|Q|}$$

$$M_{abb} = \begin{array}{c} \\ t \\ q \\ r \end{array} \begin{array}{c} \begin{array}{ccc} t & q & r \end{array} \\ \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \end{array}$$

Transition monoid of a Finite state automaton

[Schutzenberger'65]



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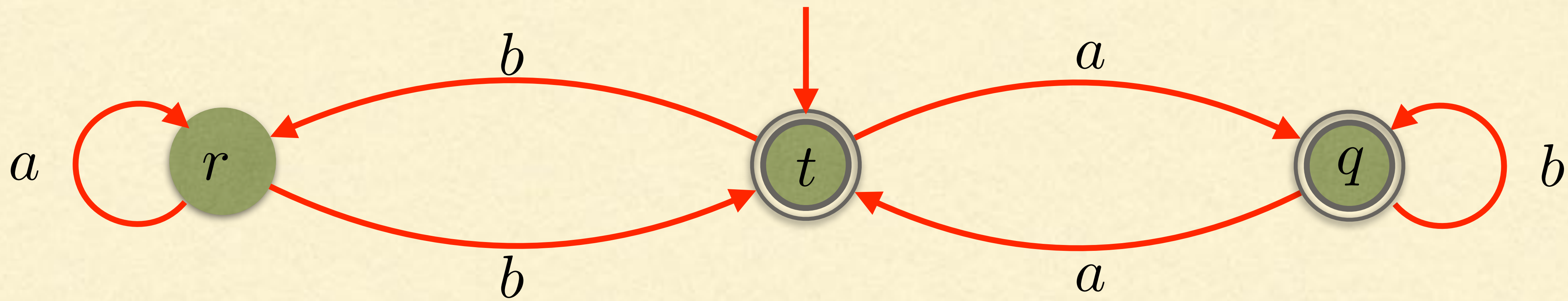
Transition monoid of a Finite state automaton

[Schutzenberger'65]



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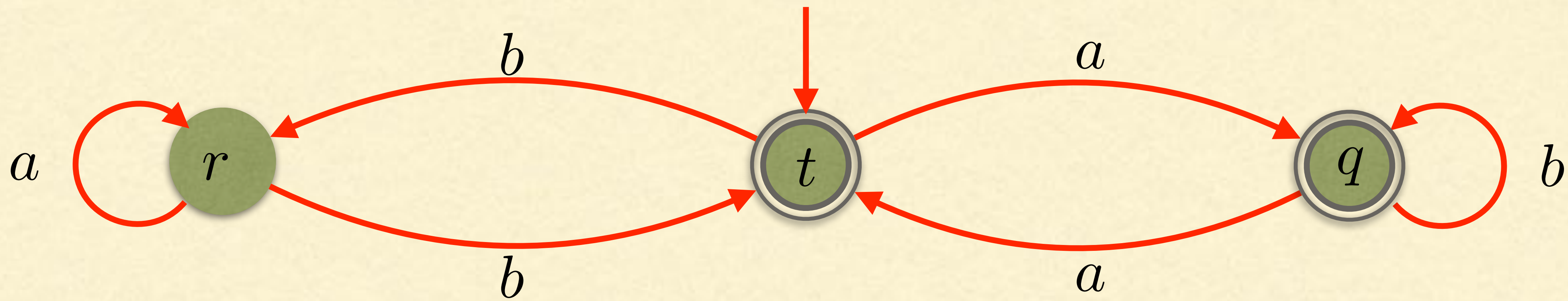
$$M_{w_1 w_2} = M_{w_1} \times M_{w_2}$$

Transition monoid of a Finite state automaton

[Schutzenberger'65]



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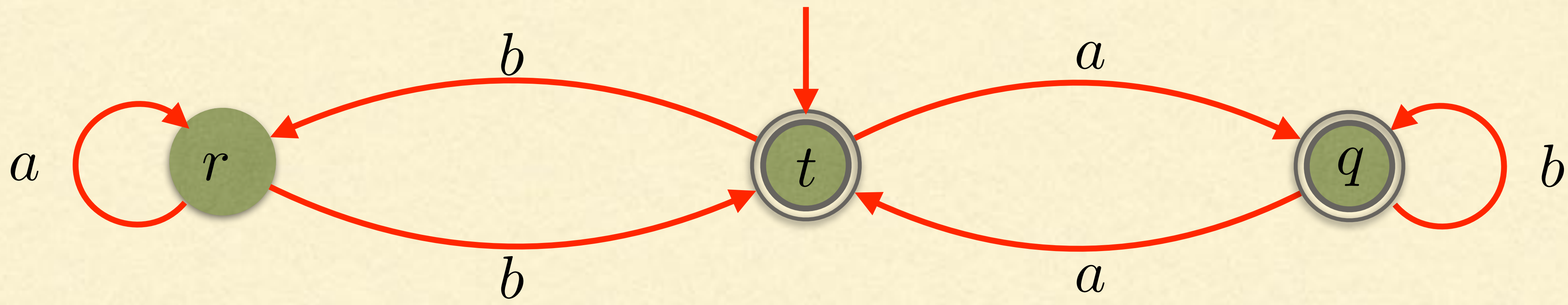
$$M_{w_1 w_2} = M_{w_1} \times M_{w_2}$$

monoid is aperiodic if $\forall m \in M_{\mathcal{A}}\ \exists x \in \mathbb{N}$ s.t. $m^x = m^{x+1}$

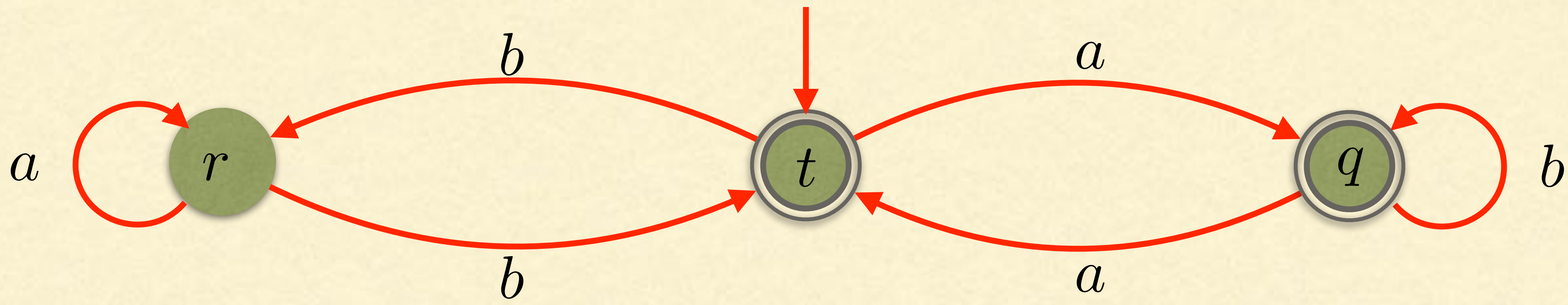
Transition monoid for Muller automaton example

Transition monoid for Muller automaton example



$$\mathcal{F} = \{\underset{F_1}{\{q\}}, \underset{F_2}{\{q, t\}}\}$$

Transition monoid for Muller automaton example



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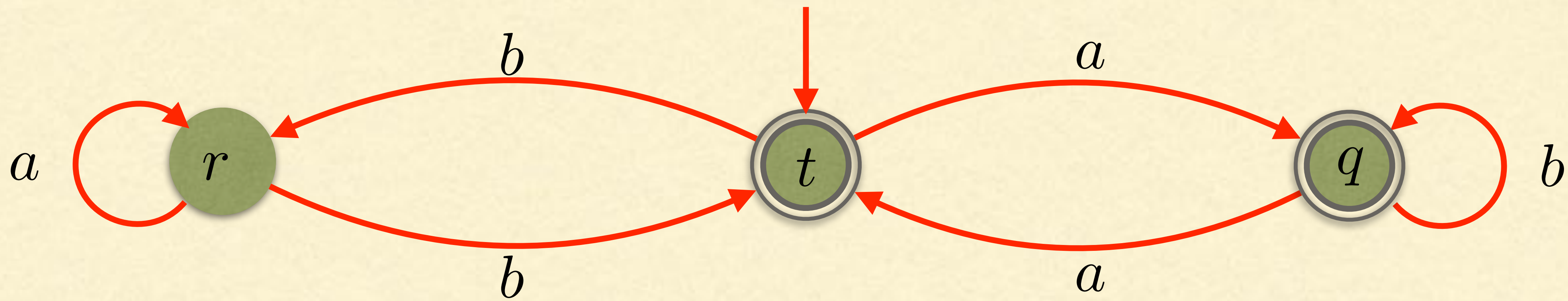
Transition monoid for Muller automaton example



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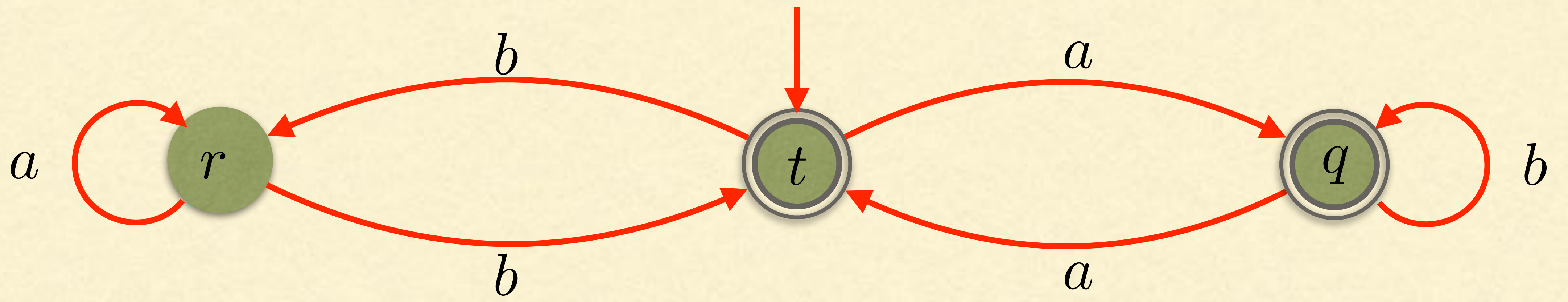
$M_{\mathcal{A}} - |Q| \times |Q|$ **over** $(\{0,1\} \cup 2^Q)^n \cup \{\perp\}$.

Transition monoid for Muller automaton example



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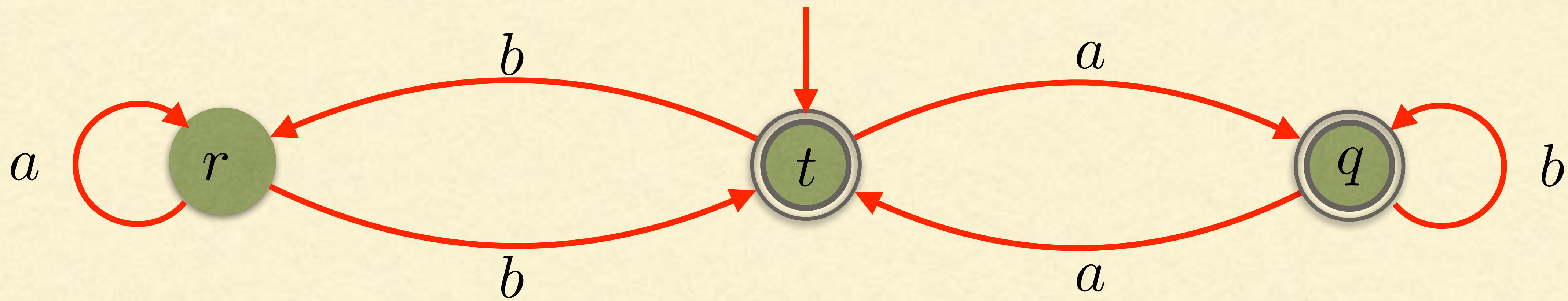

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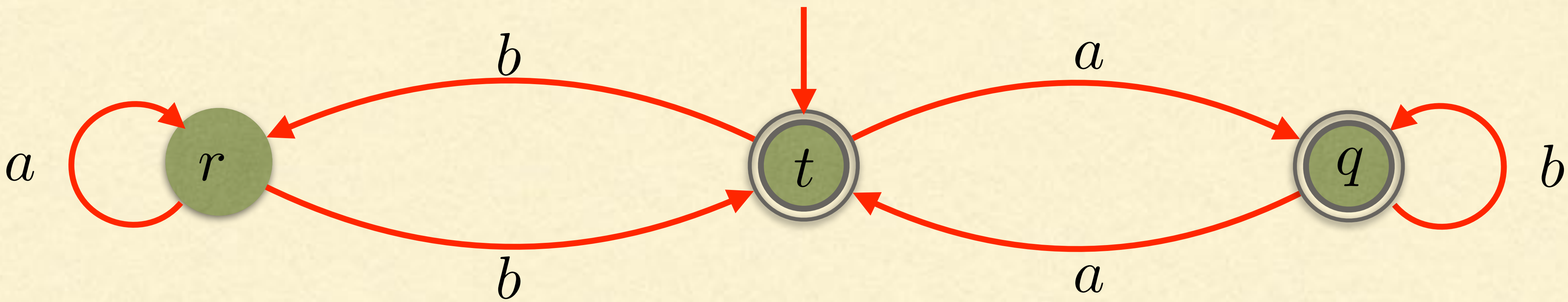

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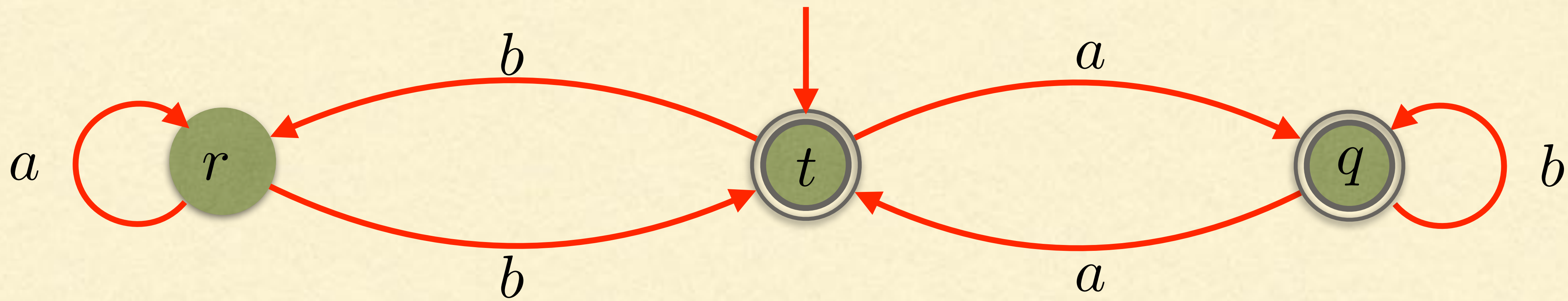

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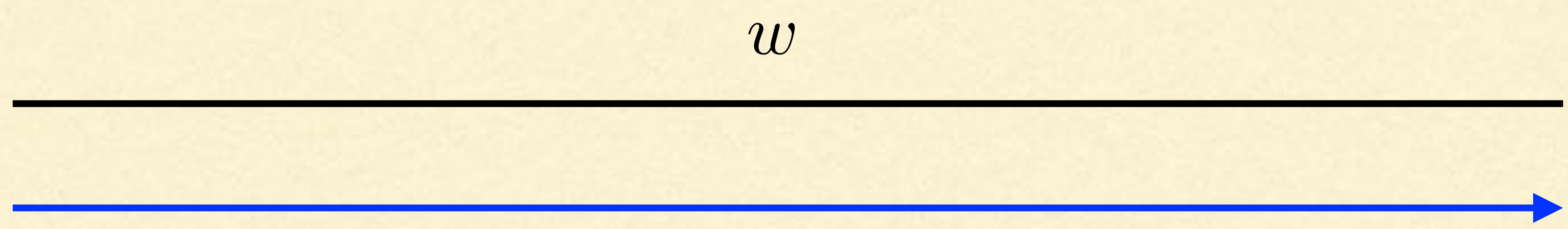
Partial runs for two way automaton

Partial runs for two way automaton

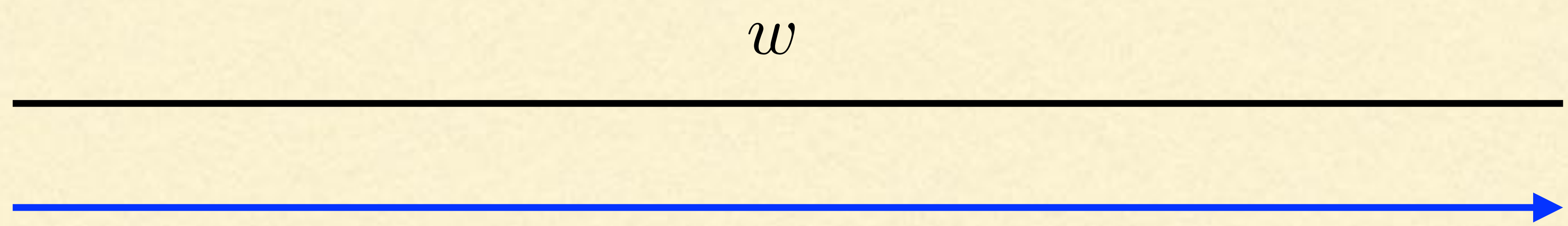
w

Partial runs for two way automaton



one way:

Partial runs for two way automaton



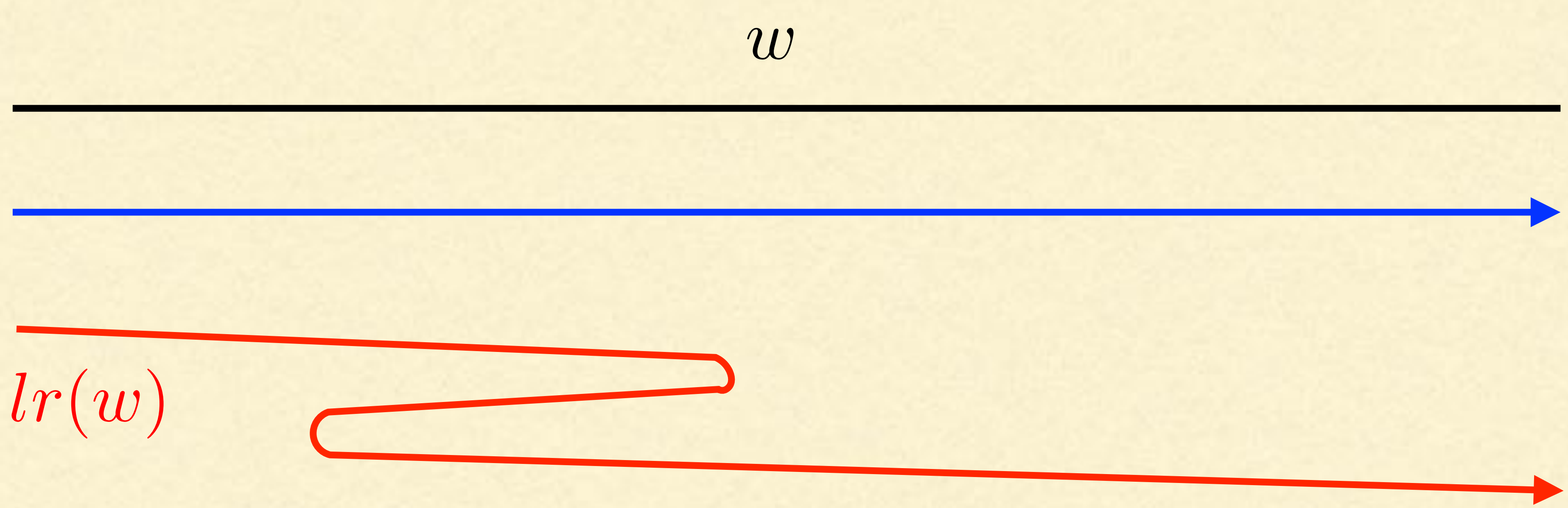
one way:

two way:

Partial runs for two way automaton

one way:

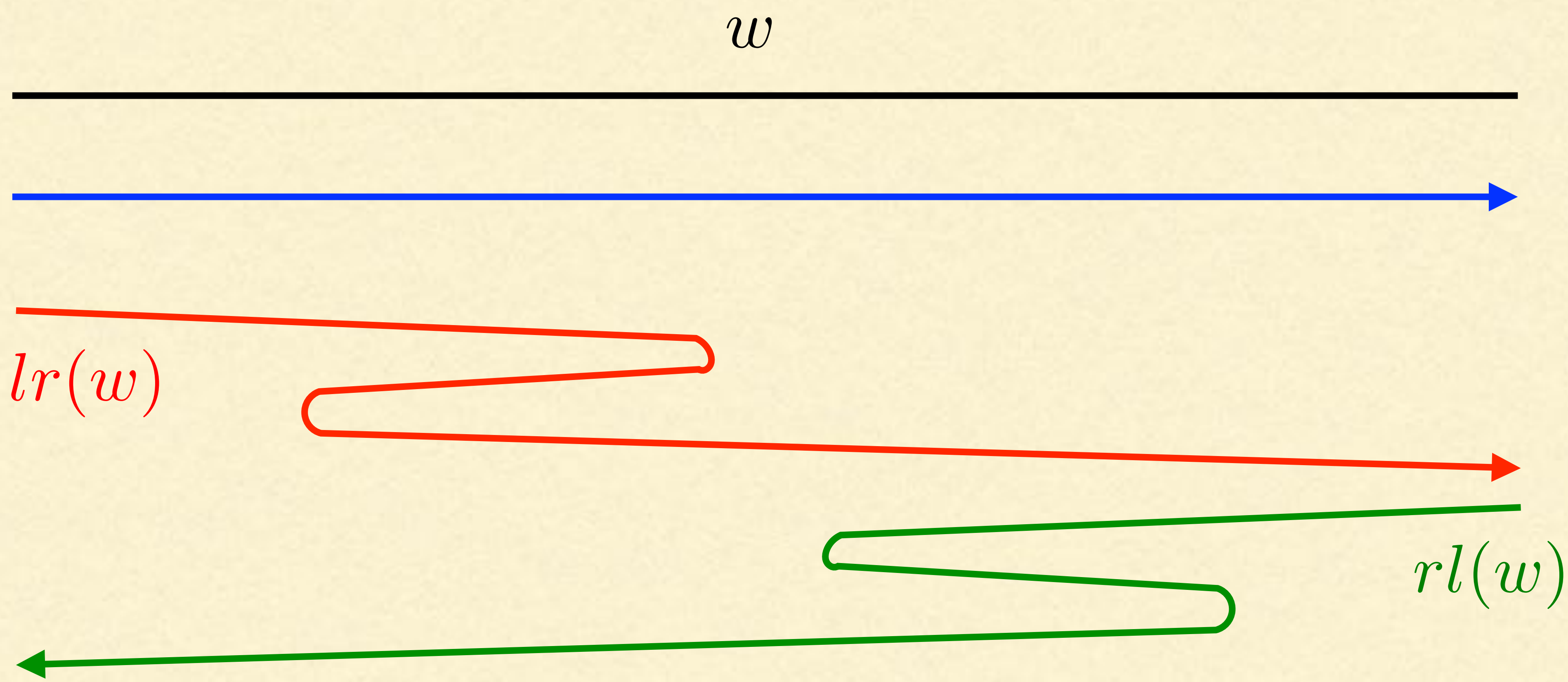
two way:



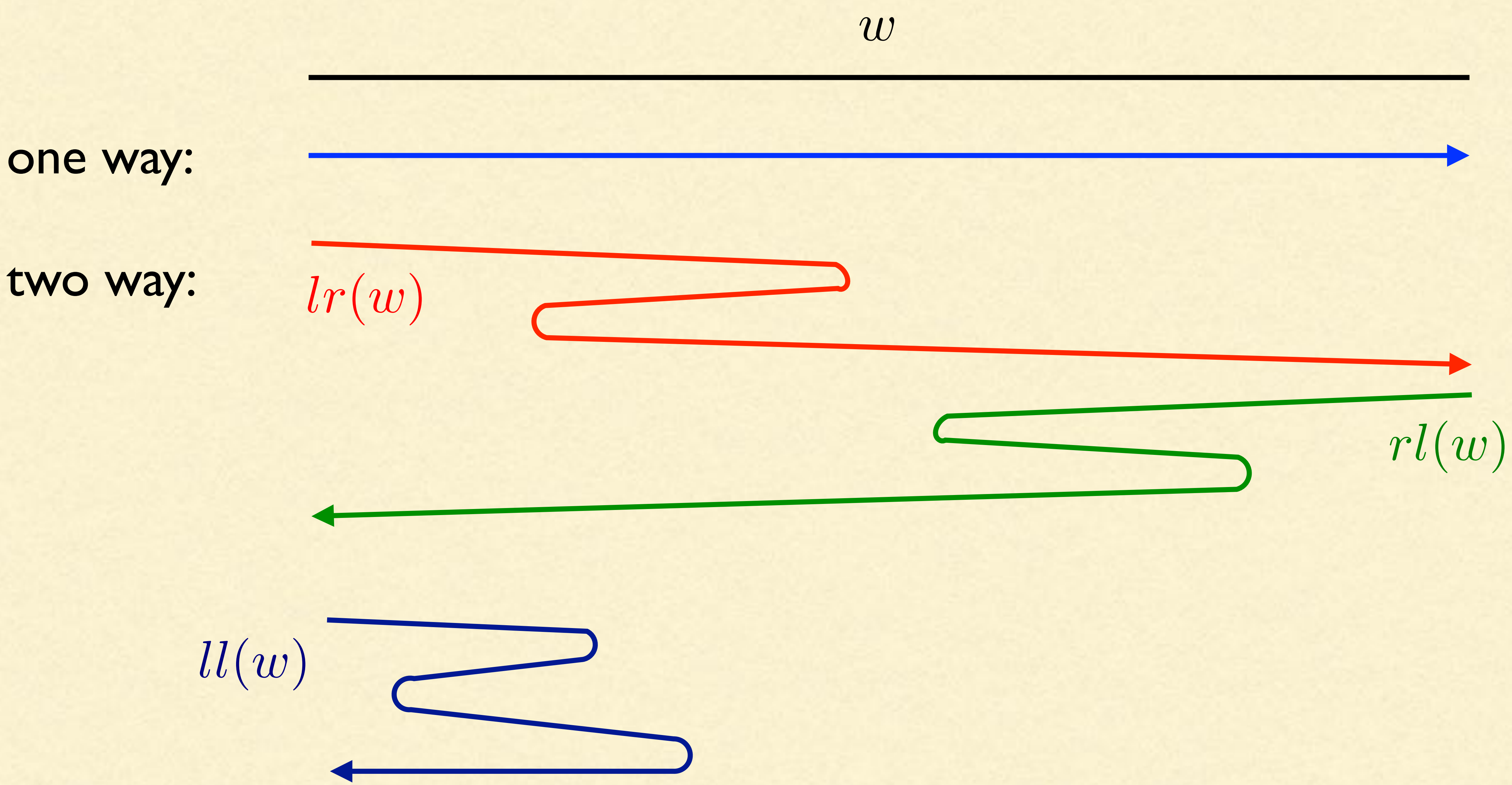
Partial runs for two way automaton

one way:

two way:



Partial runs for two way automaton



Partial runs for two way automaton

one way:

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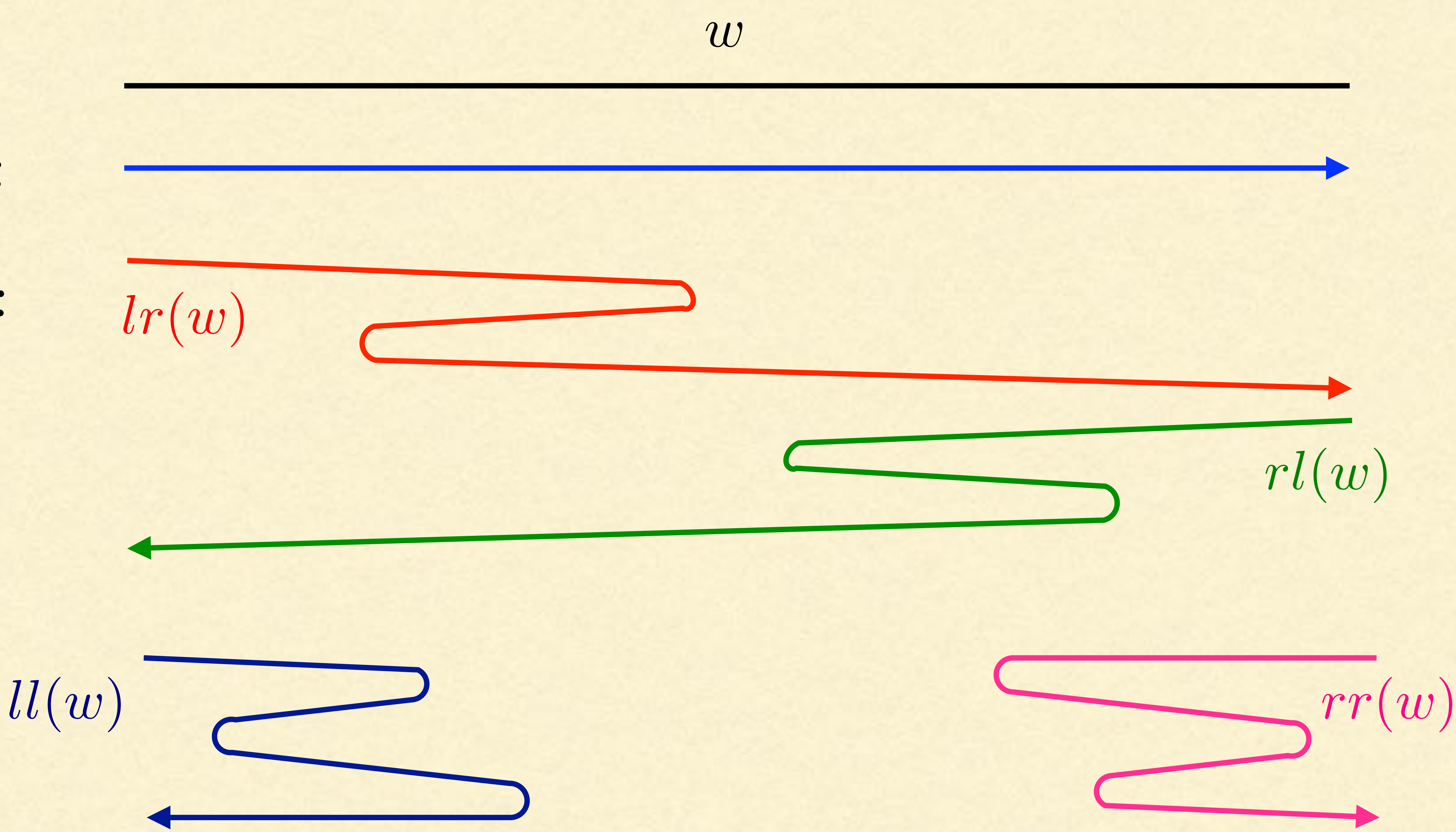
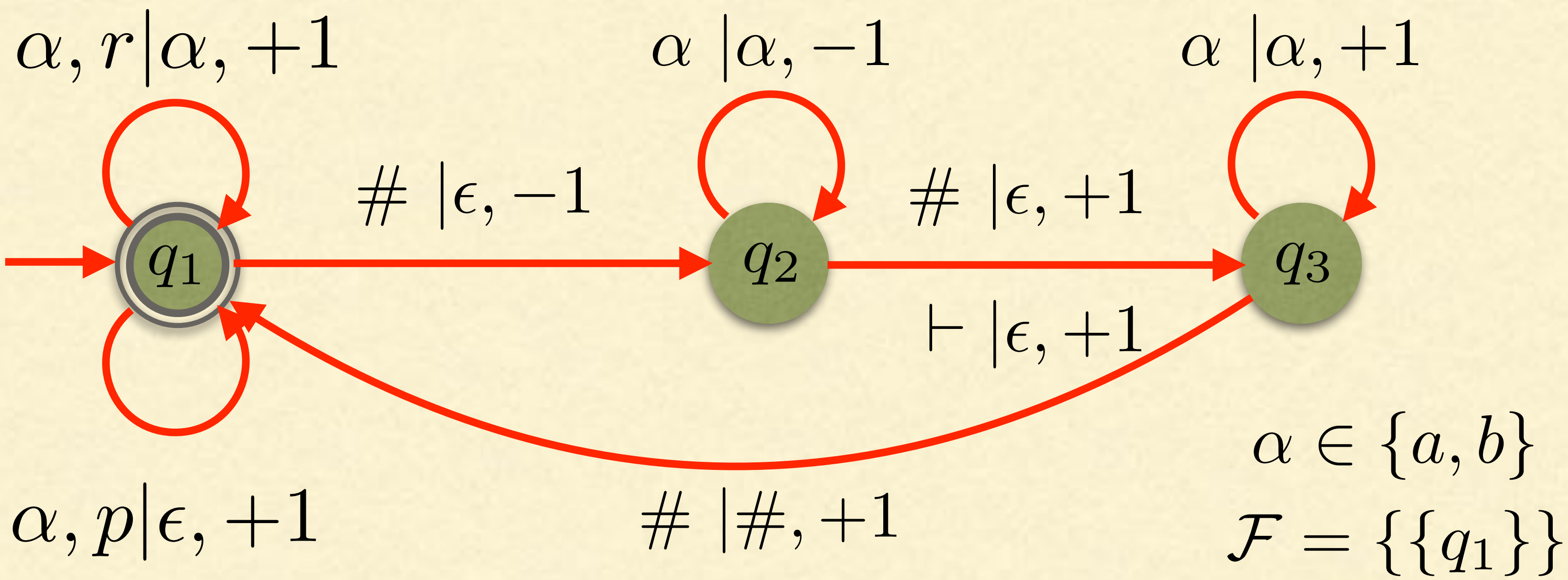


Transition monoid for Two Way Transducer for ω strings

Transition monoid for Two Way Transducer for ω strings



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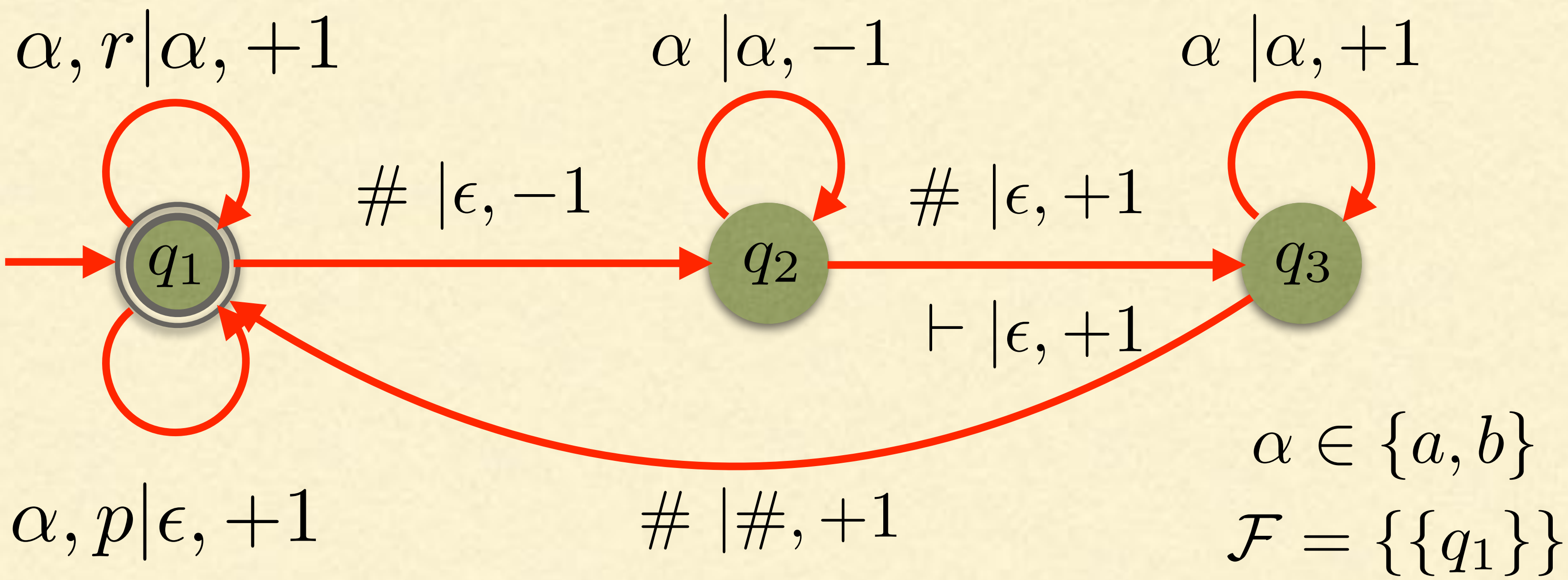
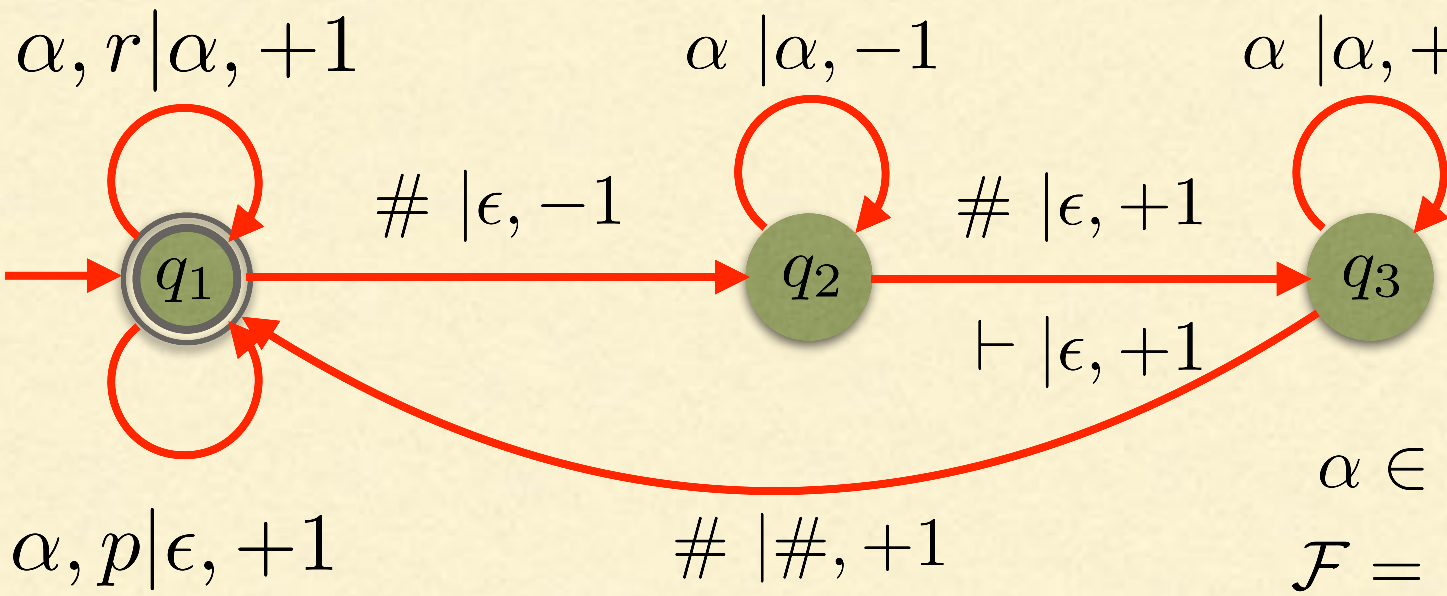


$$M_s = \begin{pmatrix} M_s^{ll} & M_s^{lr} \\ M_s^{rl} & M_s^{rr} \end{pmatrix}$$

Transition monoid for Two Way Transducer for ω strings



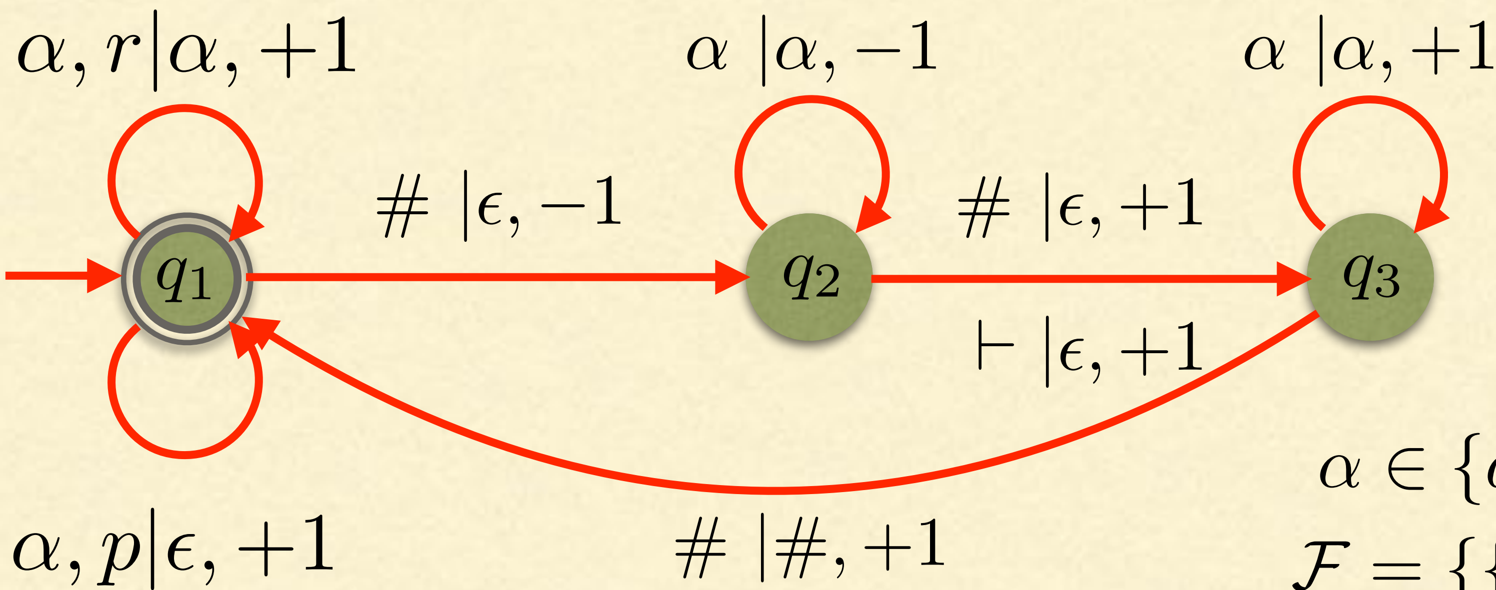
$\alpha \in \{a, b\}$

$\mathcal{F} = \{\{q_1\}\}$

$$M_s = \begin{pmatrix} M_s^{ll} & M_s^{lr} \\ M_s^{rl} & M_s^{rr} \end{pmatrix}$$

$$M_{ab\#}^{lr} = \begin{array}{c} \\ q_1 \\ q_2 \\ q_3 \end{array} \begin{array}{c} \begin{array}{ccc} q_1 & q_2 & q_3 \end{array} \\ \begin{pmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{pmatrix} \end{array}$$

Transition monoid for Two Way Transducer for ω strings



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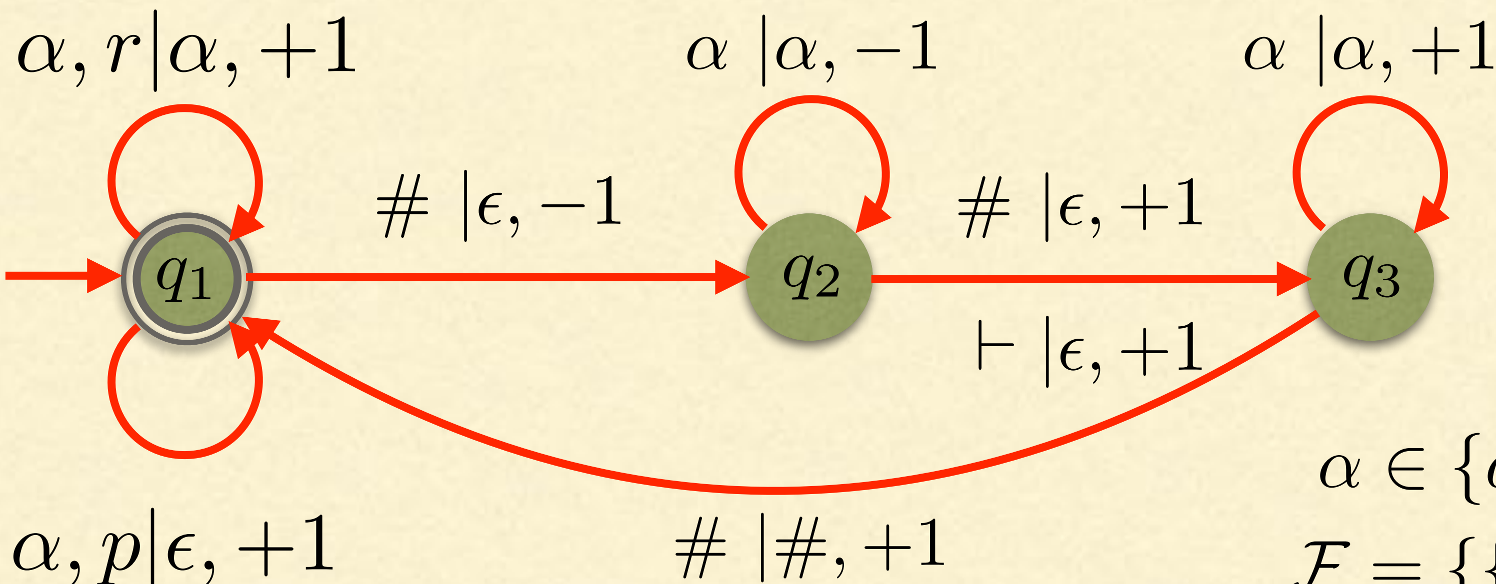

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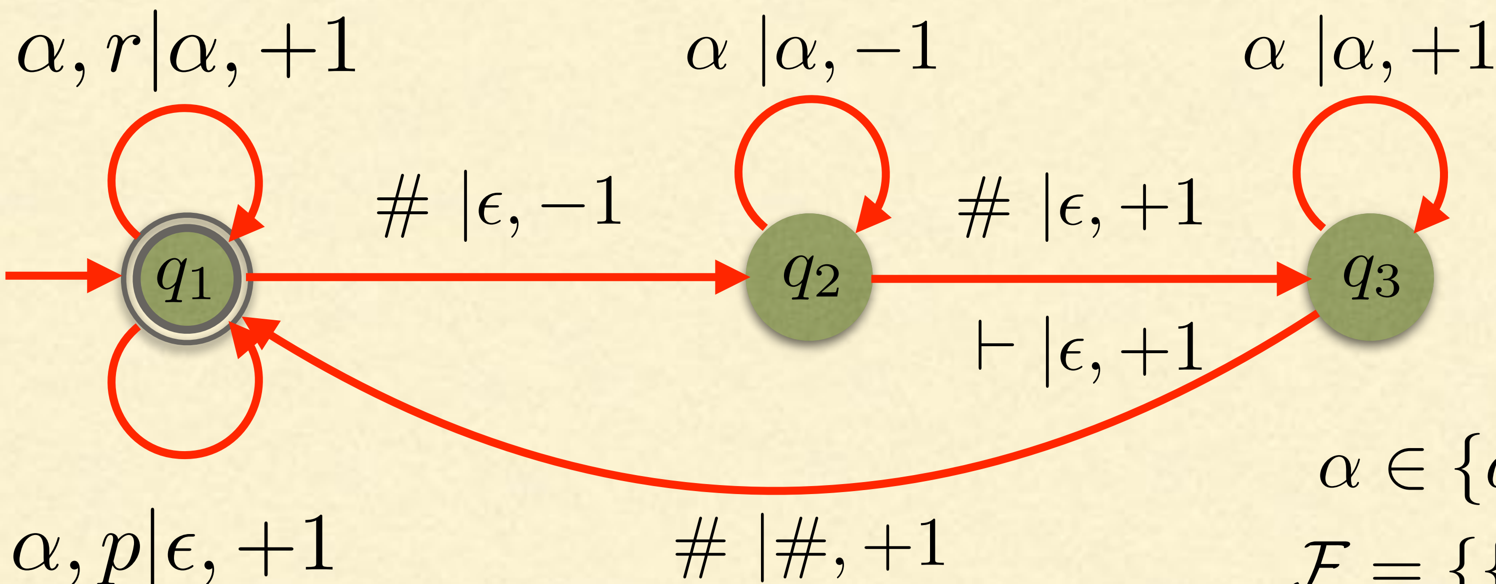
Transition monoid for Two Way Transducer for ω strings



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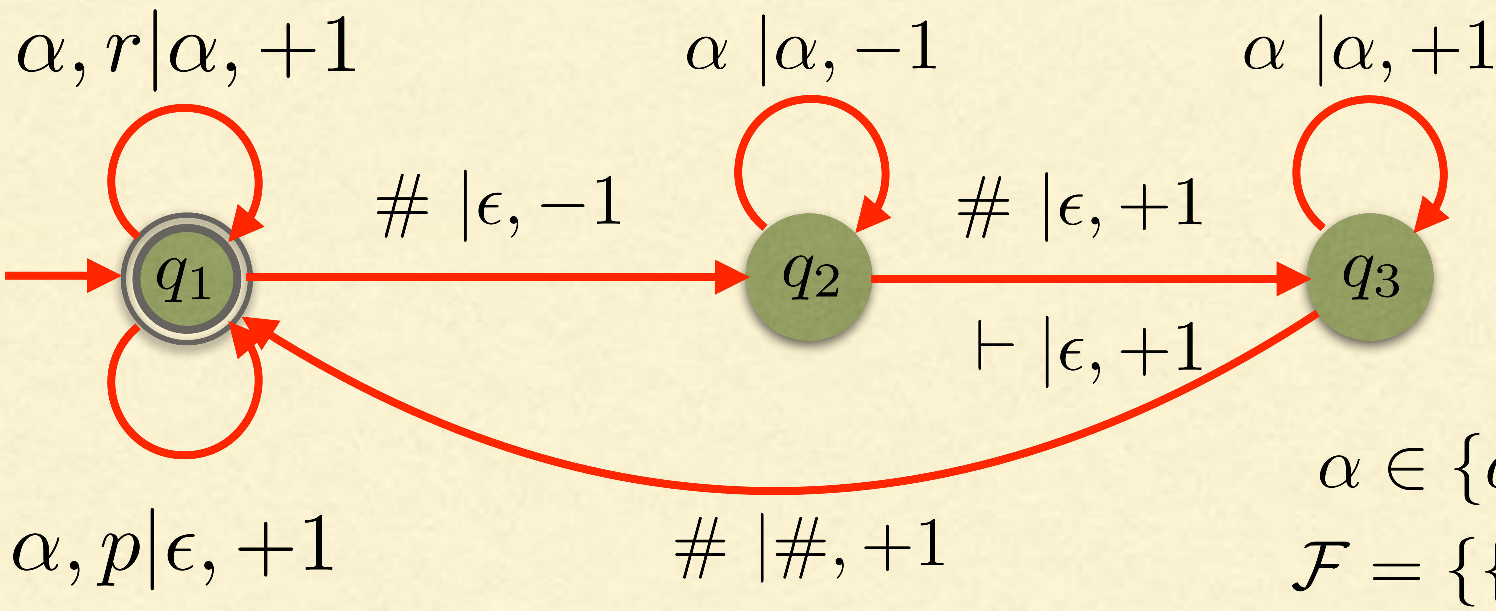
Transition monoid for Two Way Transducer for ω strings



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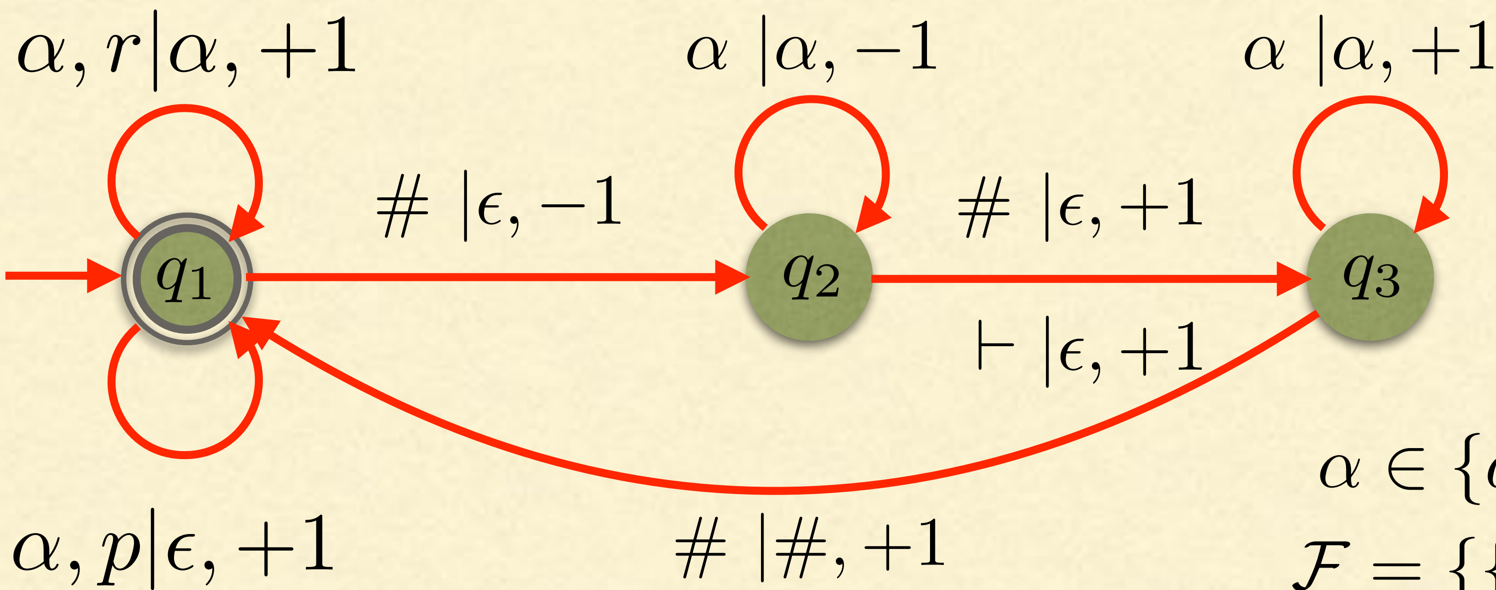
Transition monoid for Two Way Transducer for ω strings



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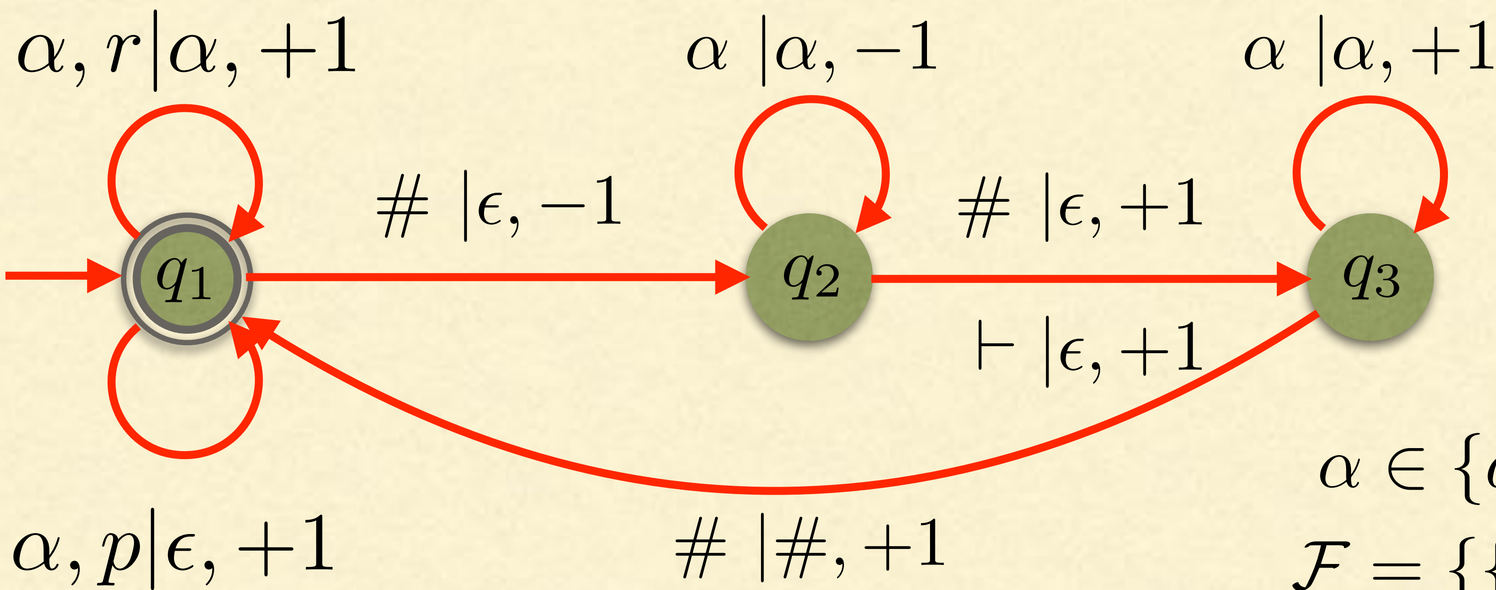
$\alpha \in \{a, b\}$

$\mathcal{F} = \{\{q_1\}\}$

$$M^{lr}_{ab\#} = \begin{array}{c} \\ q_1 \\ q_2 \\ q_3 \end{array} \begin{array}{c} \begin{array}{ccc} q_1 & q_2 & q_3 \end{array} \\ \begin{pmatrix} (0) & \perp & \perp \\ (0) & \perp & \perp \\ \perp & \perp & \perp \end{pmatrix} \end{array}$$

$$M^{rr}_{ab\#} = \begin{array}{c} \\ q_1 \\ q_2 \\ q_3 \end{array} \begin{array}{c} \begin{array}{ccc} q_1 & q_2 & q_3 \end{array} \\ \begin{pmatrix} & \perp & \perp \\ & & \\ & & \end{pmatrix} \end{array}$$

Transition monoid for Two Way Transducer for ω strings



$$M_s = \begin{pmatrix} M_s^{ll} & M_s^{lr} \\ M_s^{rl} & M_s^{rr} \end{pmatrix}$$

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$$M_{ab\#}^{lr} = \begin{array}{c} \\ q_1 \\ q_2 \\ q_3 \end{array} \begin{array}{c} \begin{array}{ccc} q_1 & q_2 & q_3 \end{array} \\ \begin{pmatrix} (0) & \perp & \perp \\ (0) & \perp & \perp \\ \perp & \perp & \perp \end{pmatrix} \end{array} \qquad M_{ab\#}^{rr} = \begin{array}{c} \\ q_1 \\ q_2 \\ q_3 \end{array} \begin{array}{c} \begin{array}{ccc} q_1 & q_2 & q_3 \end{array} \\ \begin{pmatrix} (0) & \perp & \perp \\ & & \\ & & \end{pmatrix} \end{array}$$

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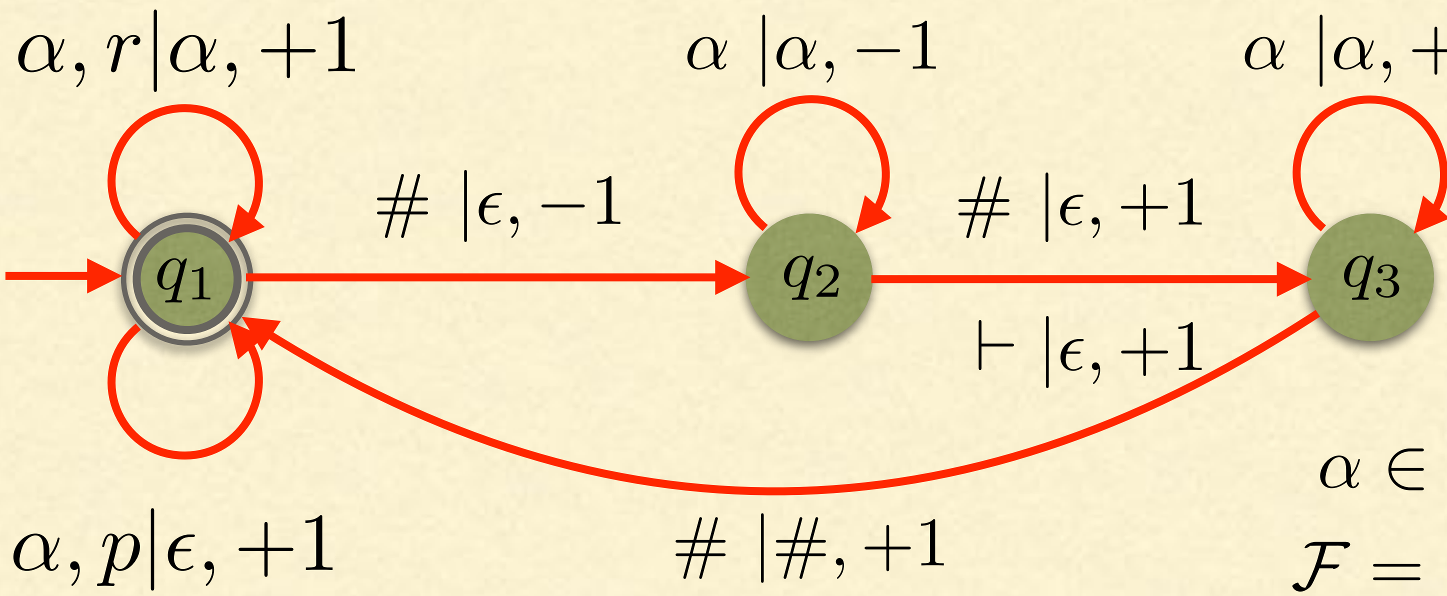

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$$M^{rr}_{ab\#} = \begin{array}{c} \\ q_1 \\ q_2 \\ q_3 \end{array} \begin{array}{c} \begin{array}{ccc} q_1 & q_2 & q_3 \end{array} \\ \begin{pmatrix} (0) & \bot & \bot \\ (0) & \bot & \bot \\ \bot & (0) & \bot \end{pmatrix} \end{array}$$

Transition monoid for Streaming String Transducer for ω strings

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$$f(a^n b^\omega) = a^{\lceil n/2 \rceil} b^\omega$$

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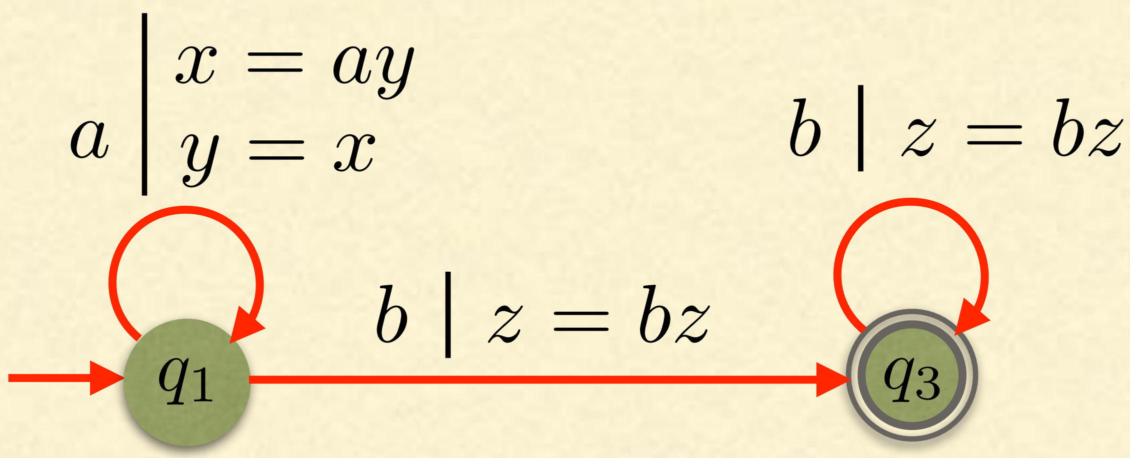

$$F(\{q_3\}) = xz$$

Transition monoid for Streaming String Transducer for ω strings

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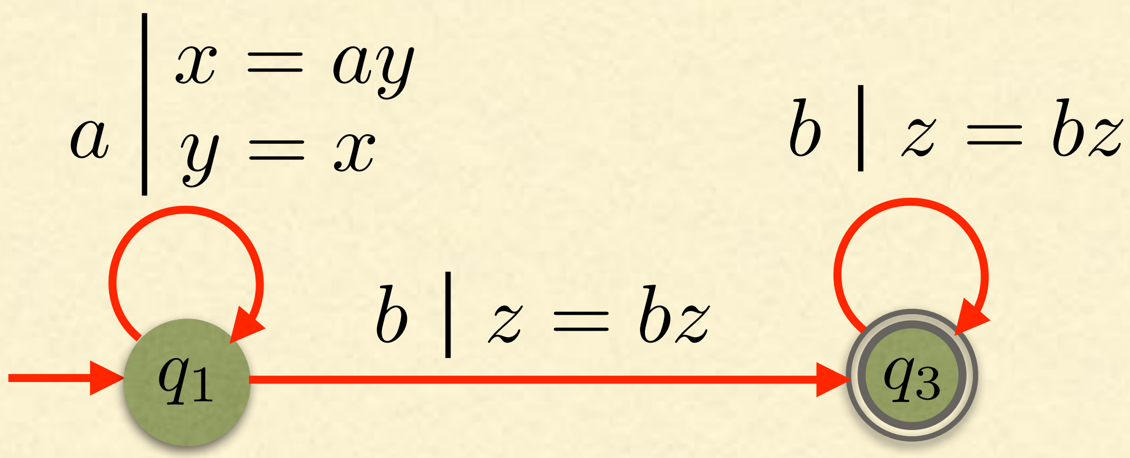

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domain language : $a^* b^\omega$ aperiodic ✔

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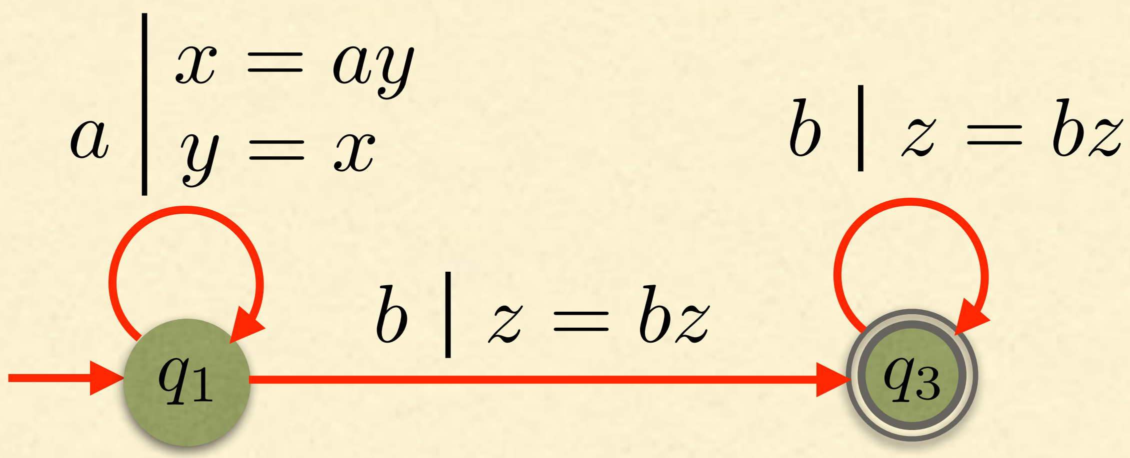

$$F(\{q_3\}) = xz$$

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transformation : FO- definable ✘

Transition monoid for Streaming String Transducer for ω strings

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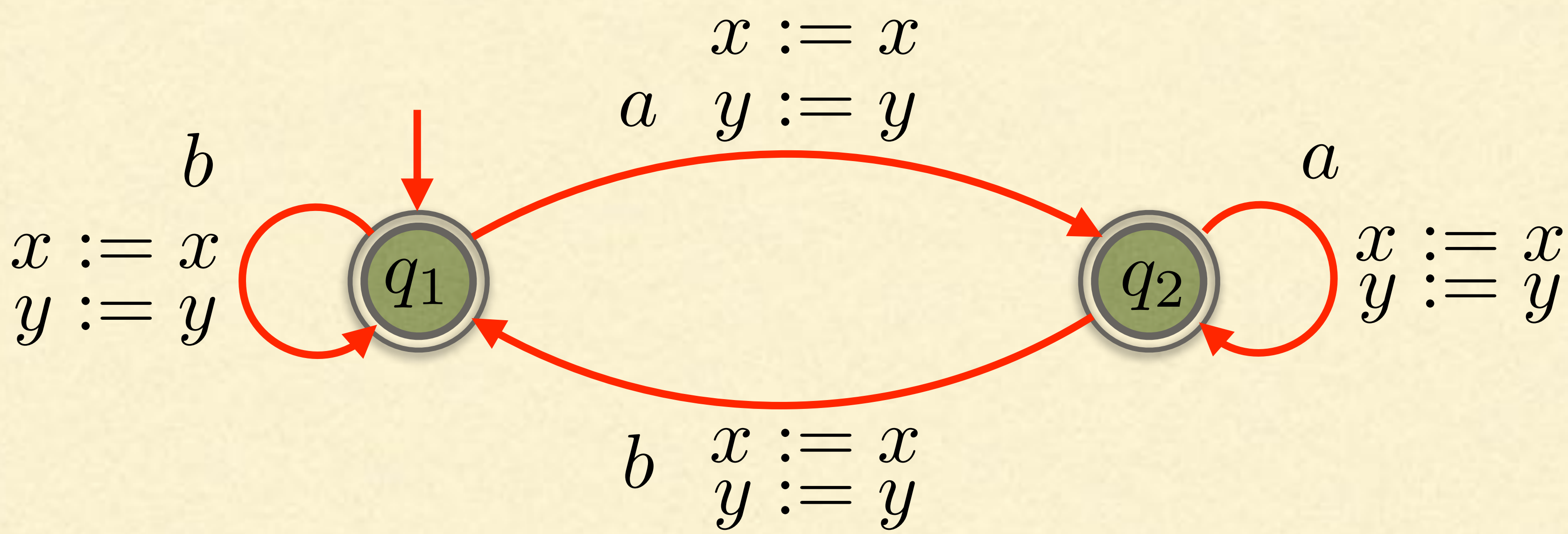
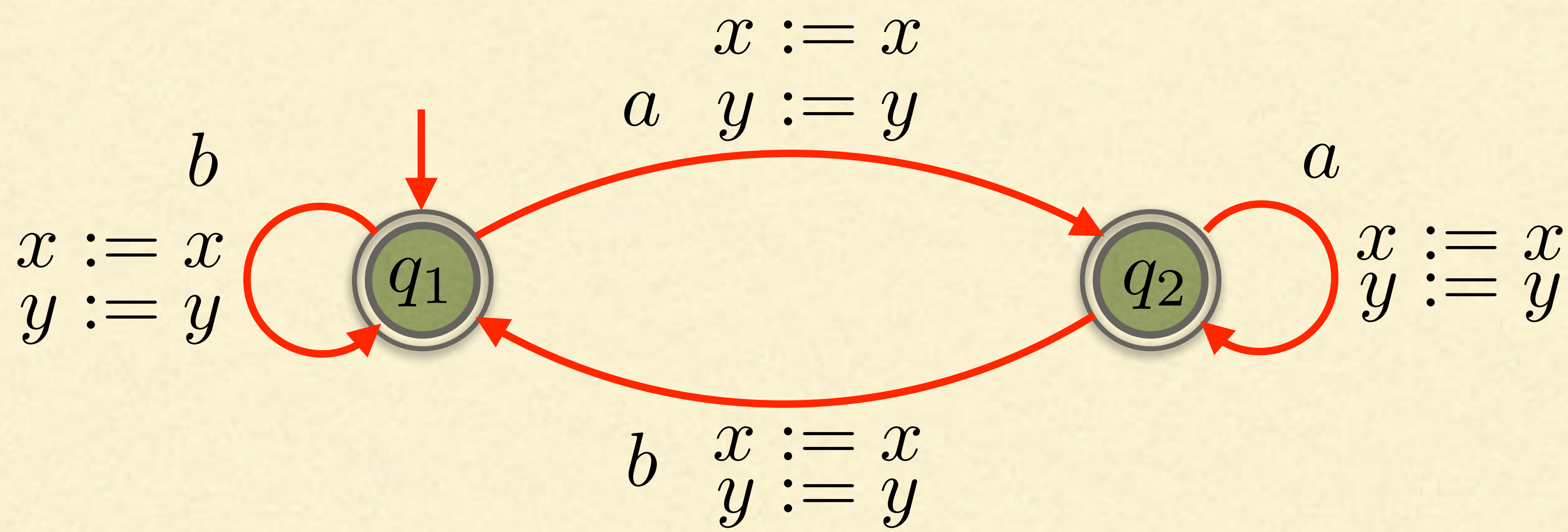


$\mathcal{F} = \{\{q_1, q_2\}\}$

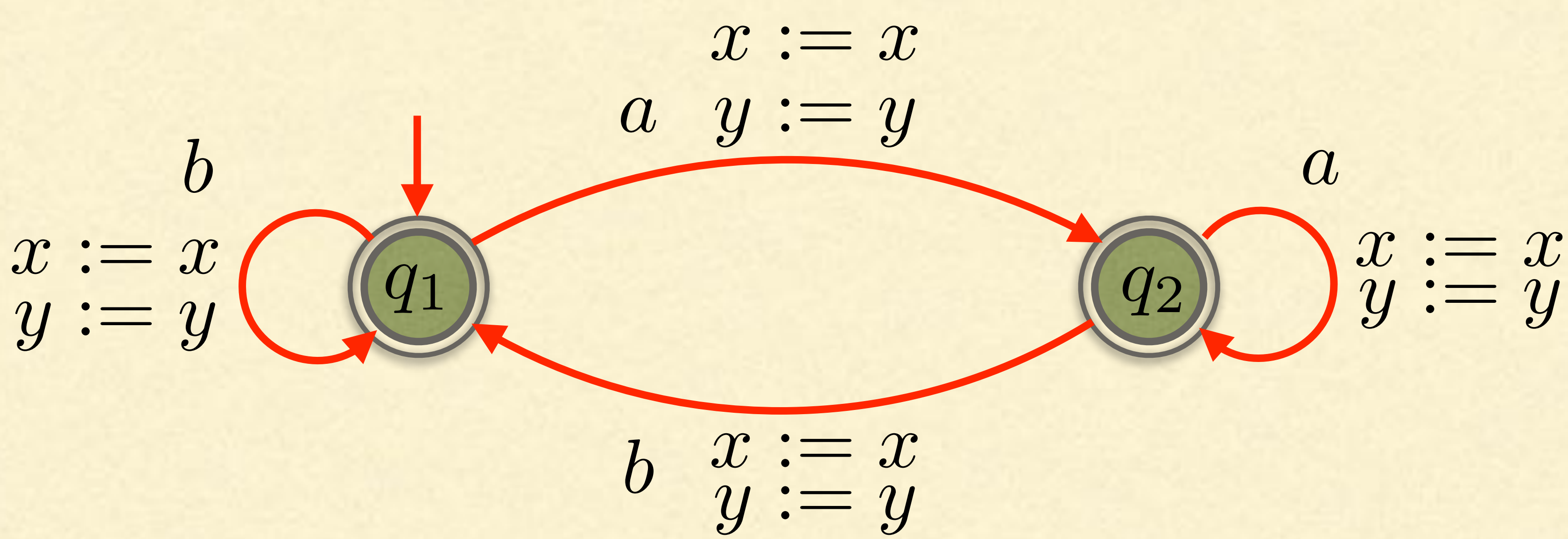
Transition monoid for Streaming String Transducer for ω strings



$$\mathcal{F} = \{\{q_1, q_2\}\}$$

$$M_b = \begin{array}{c|cccc} & (q_1, x) & (q_1, y) & (q_2, x) & (q_2, y) \\ \hline (q_1, x) & & & & \\ (q_1, y) & & & & \\ (q_2, x) & & & & \\ (q_2, y) & & & & \end{array}$$

Transition monoid for Streaming String Transducer for ω strings



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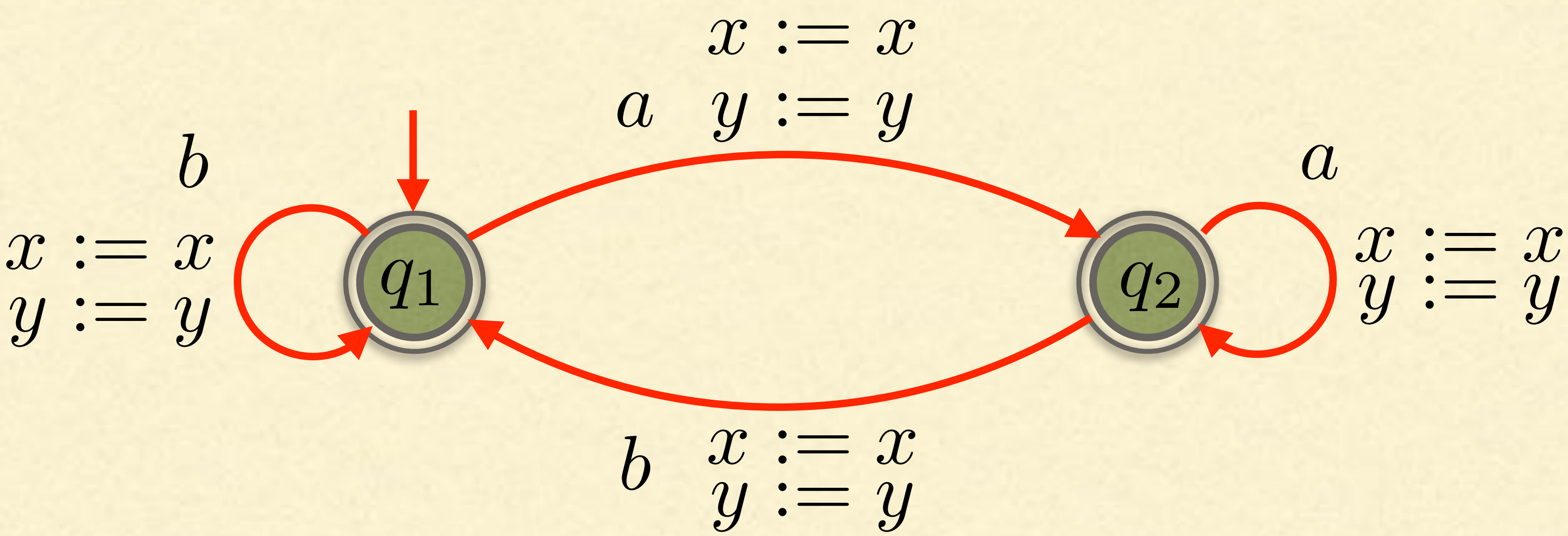
$$M_b = \begin{array}{c c} & \begin{array}{cccc} (q_1,x) & (q_1,y) & (q_2,x) & (q_2,y) \end{array} \\ \begin{array}{c} (q_1,x) \\ (q_1,y) \\ (q_2,x) \\ (q_2,y) \end{array} & \begin{pmatrix} & & \bot & \bot \\ & & \bot & \bot \\ & & & \\ & & & \end{pmatrix} \end{array}$$

Transition monoid for Streaming String Transducer for ω strings



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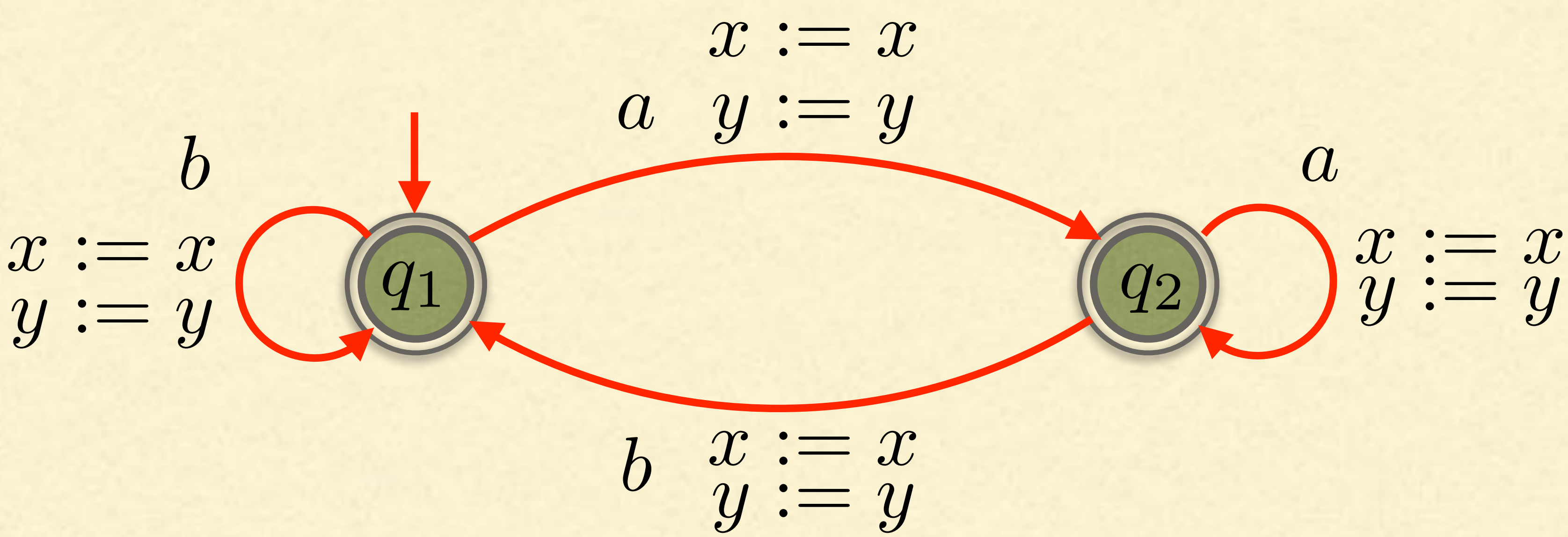
$$M_b = \begin{array}{c} \\ (q_1,x) \\ (q_1,y) \\ (q_2,x) \\ (q_2,y) \end{array} \begin{array}{c} \begin{array}{cccc} (q_1,x) & (q_1,y) & (q_2,x) & (q_2,y) \end{array} \\ \left(\begin{array}{cccc} 1(\{q_1\}) & 0(\{q_1\}) & \perp & \perp \\ 0(\{q_1\}) & 1(\{q_1\}) & \perp & \perp \\ & & & \\ & & & \end{array} \right) \end{array}$$

Transition monoid for Streaming String Transducer for ω strings



$$\mathcal{F} = \{\{q_1, q_2\}\}$$

$$M_b = \begin{array}{c} \\ (q_1, x) \\ (q_1, y) \\ (q_2, x) \\ (q_2, y) \end{array} \begin{array}{c} \begin{array}{cccc} (q_1, x) & (q_1, y) & (q_2, x) & (q_2, y) \end{array} \\ \begin{pmatrix} 1(\{q_1\}) & 0(\{q_1\}) & \perp & \perp \\ 0(\{q_1\}) & 1(\{q_1\}) & \perp & \perp \\ 1(1) & 0(1) & \perp & \perp \\ 0(1) & 1(1) & \perp & \perp \end{pmatrix} \end{array}$$

Outline

- Introduction
  - Three formalisms for transductions
  - Related work
- Aperiodic transformations for Infinite strings
  - Aperiodic two way transducer
  - Aperiodic streaming string transducer
- Equivalence results and Proof ideas
  - $SST_{sf} \subset FOT = 2WST_{sf} \subset SST_{sf}$
- Conclusion

Equivalence results

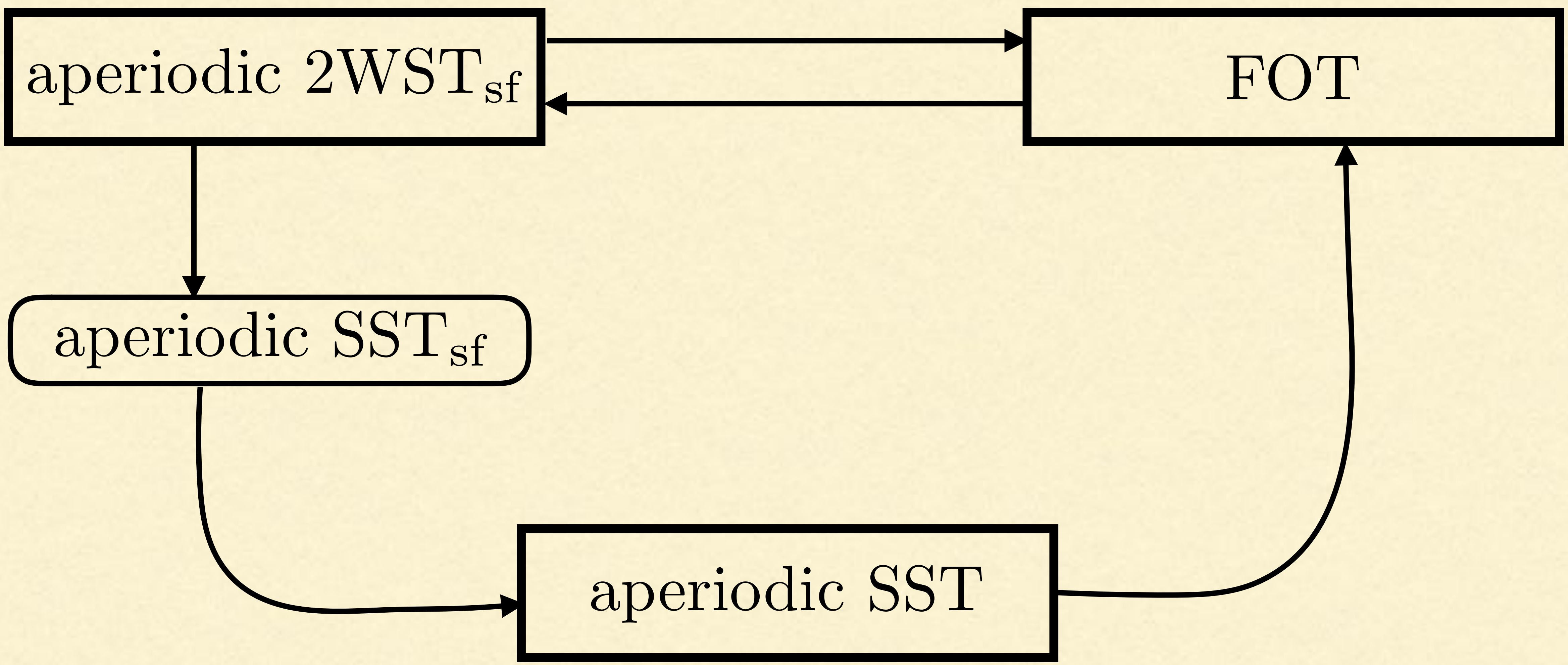
Equivalence results

aperiodic $2\mathrm{WST}_{\mathrm{sf}}$

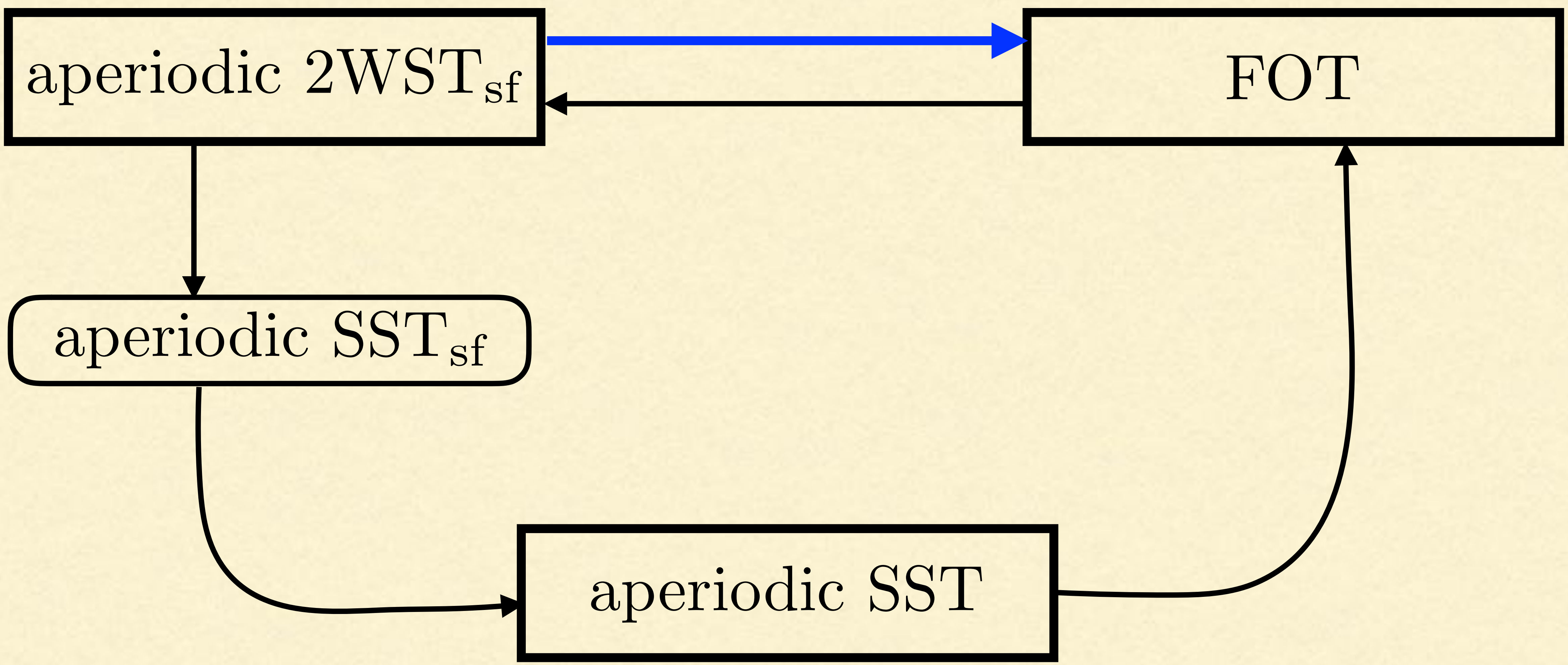
FOT

aperiodic SST

Equivalence results



Results



aperiodic $2WST_{sf} \subseteq$ FOT

aperiodic $\mathrm{2WST_{sf}} \subseteq$ FOT

input

aperiodic $2\mathrm{WST}_{\mathrm{sf}} \subseteq$ FOT

input	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8

aperiodic $2\mathrm{WST}_{\mathrm{sf}} \subseteq$ FOT

input a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 $\overset{\Sigma}{\phi_{dom}}$ ✔

aperiodic $2\mathrm{WST}_{\mathrm{sf}} \subseteq \mathrm{FOT}$

input $\boxed{a_1} \to \boxed{a_2} \to \boxed{a_3} \to \boxed{a_4} \to \boxed{a_5} \to \boxed{a_6} \to \boxed{a_7} \to \boxed{a_8}$ Σ ϕ_{dom} ✔

aperiodic $2\mathrm{WST}_{\mathrm{sf}} \subseteq$ FOT

input $\boxed{a_1} \to \boxed{a_2} \to \boxed{a_3} \to \boxed{a_4} \to \boxed{a_5} \to \boxed{a_6} \to \boxed{a_7} \to \boxed{a_8}$

Σ

ϕ_{dom} ✔

$\Downarrow$

output

aperiodic $2\mathrm{WST}_{\mathrm{sf}} \subseteq$ FOT

input $a_1 \to a_2 \to a_3 \to a_4 \to a_5 \to a_6 \to a_7 \to a_8$

Σ

ϕ_{dom} ✔

$\Downarrow$

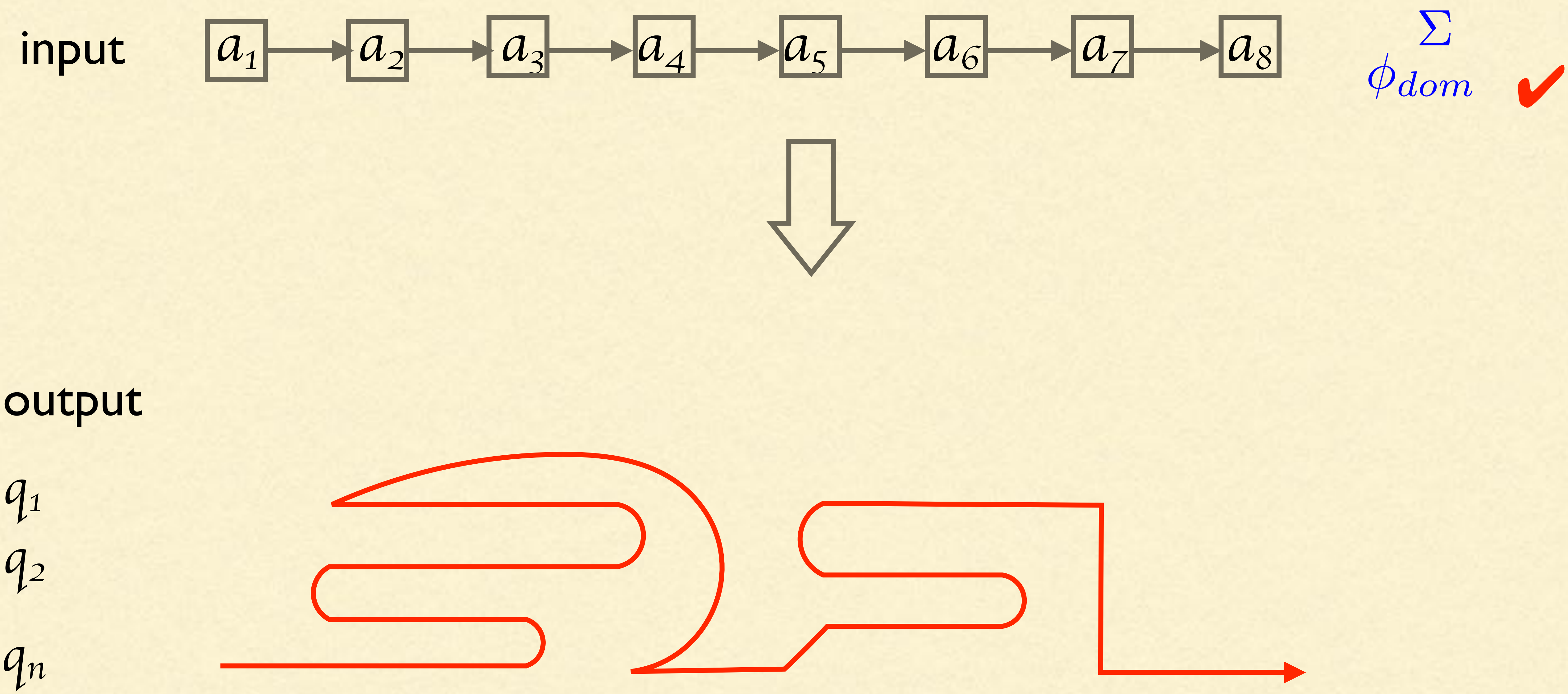
output

q_1

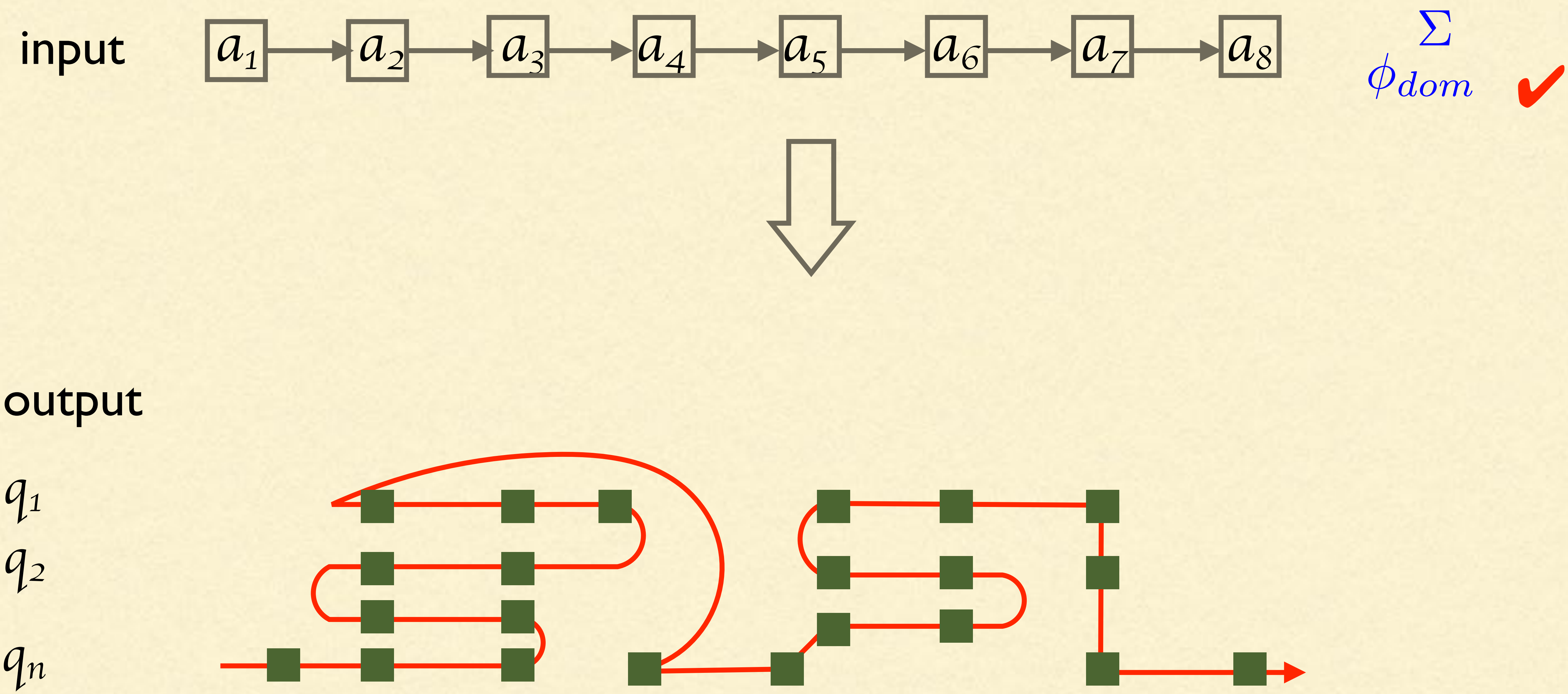
q_2

q_n

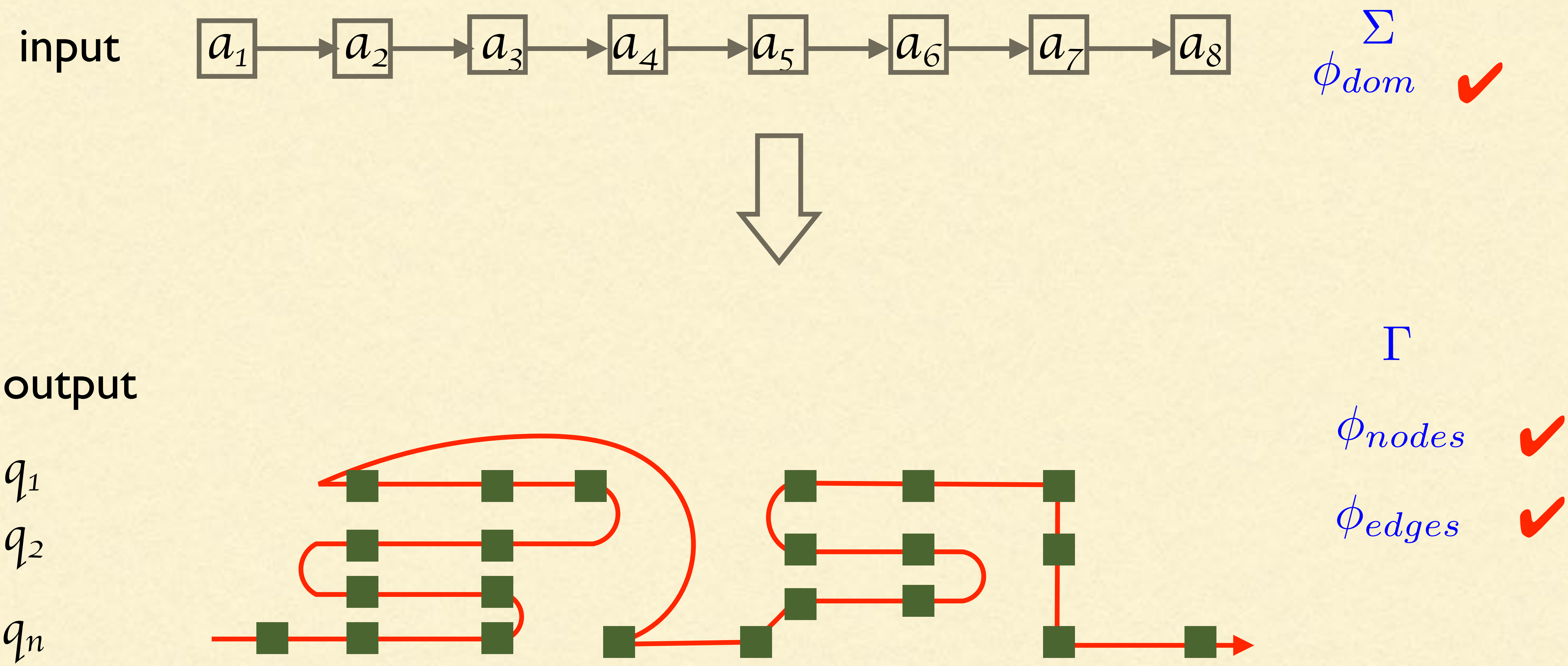
aperiodic $\mathrm{2WST}_{\mathrm{sf}} \subseteq$ FOT



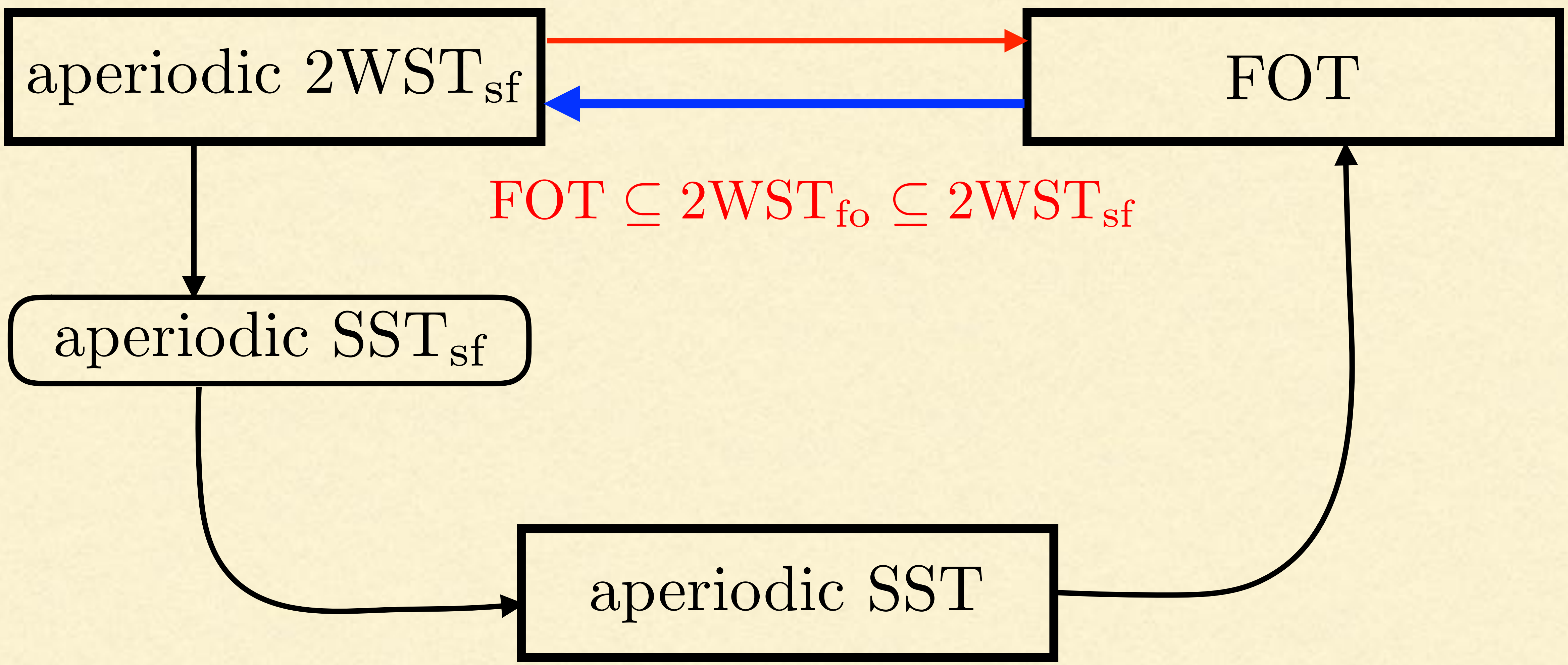
aperiodic $2\mathrm{WST}_{\mathrm{sf}} \subseteq$ FOT



$\text{aperiodic } 2\text{WST}_{\text{sf}} \subseteq \text{ FOT}$

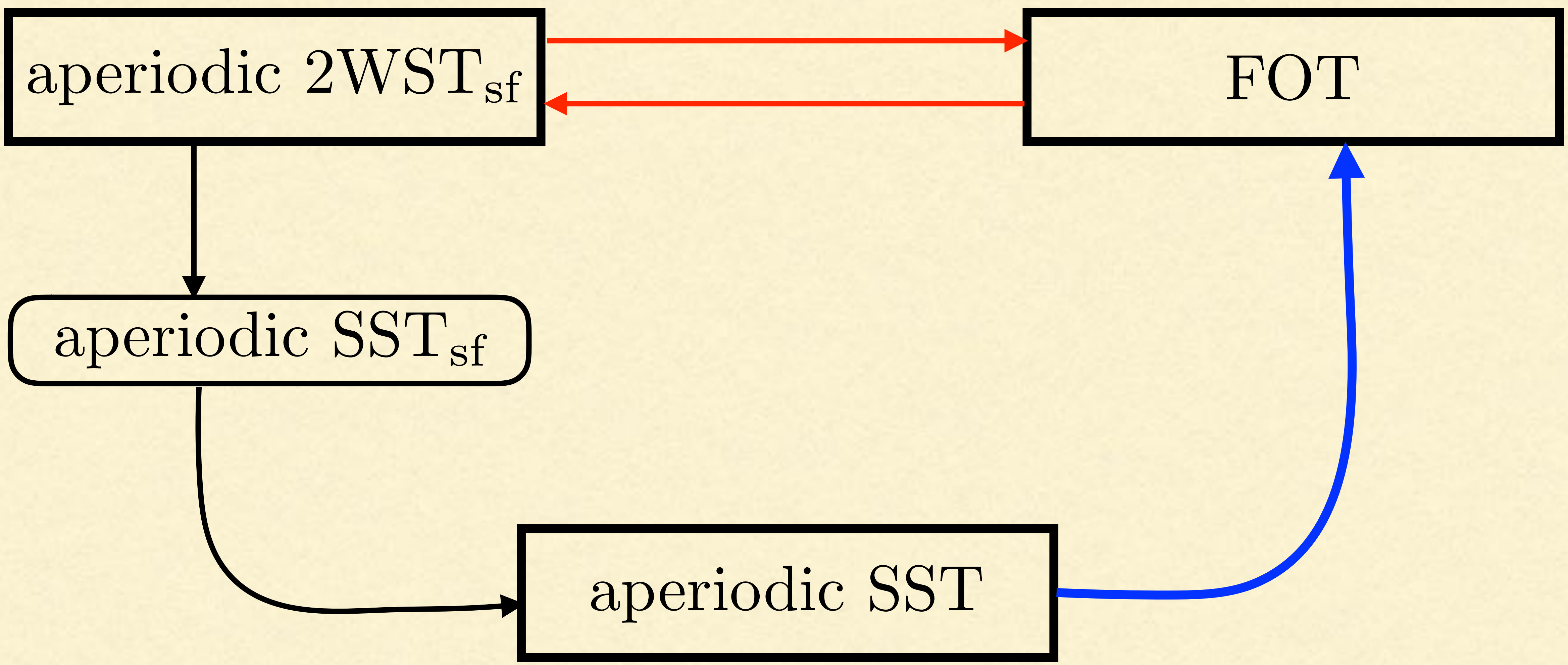


Results



aperiodic $2WST_{sf}$
FOT
$FOT \subseteq 2WST_{fo} \subseteq 2WST_{sf}$
aperiodic SST_{sf}
aperiodic SST

Results



Aperiodic SST $\subset$ FOT

[FKT'14]

Aperiodic SST $\subset$ FOT

SST output structure : [FKT'14]

Aperiodic SST $\subset$ FOT

SST output structure :

[FKT'14]

$$\text{run} \quad q_0 \xrightarrow{\substack{X = aXb \\ Y = aaa \\ Z = Zc}} q_1 \xrightarrow{\substack{X = c \\ Y = Y \\ Z = dZc}} q_2 \xrightarrow{\substack{X = aXbYc \\ Y = bZc \\ Z = b}} q_3$$

Aperiodic SST $\subset$ FOT

SST output structure : [FKT'14]

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X^{in}

X^{out}

Y^{in}

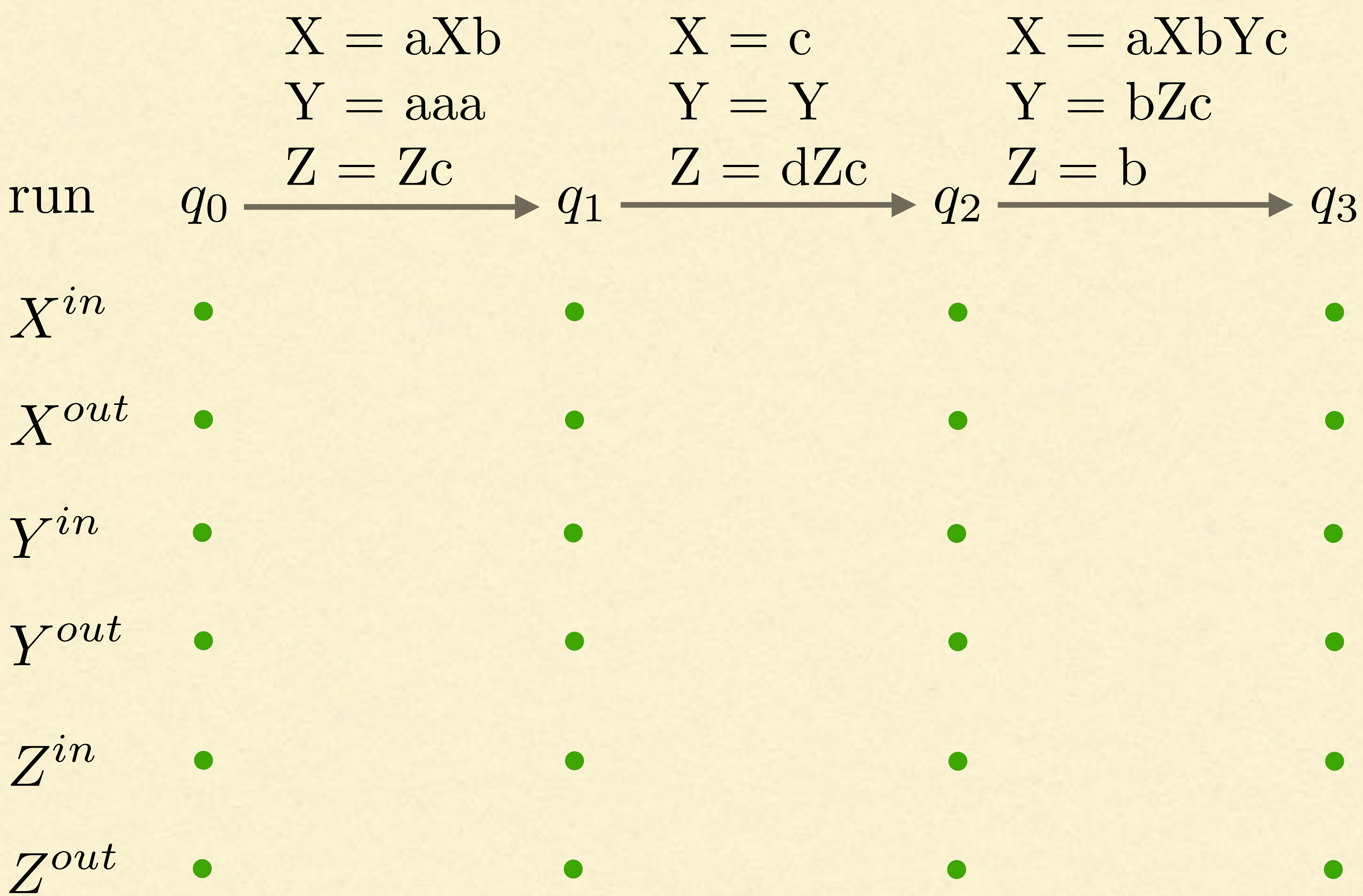
Y^{out}

Z^{in}

Z^{out}

Aperiodic SST $\subset$ FOT

SST output structure : [FKT'14]



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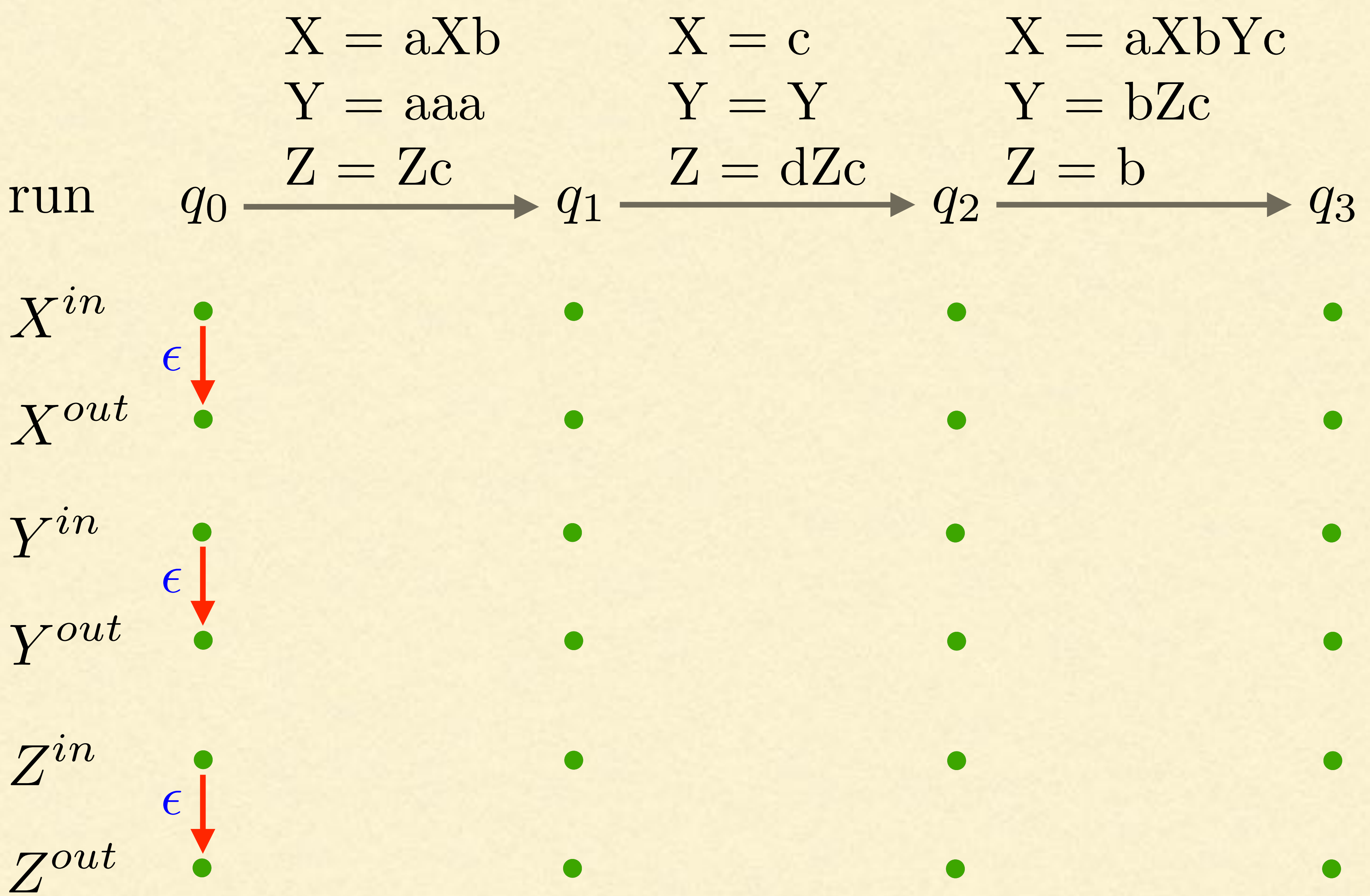

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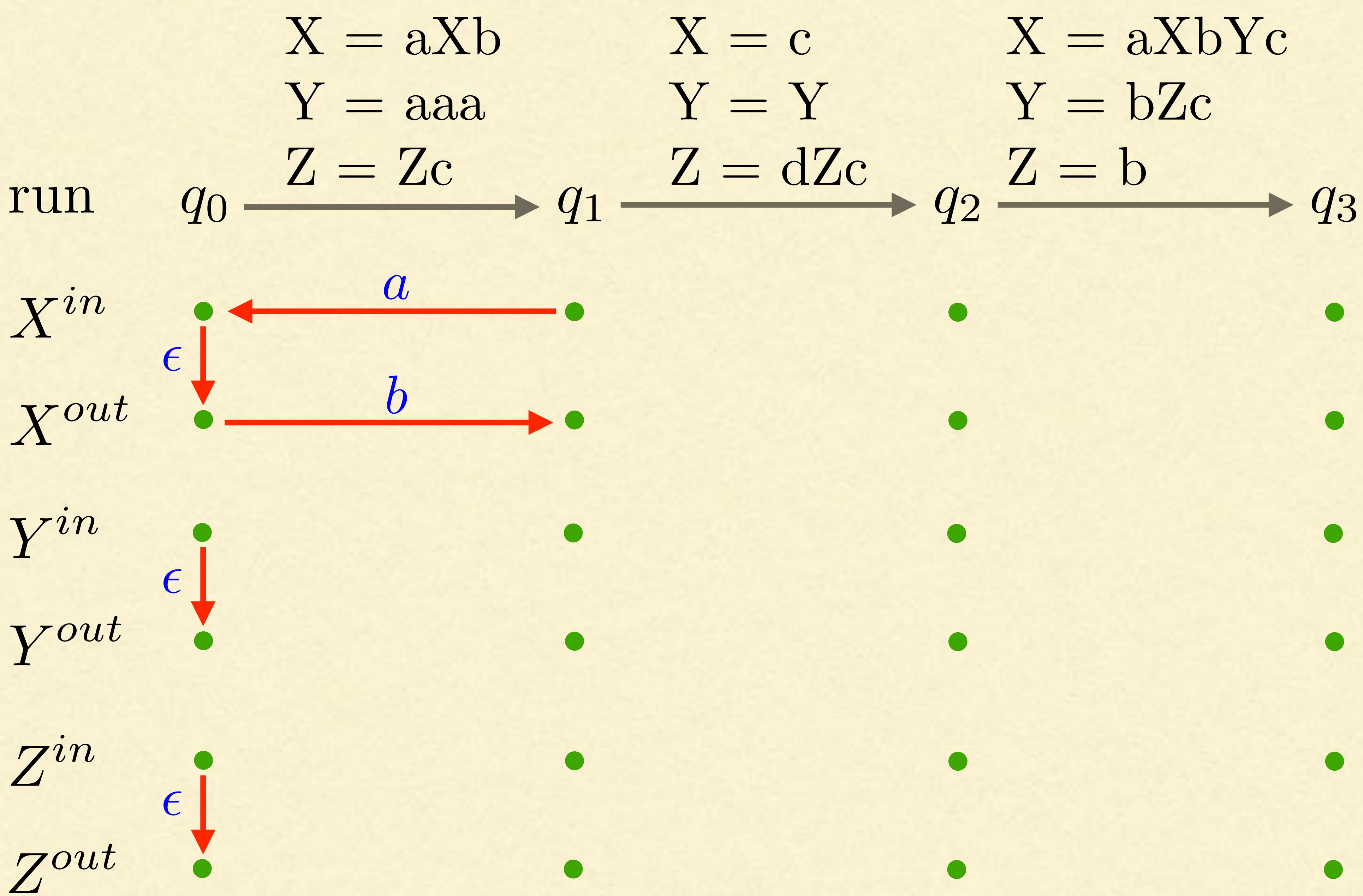
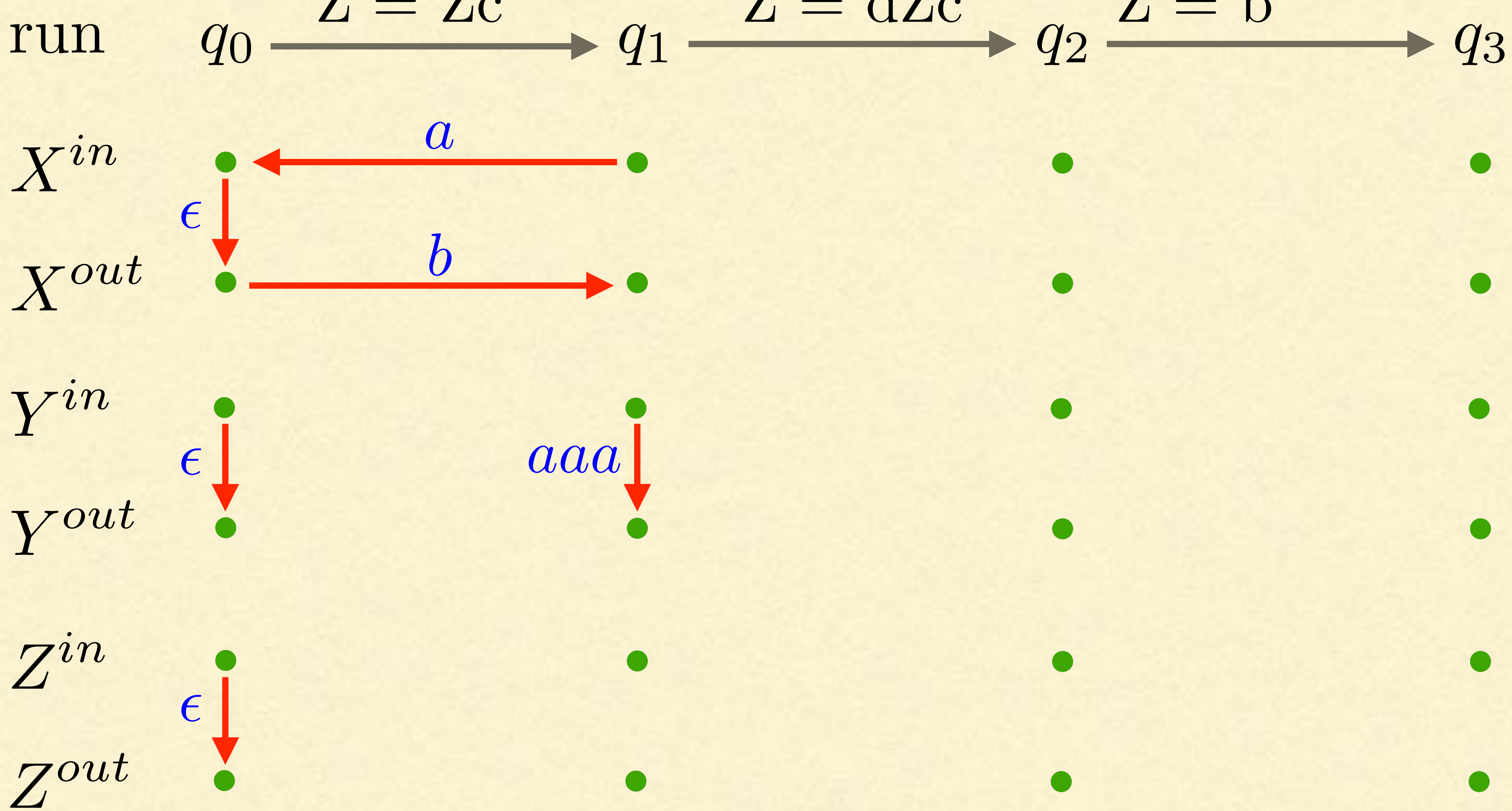


Aperiodic SST $\subset$ FOT

SST output structure : [FKT'14]

$X = aXb$	$X = c$	$X = aXbYc$
$Y = aaa$	$Y = Y$	$Y = bZc$
$Z = Zc$	$Z = dZc$	$Z = b$



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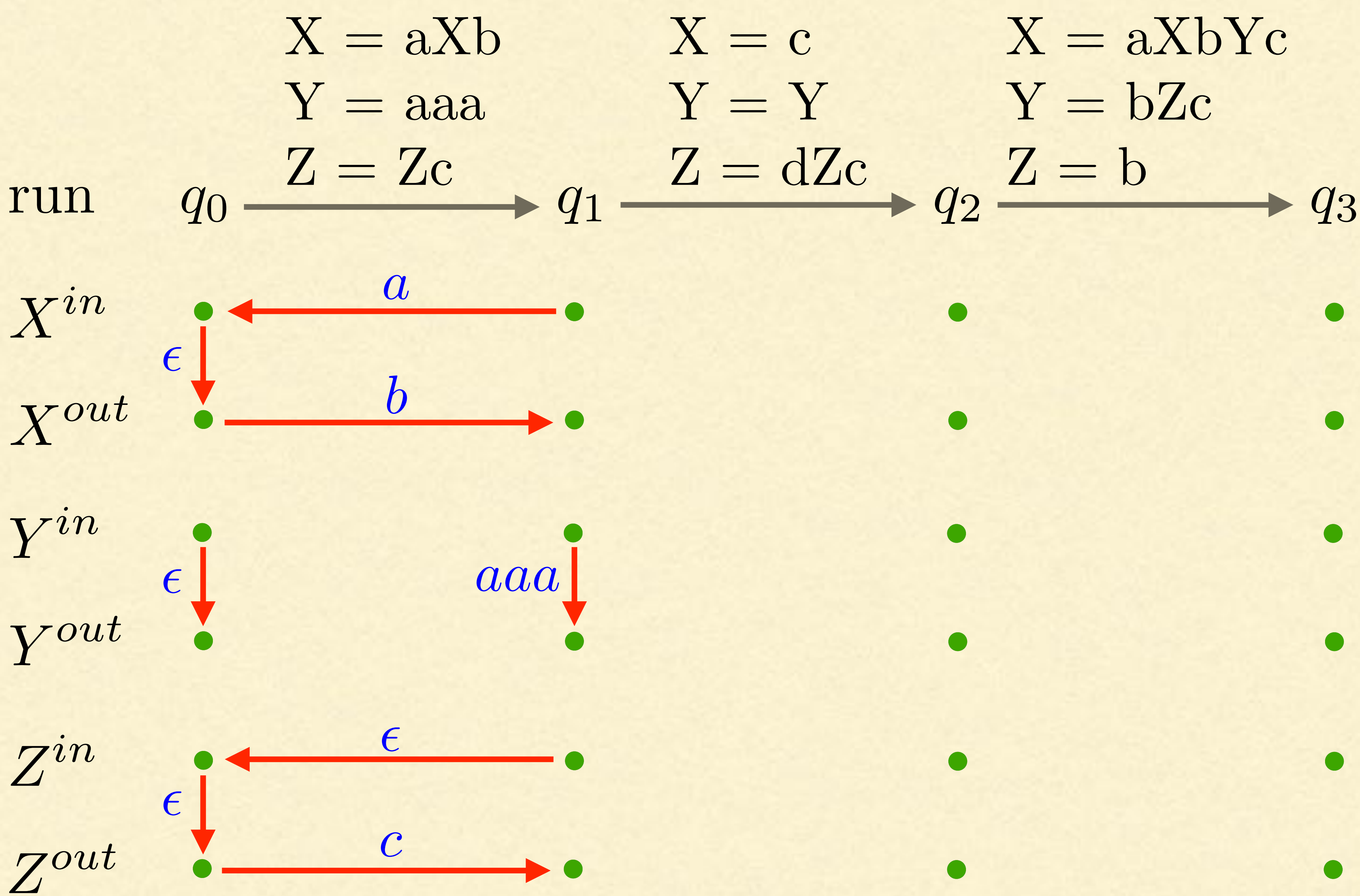

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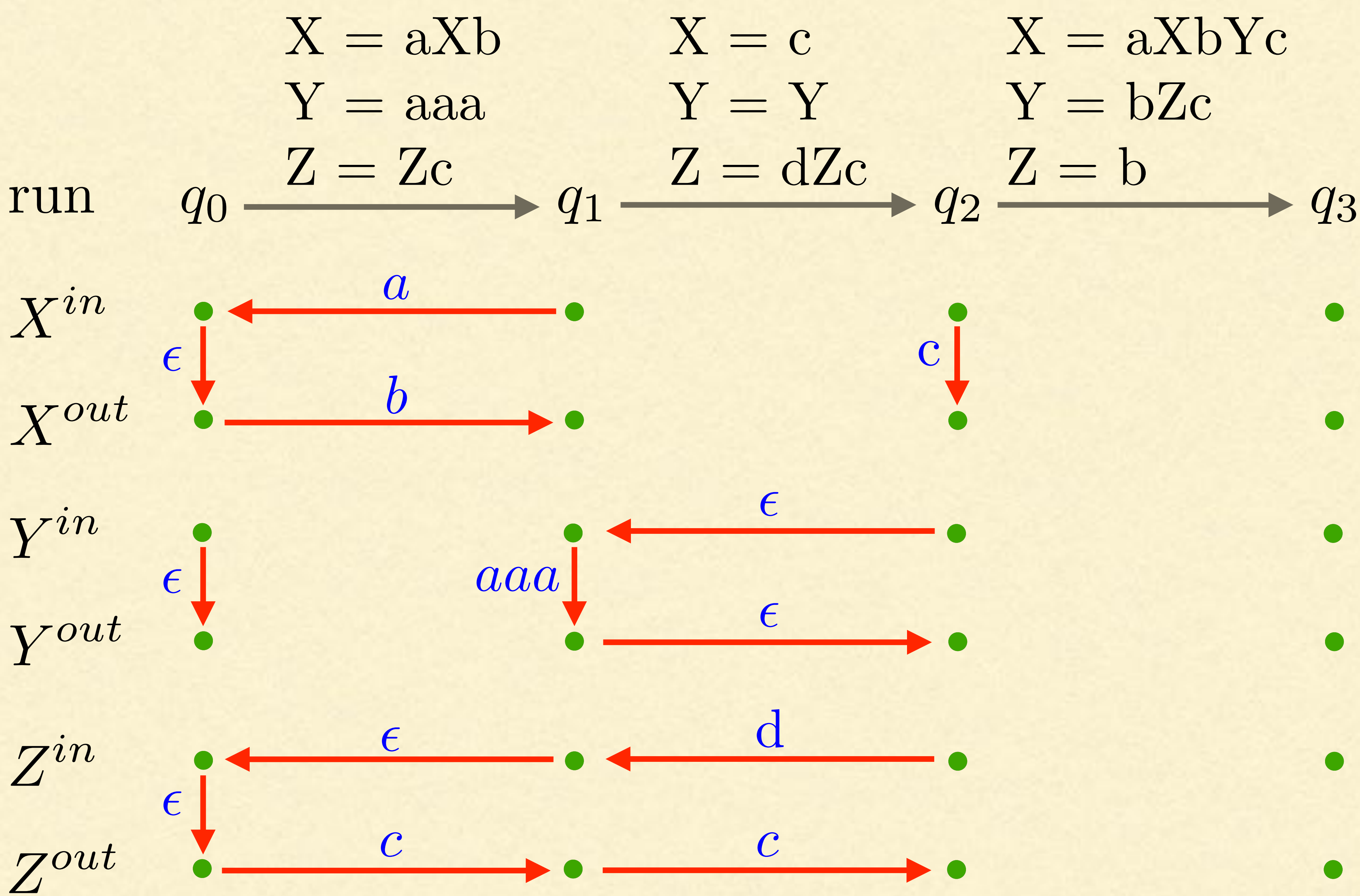
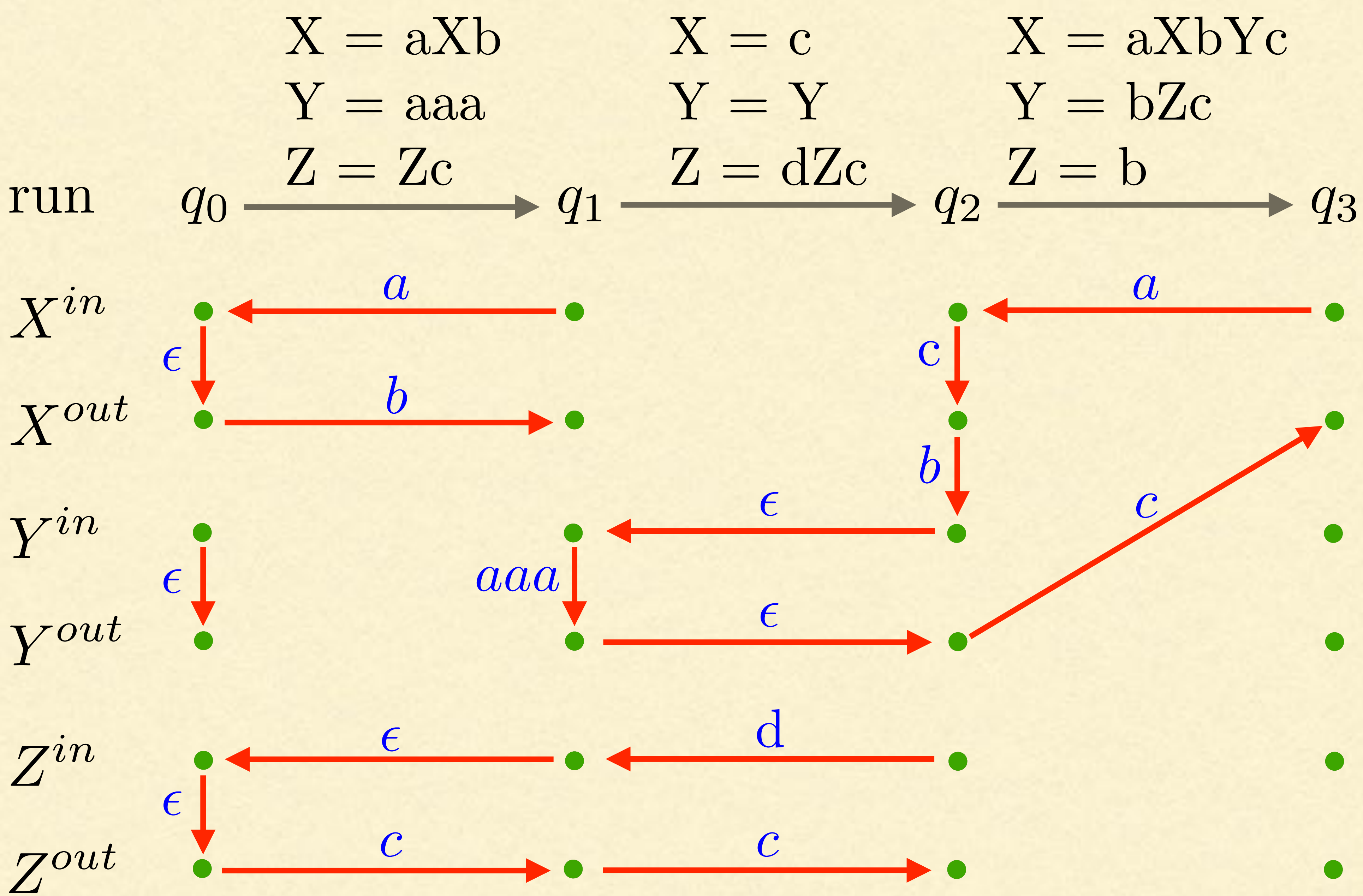


Aperiodic SST $\subset$ FOT

SST output structure :

[FKT'14]



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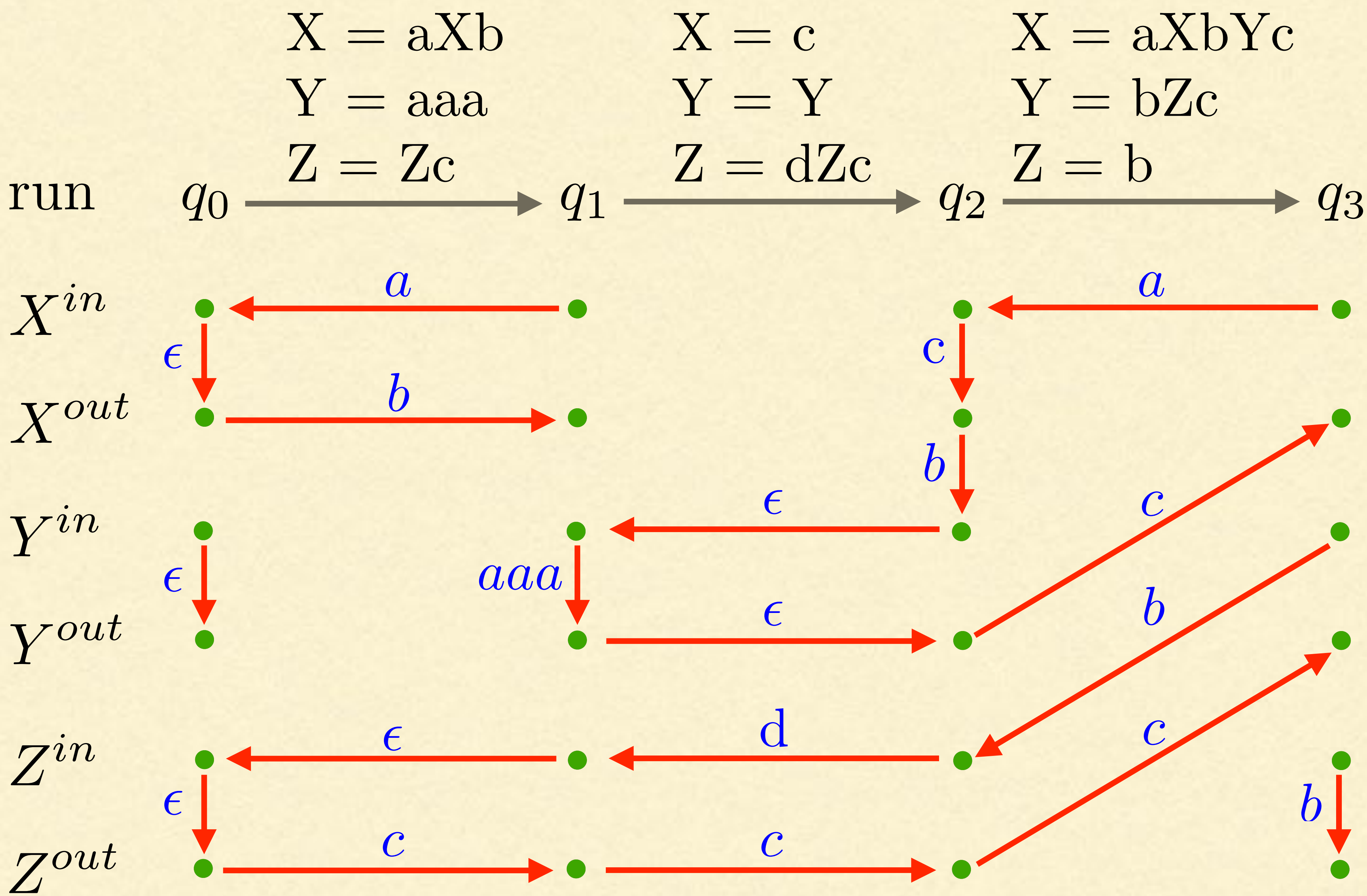
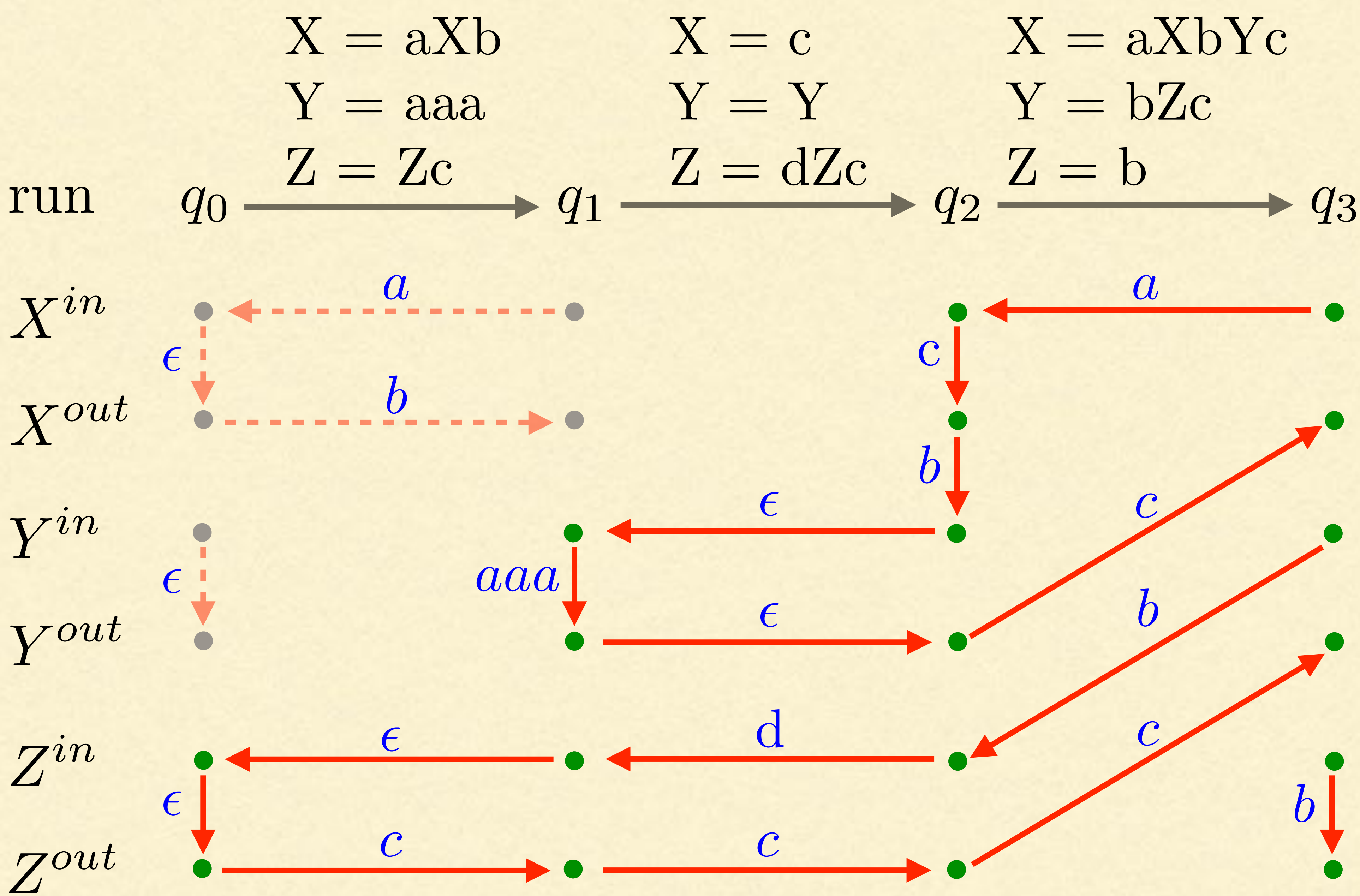


Aperiodic SST $\subset$ FOT

SST output structure :



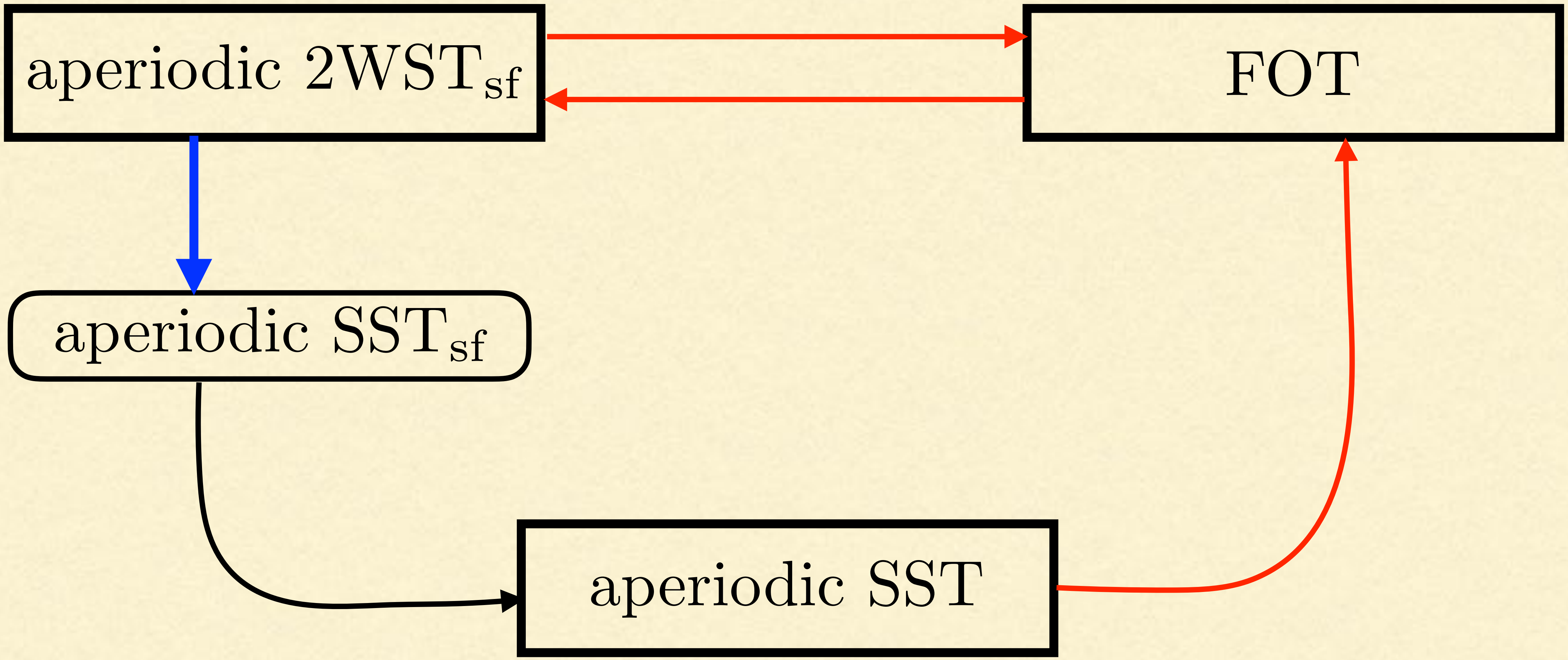
Aperiodic SST $\subset$ FOT

Claim 1: $\forall X, Y \in \mathcal{X},\ \forall d, e \in \{in, out\},\ \forall s \in \mathrm{dom(T)},\ \forall i, j \in \mathrm{dom(s)},$

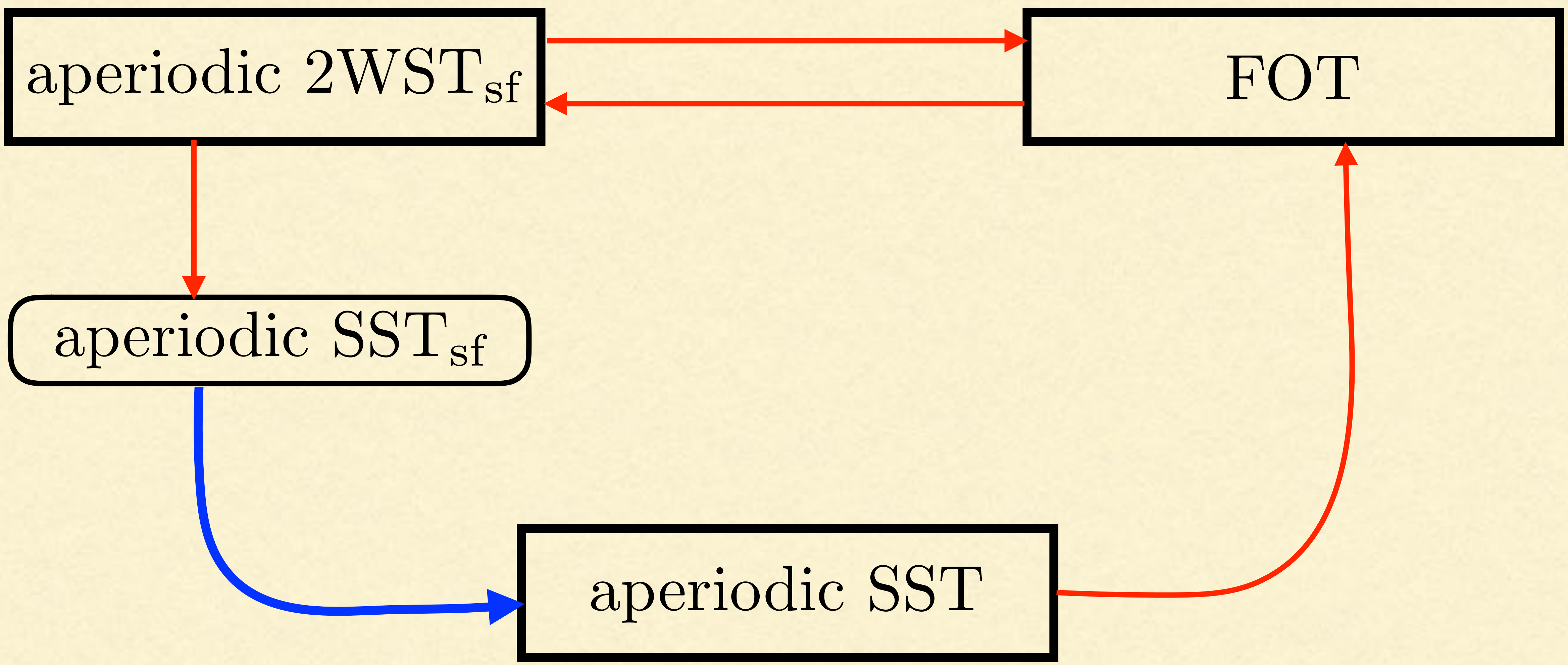
$(X^d, i) \rightsquigarrow (Y^e, j)$ is FO-definable.

Claim 2: $\phi_q(x)$ is FO definable

Results


aperiodic 2WST$_{\mathrm{sf}}$
FOT
aperiodic SST$_{\mathrm{sf}}$
aperiodic SST

Results



Conclusion

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Language : $\boxed{\text{FO logic}} \cong \boxed{\text{Aperiodic Automata}}$

Transducers : $\boxed{\text{FO transducer}} \cong \boxed{\text{Aperiodic Transducer}}$

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